Objectives: (things you should know and/or be able to do)

- Be familiar with the concept of "capillary pressure" for tubes as well as for porous media—and be able to derive the capillary pressure relation for fluid rise in a tube:

\[ p_c = 2 \gamma_{OW} \cos(\theta) \frac{1}{r} \]
Objectives: (things you should know and/or be able to do)

- Be familiar with and be able to derive the Purcell-Burdine permeability and relative permeability relations for porous media using the "bundle of capillary tubes" model as provided by Nakornthap and Evans. The permeability result is given by:

\[ k = \phi^*^3 \frac{\gamma_{ow}^2}{2} \frac{\beta}{n} \int_0^1 \frac{1}{p_c^2} dS_w^* \]

- Be able to derive the "field units" form of the Purcell-Burdine permeability equation \((k \text{ in md, } \gamma_{ow} \text{ in dyne/cm and, } p_c \text{ in psia})\). The Purcell-Burdine permeability equation as provided by Nakornthap and Evans is given in terms of absolute (i.e., "Darcy") units. The "field units" result is given by:

\[ k = 10.66 \phi^*^3 \frac{\gamma_{ow}^2}{2} \frac{\beta}{n} \int_0^1 \frac{1}{p_c^2} dS_w^* \text{ where } \phi^* = \phi (1-S_{wi}). \]
Objectives: (things you should know and/or be able to do)

- Be familiar with and be able to derive the Brooks-Corey-Burdine equation for permeability based on the Purcell-Burdine permeability equation (as given above). This result is given by:

\[ k = \phi^* \frac{3}{n} \frac{\beta}{\gamma_{ow}} \frac{2}{p_d} \left[ \frac{\lambda}{\lambda + 2} \right] \] or \[ k = 10.66 \phi^* \frac{3}{n} \frac{\beta}{\gamma_{ow}} \frac{2}{p_d} \left[ \frac{\lambda}{\lambda + 2} \right] \] (field units)

- Where the Brooks-Corey capillary pressure model is given by:

\[ p_c = p_d S_w^* \lambda \] (the normalized saturation is given by \[ S_w^* = \frac{S_w - S_{wi}}{1 - S_{wi}} = 1 - S_{WD} \].)

- Be able to discuss the possible applications for the Brooks-Corey-Burdine permeability equation.

- Be familiar with and be able to derive a type curve matching approach for capillary pressure data based on the Brooks-Corey model for capillary pressure and saturation given below.

\[ p_D = (1 - S_{WD}) \lambda \] where \[ p_D = \frac{p_c}{p_d} \] and \[ S_{WD} = \frac{1 - S_w}{1 - S_{wi}} = 1 - S_w^* \].
Schematic Development of Capillary Pressure in Porous Media
Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs
Petrophysics Lecture 4 — Capillary Pressure

Core Laboratories, Inc., Dallas (1972, 79, 89).
Pore Size and Distribution (Textural) Effect
Saturation History Effect ("Hysteresis")

Core Laboratories, Inc., Dallas (1972, 79, 89).
Fluid distribution in a homogeneous reservoir.

Fluid Distribution in the "Pendular" Region.

Schematic capillary pressure curves.

Definition:

The Leverett "J-function" was "derived" from dimensional analyses.

Evidently the group

\[
\frac{\Delta \rho gh}{\gamma \sqrt{K}}
\]

is interfacial curvature multiplied by a factor that is a property of the sand alone.

\[
J(S_w) = \frac{1}{\sigma \cos(\theta)} \sqrt{\frac{k}{\phi}} p_c
\]


Derivation of the Capillary Pressure Relation
or Fluid Rise in a Single Tube
Derivation of the Capillary Rise
Relating to a Single Capillary Tube in an Oil-Water System.

In this work, we begin with a schematic diagram of an oil-water system:

Figure 1 - Schematic Diagram of Capillary Rise for a Single Tube in an Oil-Water System.

The system in Figure 1 is in equilibrium, therefore the force balance is given by:

\[ \sum F = F_{up} - F_{down} = 0 \]  

where Eq. 1 can be rewritten as

\[ F_{up} = F_{down} \]  

where:

- \( F_{up} \) = Adhesive force (pulling up)
- \( F_{down} \) = Weight of the fluid (pulling down)

What is the "adhesive force"?

\[ F_{op} = \gamma t (2\pi r) \]  

where:

- \( \gamma \) = Adhesion tension, dyne/cm
- \( 2\pi r \) = Perimeter (or circumference) of the capillary, cm

How is the adhesion tension, \( \gamma \), defined? Consider the following diagram:

Figure 2 - Schematic Diagram and Force Balance for a Water-Oil-Solid System.

