Objectives: (things you should know and/or be able to do)

- Be familiar with the "fracture" and "matrix" models developed by Warren and Root\textsuperscript{1} and also be familiar with the Laplace and real domain results given by Warren and Root for pseudosteady-state interporosity flow. These relations are:

- **Laplace domain results:**
  - Warren and Root "interporosity flow function:"
    \[
    f(u) = \frac{\lambda + \omega(1-\omega)u}{\lambda + (1-\omega)u}
    \]
  - Cylindrical source formulation: W&R Eq. 14
    \[
    \overline{p}_D(u, r_D, \omega, \lambda, s) = \frac{1}{u} \frac{K_0(\sqrt{uf(u)} r_D)}{K_1(\sqrt{uf(u)})} + \frac{s}{u}
    \]
  - Line source formulation:
    \[
    \overline{p}_D(u, r_D, \omega, \lambda, s) = \frac{1}{u} K_0(\sqrt{uf(u)} r_D) + \frac{s}{u}
    \]
  - "Log approximation" formulation:
    \[
    \overline{p}_D(u, r_D, \omega, \lambda, s) = \frac{1}{2u} \ln \left[ \frac{4}{e^{2\gamma r_D^2 \sqrt{uf(u)}}} \right] + \frac{s}{u}
    \]

- **Real domain results:**
  - Line source solution: (W&R Eq. 15—including the skin factor, s)
    \[
    p_D(t_D, r_D, \omega, \lambda, s) = \frac{1}{2} \ln \left[ \frac{4}{e^{2\gamma r_D^2 t_D}} \right] - \frac{1}{2} E_1 \left[ \frac{\lambda}{\omega(1-\omega)} t_D \right] + \frac{1}{2} E_1 \left[ \frac{\lambda}{(1-\omega)} t_D \right] + s
    \]
  - Well testing derivative of the time domain solution:
    \[
    p_D'(t_D, r_D, \omega, \lambda) = \frac{1}{2} + \frac{1}{2} \exp \left[ \frac{-\lambda}{\omega(1-\omega)} t_D \right] - \frac{1}{2} \exp \left[ \frac{-\lambda}{(1-\omega)} t_D \right]
    \]
Objectives: (things you should know and/or be able to do)

- You should be able to distinguish between the pseudosteady-state and transient interporosity flow models (the Warren and Root\(^1\), and de Swaan\(^2\)/Najurieta\(^3\) models, respectively). (You should also be aware of the various interporosity flow models proposed by Moench.\(^4\))

- Be familiar with and be able to define the following parameters for dual porosity reservoir systems:

  **Interporosity Flow Parameter:**
  
  \[ \lambda = \alpha r_w \frac{k_m}{k_f} \]
  (where \(\alpha\) is the matrix block shape parameter)

  **Storativity Ratio:**
  
  \[ \omega = \frac{(\phi V c)_f}{[(\phi V c)_f + (\phi V c)_m]} \]
Pressure Behavior of a Well
in an Infinite-Acting
Dual Porosity (Naturally Fractured) Reservoir

from Department of Petroleum Engineering Course Notes (1994)
Pressure Behavior of a Well in an Infinite-Acting Dual Porosity (Naturally Fractured) Reservoir

- Models for Regional Fracturing (Reservoir Schematics)
- Conceptual Models for Dual Porosity (Naturally Fractured) Reservoir Systems
- Performance of Naturally Fractured Reservoir Systems—Transient Flow (No Wellbore Storage or Skin Effects)
  - Pseudosteady-State Interporosity Flow
  - Transient Interporosity Flow (Various)
- Performance Models for Dual Porosity (or "Naturally Fractured") Reservoirs

2.11 - Composite core of a fractured reservoir

Fracture Pattern 1: $\sigma_1, \sigma_3$ acting in the bedding plane and $\sigma_1$ acting normal to the bedding plane ($\sigma_1$ dip direction, $\sigma_3$ strike direction). (Stearns\textsuperscript{3}, Courtesy AAPG).

Fracture pattern 2: $\sigma_1, \sigma_3$ acting in the bedding plane and $\sigma_1$ acting normal to the bedding plane ($\sigma_1$ dip direction, $\sigma_3$ strike direction). (Stearns\textsuperscript{3}, Courtesy AAPG.)

Various types of fractures generated by folding (courtesy of Leroy\textsuperscript{3}).


FIG. 1 — IDEALIZATION OF THE HETEROGENEOUS POROUS MEDIUM.

Fig. 1—Schematic of reservoir with rectangular matrix elements.

 Idealized reservoir: model 1 - stratified matrix; model 2 - blocks matrix.