From Figure 2, we have:

\[ \gamma = \sigma_{os} - \sigma_{ws} = \sigma_{os} \cos(\theta) \]  

Substituting Eq. 4 into Eq. 3, we obtain

\[ F_{op} = \sigma_{os} \cos(\theta) (2\pi r) \]  

(Derivation of a Capillary Pressure for a Single Capillary Tube)
Derivation of a Capillary Pressure for a Single Capillary Tube

What is the "weight" force?

\[ \text{Down} = (\text{phyd}_w - \text{phyd}_o) \pi r^2 \]

where:
- \( \text{phyd}_w \) = Hydrostatic pressure in water (\( \text{A-8} \)), dynes/cm\(^2\)
- \( \text{phyd}_o \) = Hydrostatic pressure in oil (\( \text{A-8} \)), dynes/cm\(^2\)
- \( \pi r^2 \) = Cross-sectional area of the tube, cm\(^2\)

Specifically:
- \( \text{phyd}_w \) = \( \frac{9}{q_e} \) (\( q_e \) included for AES units) \( \text{(7)} \)
- \( \text{phyd}_o \) = \( \frac{9}{q_e} \) (\( q_e \) included for AES units) \( \text{(8)} \)
- \( \pi r^2 \) = \( \frac{q_e}{q_c} \) \( \text{(9)} \)

Substituting Eqs. 2-9 into Eq. 6, gives us:

\[ \text{Down} = \frac{9}{q_e} \pi r^2 h \left( \frac{9}{q_c} - \frac{9}{q_e} \right) \]

Substituting Eqs. 5 and 10 into Eq. 2, and solving for \( h \), we obtain:

\[ h = \frac{2 \text{ew} \cos(\theta)}{r} \frac{\frac{9}{q_e}}{\left( \frac{9}{q_c} - \frac{9}{q_e} \right)} \]

Solving Eq. 11 for \( \frac{9}{q_c} \frac{9}{q_e} h \), we have:

\[ \frac{9}{q_c} \frac{9}{q_e} h = 2 \text{ew} \cos(\theta) = 2 \text{Ar} \]

Eq. 11 gives us the "capillary rise" for a single tube - this result can be used as an analogy for a porous media. However, most efforts will be focused on a "bundle of capillary tubes," rather than a single, equivalent tube.

Returning our attention to Figure 1, we again state that the system is in equilibrium, therefore, we can write:

\[ \text{P_w} = \text{P_m} \left[ \frac{\text{Hydrostatic pressure}}{\text{balance}} \right] \]

Writing relations for each phase at point B, we have:

\[ \text{P_w} = \text{P_m} - \frac{q_w}{q_c} \frac{9}{q_c} h \]

\[ \text{P_o} = \text{P_m} - \frac{q_o}{q_c} \frac{9}{q_c} h \]

Substituting Eqs. 14 and 15 into Eq. 13, we obtain:

\[ \text{P_w} + \frac{q_w}{q_c} \frac{9}{q_c} h = \text{P_o} + \frac{q_o}{q_c} \frac{9}{q_c} h \]

or, solving for \( \text{P_w} - \text{P_o} \), we have:

\[ \text{P_w} - \text{P_o} = \left( \frac{q_w}{q_c} - \frac{q_o}{q_c} \right) \frac{9}{q_c} h \]

where we define the difference in phase pressures, i.e., \( \text{P_w} - \text{P_o} \), as the "capillary pressure," \( \text{P_c} \).

\[ \text{P_c} = \text{P_w} - \text{P_o} = \left( \frac{q_w}{q_c} - \frac{q_o}{q_c} \right) \frac{9}{q_c} h \]

Substituting Eq. 17 into Eq. 18, we have:

\[ \text{P_c} = \frac{2 \text{Ar}}{r} \]

Substituting Eq. 18 into Eq. 11, we obtain:

\[ h = \frac{1}{\left( \frac{9}{q_c} - \frac{9}{q_e} \right)} \]

where Eq. 19 can be used to convert \( \text{P_c} \) to \( h \), or vice-versa.
Recalling the capillary pressure relation, Eq. 18, we have

$$ P = \frac{1}{2} \rho \omega \cos(\theta) $$

Solving this relation for $\frac{r}{h}$ we have

$$ \frac{r}{h} = \frac{\rho \omega \cos(\theta)}{P_c} \tag{20} $$

We immediately note that, for the same tube, changing the wetting characteristics would still yield the same relationship (i.e., $\frac{r}{h}$ would be constant for a given tube).

Using Eq. 20, we can write the following relation by induction

$$ \frac{\sigma \cos(\theta_1)}{P_1} = \frac{\sigma \cos(\theta_2)}{P_2} $$

Or

$$ P_2 = \frac{\sigma \cos(\theta_2)}{\sigma \cos(\theta_1)} P_1 \tag{21} $$

Eq. 21 has found considerable use as a "conversion" formula for "converting" Mercury capillary pressures, $P_1$, to an equivalent water-oil, air-oil, or water-air system.

Typical values of interfacial tension ($\sigma$) and contact angle ($\theta$) are summarized below:

<table>
<thead>
<tr>
<th>System</th>
<th>Interfacial Tension (dynes/cm)</th>
<th>Contact Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hg-air</td>
<td>460</td>
<td>140</td>
</tr>
<tr>
<td>Water-oil</td>
<td>~24</td>
<td>90-30</td>
</tr>
<tr>
<td>Water-gas</td>
<td>70-75</td>
<td>0</td>
</tr>
<tr>
<td>Oil-gas</td>
<td>~50</td>
<td>0</td>
</tr>
</tbody>
</table>

(Derivation of a Capillary Pressure for a Single Capillary Tube)
Derivation of the Leverett "J-Function"
The Leverett "J-Function":

The Leverett "J-Function" is one of the most popular techniques for "averaging" capillary pressure data. While the "J-Function" is essentially empirical, it is also virtually the only technique for "correlating" or "averaging" capillary pressure data.

Retracing some of Leverett's thoughts, we return to Eq. 30, and solve for the constant, \( z \). This gives:

\[
\frac{z}{\sigma \cos(\theta)} = \rho \tag{22}
\]

where,

- \( r \) = radius of tube, cm
- \( \sigma \) = interfacial tension, dynes/cm
- \( \rho \) = capillary pressure, dynes/cm²

Substituting units, we have

\[
(1) \left( \frac{\text{cm}}{\text{dynes/cm}} \right) (1) = \left( \frac{\text{dynes/cm}}{\text{cm}} \right) (1)
\]

which is consistent in terms of units.

But how do we extend Eq. 22 to porous media? What is the appropriate radius, \( r \)? Leverett proposed:

\[
r = \sqrt{\frac{k}{\phi}} \tag{23}
\]

where,

- \( k \) = permeability, D (1 D = 9.86923 x 10⁻¹⁹ cm²)
- \( \phi \) = porosity, fraction

\( r = \sqrt{k/\phi} \) is not a real "radius" as much as it is a "characteristic length."

Substituting Eq. 23 into Eq. 22, we have the definition of the Leverett "J-Function", \( J_q(s_w) \).

\[
J_q(s_w) = \frac{1}{\sigma \cos(\theta)} \sqrt{\frac{k}{\phi}} \rho \tag{24}
\]

Eq. 24 is only truly "dimensionless" if permeability is given in cm², but this is a minor issue.

The question is... does this concept unify (or correlate) capillary pressure data? We note that \( J_q(s_w) \) is a function of saturation and not constant as Eq. 22 suggests. However, the concept is that \( J_q(s_w) \) would be constant at a given saturation, hence all capillary pressure data should be correlated at a given saturation... creating a "universal" capillary pressure curve. Does this work? Sort of... you can use \( J_q(s_w) \) as a correlating function...
Derivation of Permeability Using the Purcell Approach
Calculation of Permeability from Capillary Pressure Data

(Purcell)

Three basic considerations:

1. Capillary pressure in a capillary,

\[ P_c = \frac{2 \sigma \cos \theta}{r} \]

2. Capillary flow: Poiseuille's law

\[ Q_i = \frac{\pi r_i^4 \Delta p}{8 \mu L} \]

3. Darcy's equation,

\[ Q_t = \frac{k A \Delta p}{\mu L} \]

Let \( V_i = \pi r_i^2 L \), the volume of a capillary,

\[ Q_i = \frac{V r_i^2 \Delta p}{8 \mu L^2} \quad V = \text{cm}^3 \]

Since, \( r_i = \frac{2 \sigma \cos \theta}{\rho} \)

\[ Q_i = \frac{(\sigma \cos \theta)^2}{(P_{ci})^2} \cdot \frac{V_i}{2 \mu L^2} \cdot \Delta p \]

For a bundle of \( n \) capillary tubes,

\[ Q_t = \sum_{i=1}^{n} Q_i = \frac{(\sigma \cos \theta)^2 \Delta p}{2 \mu L^2} \sum_{i=1}^{n} \frac{V_i}{(P_{ci})^2} \]

Since,

\[ Q_t = \frac{k A \Delta p}{\mu L} \]

\[ k = \frac{(\sigma \cos \theta)^2}{2 A L} \sum_{i=1}^{n} \frac{V_i}{(P_{ci})^2} \]

\( P_c = \frac{\text{dynes}}{\text{cm}^2} \quad k = \text{cm}^2 \)

\( \sigma = \frac{\text{dyne}}{\text{cm}} \quad A = \text{cm}^2 \)

\( r = \text{cm} \quad L = \text{cm} \)

\( Q = \text{cm}^3/\text{sec} \quad \mu = \text{poise} = \frac{\text{dynes-sec}}{\text{cm}^2} \)

\( \Delta p = \text{dyne/cm}^2 \)
Define the fractional volume of ith capillary

\[ S_i = \frac{V_i}{V_t}, \quad S_i = \text{fraction} \]

and \[ \phi = \frac{V_t}{A L}, \quad \phi = \text{fraction} \]

\[ k = \frac{(\sigma \cos \theta)^2}{2} \phi \sum_{i=1}^{n} \frac{S_i}{(P_{ci})^2} \]

Introducing a lithology factor \( \lambda \) for deviation of the actual pore space,

\[ k = \frac{(\sigma \cos \theta)^2}{2} \phi \lambda \sum_{i=1}^{n} \frac{S_i}{(P_{ci})^2} \]

In differential form,

\[ k = \frac{(\sigma \cos \theta)^2}{2} \phi \lambda \int_{S=0}^{S=1} \frac{dS}{P_c^2} \]
Type Curve Matching using the Brooks and Corey Model
Brooks-Corey/Burdine Permeability Relation and Capillary Pressure

Type Curve Matching using the Brooks and Corey Model

The permeability of a porous medium can be related to the capillary pressure profile as proposed by Brooks (ASME, Trans, 1944) and Burdine (AIIME, Trans, 1954) and given in detail by Mortier and Evans (SPEEE, May 1986). This relation is given as

$$ k = \frac{\phi^2}{n} \left( \frac{1}{2} \right) \int_0^{1/2} d\phi_k $$

where,

- $k = \text{permeability, cm}^2$ (not md)
- $\phi = \text{porosity}$
- $n = \text{fraction of bulk volume}$
- $S_w = \text{minimum irreducible wetting-phase saturation}$
- $\phi_n = \text{oil-water interfacial tension, dynes/cm}$
- $\phi = \text{dimensionless pore radius distribution, } \geq 1$ ($\phi = \lambda^2$)
- $n = \text{number of flow exits per pore (n=1, n=1/(1-S_w))}$
- $k$ = capillary pressure function, dynes/cm^2
- $S_w$ = wetting-phase saturation, fraction
- $\phi_k = S_w - S_w^{*} = \text{"effective" saturation, fraction}$

The Brooks and Corey model for relating capillary pressure and effective saturation is

$$ S_w^{*} = \phi_k \phi_w^{1/2} $$

where $\phi$ is the "displacement" or "threshold" pressure. Rearranging into a "dimensionless" form we have

$$ S_w^{*} = \phi_k \phi_w^{1/2} \frac{1}{\phi} $$

Defining a dimensionless saturation, $S_w^{*}$, we have

$$ S_w^{*} = \frac{1 - S_w}{1 - S_w^{*}} $$

or

$$ S_w^{*} = \frac{1 - S_w}{1 - S_w^{*}} $$

or

$$ S_w^{*} = \frac{1 - S_w}{1 - S_w^{*}} $$

Substituting Eq.2 into Eq.8 gives

$$ I = \int_0^{1/2} \frac{1}{\phi} \frac{1}{\phi_k} \phi_w^{1/2} d\phi_k $$

or

$$ I = \frac{1}{\phi_k^2} \int_0^{1/2} \phi_w^{1/2} (1 + z/\lambda) \left[ (1 + z/\lambda)^{-1} - (0)^{-1} \right] $$

which gives

$$ I = \frac{1}{\phi_k^2} \left[ \frac{\lambda}{\lambda + z} \right] $$

Substituting Eq.5 into Eq.3 gives

$$ k = \left( \frac{1}{2} \right)^{1/2} \int_0^{1/2} \frac{1}{\phi_k} \phi_w^{1/2} d\phi_k $$

Converting Eq.1 to field units ($k$ in md, $p_c$ in psia) gives

$$ k(\text{md}) \left[ \frac{9.807 \times 10^{-12} \text{ cm}^2}{\text{md}} \right] = \frac{1}{\phi_k^2} \phi_w^{1/2} \left[ \frac{1}{\text{psia}} \right] \int_0^{1/2} \frac{1}{\phi_k^2} \phi_w^{1/2} d\phi_k $$

or

$$ k(\text{md}) = \left( \frac{1}{\phi_k^2} \phi_w^{1/2} \left[ \frac{1}{\text{psia}} \right] \int_0^{1/2} \frac{1}{\phi_k^2} \phi_w^{1/2} d\phi_k \right) $$

reducing constants

$$ k(\text{md}) = 10.66 \left[ \frac{\text{md}}{\text{psia}} \right] \phi_n \phi_w^{1/2} \left[ \frac{1}{\text{cm}^2} \right] \left[ \frac{1}{\text{psia}} \right] \int_0^{1/2} \frac{1}{\phi_k^2} \phi_w^{1/2} d\phi_k $$

or finally

$$ k = 10.66 \phi_n \phi_w^{1/2} \left[ \frac{1}{\text{cm}^2} \right] \left[ \frac{1}{\text{psia}} \right] \int_0^{1/2} \frac{1}{\phi_k^2} \phi_w^{1/2} d\phi_k $$

Again where $k$ in md, $p_c$ in psia, and $\phi_w$ in dynes/cm. We note that the constant 10.66 is correct, the value of 10.24 reported in several references is incorrect.

Isolating the integral in Eq.7 gives

$$ I = \int_0^{1/2} \frac{1}{\phi_k^2} \phi_w^{1/2} d\phi_k $$

Substituting Eq.2 into Eq.8 gives

$$ I = \frac{1}{\phi_k^2} \int_0^{1/2} \phi_w^{1/2} (1 + z/\lambda) \left[ (1 + z/\lambda)^{-1} - (0)^{-1} \right] $$

which gives

$$ I = \frac{1}{\phi_k^2} \left[ \frac{\lambda}{\lambda + z} \right] $$

(Type Curve Matching using the Brooks and Corey Model)
Substituting Eq. 9 into Eq. 7

\[ k = 10.66 \times \phi^3 \frac{\phi^2 \sigma_{s0w}^2}{n^2} \left[ \frac{1}{(1 - \phi) + \frac{1}{2}} \right] \]  

(10)

where Eq. 10 is fully defined except for the \( \phi \) and \( n \) terms which are essentially empirical parameters used to represent pore throat impedance and the number of pore throat exits, respectively. Efforts to quantify \( \phi \) and \( n \) are also empirical — but recent efforts (AK, University of Kuwait 1995) suggest the following models

\[ \phi = \frac{1}{\phi} \]  

(11)

and

\[ n = \frac{1}{1 - Swi} \]  

(12)

Substituting Eqs. 11 and 12 into Eq. 10 gives

\[ k = 10.66 \times \phi^3 (1 - Swi) \frac{\sigma_{s0w}^2}{\phi^2 \gamma} \left[ \frac{1}{(1 - \phi) + \frac{1}{2}} \right] \]

or

\[ k = 10.66 \times (1 - Swi) \phi^3 \sigma_{s0w}^2 \left[ \frac{1}{n^2} \right] \]

(13)

where \( \alpha \) is an empirical constant, we assume \( \alpha = 1 \) unless there are sufficient data to quantify this term.

In terms of applications, Eq. 13 (or Eq. 10) can be used to compute permeability or to correlate data. (In order to develop a core-log correlation of core and well log data).

The question of how to determine \( Swi, Ph, \) and \( \lambda \) remains.

One approach is to use Eq. 2 and solve for these parameters using capillary pressure data. Recalling Eq. 2 and substituting the definition of \( Swi \) gives

\[ \frac{12}{12} = \frac{12}{12} \left[ \frac{Swi - Swi}{1 - Swi} \right]^{-1} \lambda \]

(14)

Eq. 14 can be fitted to data using non-linear regression or we can use Eq. 6 to develop a "type curve" approach.

Recalling Eq. 6 we have

\[ \frac{P_b}{P_w} = \frac{P_e}{P_d} = (1 - Swi)^{-\alpha} \]

(16)

where

\[ P_b = \frac{P_e}{P_d} \]

and

\[ Swi = \frac{1 - Sw}{1 - Swi} \]

(17)

using the type curve approach we overlay a plot of \( \frac{P_e}{P_w} \) versus \( 1 - Swi \) over a plot of \( \frac{P_b}{P_w} \) versus \( Swi \). Once the data are "matched" onto the type curves we read the "match point" values (ratios of \( P_e/P_b \) and \( 1 - Swi/Swi \)). These relations are given by

\[ \frac{P_e}{P_b} = \frac{(P_e)_{MP}}{(P_b)_{MP}} \]

(18)

\[ Swi = 1 - \frac{(1 - Swi)_{MP}}{(Swi)_{MP}} \]

The type curve matching procedure is illustrated below.
Example Application of the Type Curve Approach: Cottage Grove S

- Sample Name: Cottage Grove #5
- Origin: <unknown>
- Rock Type: Sandstone
- Porosity: 0.28 (fraction)
- Permeability: 127 md
- Fluid System: Brine-Air
- Fluid Properties: \( \psi = 92 \) dyn cm

Capillary Pressure Data:

\[
\begin{array}{ccc}
Sw, fraction & \psi_l, psia & \psi, psia \\
0.237 & 56.500 & 5.030 \\
0.263 & 40.320 & 4.480 \\
0.293 & 30.610 & 4.010 \\
0.311 & 22.530 & 3.590 \\
0.328 & 15.910 & 3.250 \\
0.359 & 10.300 & 2.880 \\
0.391 & 10.820 & 2.640 \\
0.469 & 4.040 & 2.300 \\
0.489 & 7.930 & \\
0.487 & 6.090 & \\
\end{array}
\]

Analysis:

From the attached type curve and data plots we have:

\[
\lambda = 1 \\
\psi_l/\psi = 2.0 \text{ psia} \\
(1-Swi)\psi = 0.77 \text{ (fraction)}
\]

From Eq. 18 we obtain the displacement pressure, \( p_d \),

\[
p_d = (\psi_l)_{MP} = 2.0 \text{ psia}
\]

From Eq. 19 we obtain the irreducible wetting saturation, \( Sw_i \),

\[
Sw_i = 1 - (1-Swi)_{MP} = 1 - 0.77 = 0.23 \text{ (fraction)}
\]

From Eq. 13 we can estimate the permeability,

\[
k = 10.66 \lambda \psi_l^2 \left( \frac{\lambda}{\psi_l} \right)
\]

"Known" Data
- \( \lambda = 1 \) (assumed)
- \( \phi = 0.28 \) (fraction)
- \( \psi_l = 92 \text{ dyn cm} \)
- \( Sw_i = 0.23 \) (fraction)

"calculated" Data
- \( \lambda = 1 \)
- \( \psi_l = 2 \text{ psia} \)
- \( Sw_i = 0.23 \) (fraction)

Results from "Type Curve" analysis

Substituting gives:

\[
k (\text{md}) = 10.66 \lambda (1.0) \left[ 1 - (0.23) \right]^2 \left( \frac{(0.28)^2(92 \text{ dyn} \text{ cm})^2}{(2.0 \text{ psia})^2} \right) (1) + 2
\]

or

\[
k = 126.9 \text{ md}
\]

Summary:

This example demonstrates one approach for the analysis of capillary pressure data. In this example the calculated permeability (126.9 md) compares extraordinarily well with the known value (127 md). We should consider such agreement the exception rather than the rule.

In addition, the \( \lambda \) parameter can be used in the estimation of relative permeability trends as discussed by Nakornthap and Evans.
Type Curve for Brooks and Corey $p_c/S_{WD}$ Method

**Brooks and Corey Type Curve:**

Model: $p_D = p_c/p_d = S_w^{*(1-1/\lambda)} = (1-S_{WD})^{(1-1/\lambda)}$

1. Plot $p_c$ vs. $1-S_w$ on the same scale log grid.
2. Slide data plot horizontally and vertically until one of the $\lambda$ curves are overlaid. Record the $\lambda$ value.
3. Obtain $p_c=p_0$ at $p_D=1$.
4. Obtain $S_w=S_{wi}$ at $S_{WD}=1$.

$S_{WD} = 1 - S_w = 10^{-1}(1-S_w)/(1-S_{wi})$
Data Grid for Brooks and Corey $p_c/S_{wd}$ Method

Cottage Grove - 5 Sandstone Reservoir
$\phi = 0.28$ fraction
$k = 127$ md
Brine-Air System
($\gamma_{wg} = 72$ dyne/cm)
Type Curve Match for Brooks and Corey $p_e/S_wD$ Method -- Cottage Grove 5

Cottage Grove - 5
Sandstone Reservoir
$\phi = 0.28$ fraction
$k = 127$ md
Brine-Air System
($\gamma_{wg} = 72$ dyne/cm)

Results for Cottage Grove - 5
$\lambda = 1$ (dimensionless)
$(p_d)_{MP}/(p_D)_{MP} = 2.0$ psia
$(1-S_w)_{MP}/(S_wD)_{MP} = 0.77$ (fraction)
Generalized Correlation Approach using the Brooks and Corey Model
A Generalized Correlation Approach for Capillary Pressure and Relative Permeability Data

- The Brooks / Corey / Bunsen Equation

Capillary Pressure Correlation

The Bunsen relation is derived from a bundle of capillary tubes model and is given by

\[ k = 10.66 \frac{\phi^2}{n} \int_0^1 \frac{1}{\phi_k} ds_k \]

(1)

where,

- \( s^*_w = s_w - s_w^i \) = "effective" saturation, fraction
- \( \gamma = \sigma \cos(\theta) \) = angle of incidence
- \( \theta = \) interfacial tension, dynes/cm
- \( \phi = \) dimensionless pore radius distribution (\( \beta > 1 \)), \( \beta = 1/\phi \)
- \( n = \) number of flow exits per pore (n = 1), n = 1/(1-\( s_w^i \))
- \( s_{wi} = \) irreducible wetting phase saturation, fraction
- \( \phi = \) capillary pressure, psia
- \( \lambda = \) porosity (fraction of bulk volume), fraction

The Brooks and Corey model for capillary pressure is

\[ \beta = \frac{\phi}{\phi_k} \]

(2)

Substituting Eq. 2 into Eq. 1 gives

\[ k = 10.66 \frac{\phi^2}{n} \int_0^1 \frac{1}{\phi_k} ds_k \]

(3)

A correlating function for the \( \phi^2 \) term has recently been proposed by Sutradh, Ali of Kuwait University. This relation is given by

\[ \beta = \frac{\phi}{n} \]

(4)

where \( \lambda \) is an empirical adjustment constant (assume \( \lambda = 1 \)).

Solving Eq. 5 for the displacement pressure, \( \rho_d \), gives

\[ \rho_d = \frac{\sqrt{k_{wi}}} {\sqrt{k_w}} \frac{(1-s_{wi})^2}{\sqrt{k_w} s_{wi}^i} \frac{\phi}{\phi_k} \frac{1}{\lambda + z} \]

(6)

where Eq. 6 is a "semi-analytical" expression for relating the displacement pressure to formation properties. Eq. 6 directly provides

\[ \rho_d = \frac{\sqrt{k_{wi}}}{\sqrt{k_w}} \frac{(1-s_{wi})^2}{\sqrt{k_w} s_{wi}^i} \frac{\phi}{\phi_k} \frac{1}{\lambda + z} \]

(7)

or

\[ \rho_d = \frac{\sqrt{k_{wi}}}{\sqrt{k_w}} \frac{(1-s_{wi})^2}{\sqrt{k_w} s_{wi}^i} \frac{\phi}{\phi_k} \frac{1}{\lambda + z} \]

(8)

From Eq. 7 we can intuitively suggest that

\[ \frac{\rho_{d1}}{\rho_{d2}} \approx \frac{\rho_{e1}}{\rho_{e2}} \] (4) or \[ \frac{\rho_{e1}}{\rho_{e2}} = \frac{\rho_{d1}}{\rho_{d2}} \] (9a)

So by analogy with Eq. 8 (using \( \rho_{d1}/\rho_{d2} \approx \rho_{e1}/\rho_{e2} \)), we have

\[ \rho_{e2} = \frac{\sqrt{k_{wi2}}}{\sqrt{k_{wi1}}} \frac{(1-s_{wi2})^2}{\sqrt{k_{wi1}} s_{wi1}^i} \frac{\phi}{\phi_k} \frac{1}{\lambda_2 s_{wi2} + z} \]

(10)

where Eq. 10 could allow us to "convert" lab data to reservoir data. This concept is similar to the Leverett "J-Function" which is given by

\[ J_s(s_w) = \frac{1}{s} \sqrt{\frac{s_w}{\phi}} \]

(11)

Assuming that \( J_s(s_w) = J_s(s_w) \) we have

\[ \rho_{e2} = \frac{\sqrt{k_{wi2}}}{\sqrt{k_{wi1}}} \frac{(1-s_{wi2})^2}{\sqrt{k_{wi1}} s_{wi1}^i} \frac{\phi}{\phi_k} \frac{1}{\lambda_2 s_{wi2} + z} \]

(12)
We note that Eq. 12 is the "standard" application of the Leverett J-function. Both Eqs. 10 and 12 "rescale" capillary pressure data, in the sense that if we assume that the data have the same character (i.e., shape) then these concepts should suffice. However, we immediately recognize from Eq. 2 that capillary pressure data will not exhibit the same shape, in a general sense.

This forces us to realize that the concept of a "universal" capillary pressure curve - as Leverett believed he had established using his J-Function. Unfortunately, we cannot establish a "universal" trend, but we can develop an equality condition using the Brooks and Corey capillary pressure relation (Eq. 2). Recalling Eq. 2 we have

\[ P_e = P_e \text{ } S_w^{1/k_1} \]  

Setting Eq. 2 as an equality (i.e., \( P_e \text{ } S_w^{1/k_1} = 1 \))

\[ \frac{P_e S_w^{1/k_1}}{P_e} = \frac{P_e S_w^{1/k_1}}{P_e} \]

or solving for \( P_e \), we have

\[ P_e = \frac{P_e S_w^{1/k_1}}{P_e} \text{ } S_w^{1/k_1} \]

Recalling the definition of the "effective" saturation function, \( S_D \), we have

\[ S_D = \frac{S_w - S_W}{1 - S_W} \]

Substituting Eq. 13 into Eq. 16 we have

\[ P_e = \frac{P_e S_w^{1/k_1}}{P_e} \text{ } S_w^{1/k_1} \]

Substituting Eq. 13 into Eq. 16 we have

\[ P_e = \frac{P_e S_w^{1/k_1}}{P_e} \text{ } S_w^{1/k_1} \]

Recall that \( \lambda_1 \) and \( \lambda_2 \) are all so without any additional information we can assume that \( \sqrt{\lambda_1/\lambda_2} \approx 1 \). This gives us our final form

\[ P_e = \frac{(1 - S_w)^2}{(1 - S_W)^2} \frac{P_e}{P_e} \text{ } k_1^{1/2} \frac{1}{k_2} \frac{1}{k_1} \frac{1}{k_2} \frac{1}{k_2} \frac{1}{k_2} \]

Relative Permeability Correlation

From Nakamoto and Evans, the wetting and non-wetting phase relative permeability functions are given as

\[ \frac{k_{rw}}{k_{ro}} = 1 \]

and

\[ k_{rw} = k_{ro} \left( \frac{1}{1 - S_w} \right)^{(1 + 2/h)} \]

Rearranging Eqs. 19 and 20 we have

\[ \frac{k_{rw}}{k_{ro}} \left( \frac{1}{1 - S_w} \right)^{(1 + 2/h)} = 1 \]

and

\[ k_{rw} = k_{ro} \left( \frac{1}{1 - S_w} \right)^{(1 + 2/h)} \]

Setting the equality in Eq. 21 for two separate conditions, we have

\[ \frac{k_{rw}}{k_{ro}} \left( \frac{1}{1 - S_w} \right)^{(1 + 2/h)} = 1 \]

Solving for \( k_{rw} \)

\[ k_{rw} = k_{ro} \left( \frac{1}{1 - S_w} \right)^{(1 + 2/h)} \]
Setting the equality in Eq. 22 for two separate conditions gives
\[
\frac{k_{rm1}}{k_{rm1}} = \frac{k_{rm2}}{k_{rm2}} \frac{1}{(1-S_w 1)^2 \left[1-S_w^*(1+2/\lambda_1)\right]} \frac{1}{(1-S_w 2)^2 \left[1-S_w^*(1+2/\lambda_2)\right]}
\]

Solving for \( k_{rm2} \)
\[
k_{rm2} = \frac{k_{rm2}^0}{k_{rm1}} \frac{(1-S_w 2)^2 \left[1-S_w^*(1+2/\lambda_2)\right]}{(1-S_w 1)^2 \left[1-S_w^*(1+2/\lambda_1)\right]} k_{rm1}
\]

(24)

The application of Eqs. 23 and 24 are straightforward, but how do we \( k_{rm1}, k_{rm2}, \lambda_1 \), and \( \lambda_2 \)? \( k_{rm1} \) and \( k_{rm2} \) are probably best left to case-by-case correlations, but what of \( \lambda_1 \) and \( \lambda_2 \)? \( \lambda_1 \) and \( \lambda_2 \) can be related by correlation of capillary pressure data and permeability, where \( \lambda = f(k, S_w, \text{etc.}) \), but this is also left to a case-by-case correlation.

For completeness, we should substitute Eq. 15 into Eqs. 23 and 24, this gives
\[
k_{rw1} = \frac{k_{rw1}^0}{k_{rw1}} \frac{[S_w - S_w 1]}{[1 - S_w 1]} \left[1 - \frac{[S_w - S_w 1]}{[1 - S_w 1]} (3+2/\lambda_1)\right] k_{rw1}
\]

(25)

and
\[
k_{rm2} = \frac{k_{rm2}^0}{k_{rm1}} \frac{[S_w - S_w 2]}{[1 - S_w 2]} \left[1 - \frac{[S_w - S_w 2]}{[1 - S_w 2]} (3+2/\lambda_2)\right] k_{rm1}
\]

(26)