Solutions used for These Plots

Line source solution:

\[
p_D(t_D, r_D, \omega, \lambda, \phi, s) = \frac{1}{2} \ln \left[ \frac{4}{e^\gamma} \frac{t_D}{r_D^2} \right] - \frac{1}{2} E_1 \left[ \frac{\lambda}{\omega(1-\omega)} t_D \right] + \frac{1}{2} E_1 \left[ \frac{\lambda}{(1-\omega)} t_D \right]
\]

Well testing derivative of the time domain solution:

\[
p_D'(t_D, r_D, \omega, \lambda) = \frac{1}{2} + \frac{1}{2} \exp \left[ -\frac{\lambda}{\omega(1-\omega)} t_D \right] - \frac{1}{2} \exp \left[ -\frac{\lambda}{(1-\omega)} t_D \right]
\]
Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs
Reservoir Flow Solutions Lecture 5 — Dual Porosity Reservoirs: Warren and Root Approach

"Stewart and Alscharsobbi" Type Curve: \( p'_{wD} \) vs. \( t \sqrt{D/4} \) — Various \( \lambda \) and \( \omega \) Values

Pseudosteady-State Interporosity Flow

"Fracture" System Radial Flow Region, \( p'_D = 1/2 \)

Type Curve for an Unfractured Well in an Infinite-Acting Naturally-Fractured Reservoir with NO Wellbore Storage or Skin Effects. Plotting Format: paper SPE 18172
Stewart, G. and Alscharsobbi. "Well Test Interpretation for Naturally-Fractured Reservoirs"

"Onur, Satman, and Reynolds" Type Curve: \( p'_{wD} \) vs. \( tD/(1-\omega) \) — Various \( \lambda \) and \( \omega \) Values

Pseudosteady-State Interporosity Flow

"Fracture" System Radial Flow Region, \( p'_D = 1/2 \)

Type Curve for an Unfractured Well in an Infinite-Acting Naturally-Fractured Reservoir with NO Wellbore Storage or Skin Effects. Plotting Format: paper SPE 23835
Onur, M., and Satman, A. "New Type Curves to Determine Naturally Fractured Reservoir Parameters"

Transient Interporosity Flow

"Fracture" System Radial Flow Region, \( p'_D = 1/2 \)

Type Curve for an Unfractured Well in an Infinite-Acting Naturally-Fractured Reservoir with NO Wellbore Storage or Skin Effects. Plotting Format: paper SPE 23835
Onur, M., and Satman, A. "New Type Curves to Determine Naturally Fractured Reservoir Parameters"
Derivation of the Warren and Root Approach: Pseudosteady-State Interporosity Flow

from Department of Petroleum Engineering Course Notes (1994)
Performance of Dual Porosity/Naturally Fractured Reservoirs

The analytical modelling of reservoir performance for naturally fractured/dual porosity reservoirs was initiated by Warren and Root, "The Behavior of Naturally Fractured Reservoirs," 1958, 1959. This work gives us rigorous solutions in the Laplace domain for performance in an infinite-source reservoir, as well as approximations in the real domain.

The true and idealized physical models for our "naturally fractured" reservoir is shown below.

fig. 1 — idealization of the heterogeneous porous medium.

Consider the flow of a "slightly compressible" liquid, we can write the diffusivity equations for the "fracture" and "matrix" systems as

\[ \frac{k_f}{m} \frac{\partial P_f}{\partial t} = \left[ \frac{\partial}{\partial x} \left( k_f \frac{\partial P_f}{\partial x} \right) \right] \]

Fracture Relation (1)

\[ \frac{k_m}{m} \frac{\partial P_m}{\partial t} = \left[ \frac{\partial}{\partial x} \left( k_m \frac{\partial P_m}{\partial x} \right) \right] + q \]

Matrix Relation (2)

Solving Eqs. 1 and 2 for the flowrate, \( q \), and equating gives

\[ \frac{k_f}{m} \frac{\partial^2 P_f}{\partial x^2} + \frac{k_m}{m} \frac{\partial^2 P_m}{\partial x^2} \]

Fracture Relation (3)

where Eq. 3 is identical to Eq. 6 in the Warren and Root paper.

Warren and Root suggested that the behavior of the matrix blocks could be considered to be at pseudosteady-state flow conditions. In this case, the flowrate out of the matrix is

\[ q = -\frac{\partial P_m}{\partial t} \]

Pseudosteady-State Flow

In addition, Warren and Root suggest an "interface" condition to describe the pressure drop across the fracture face. This relation is given as

\[ q = \frac{k_m}{m} \left( \frac{P_f}{n} - \frac{P_m}{n} \right) \]

Interface Condition (5)

where \( n \) is a geometric factor given as \( n = 4n (n+1) \lambda \), where \( n \) is the number of normal sets of fractures and \( \lambda \) is a characteristic length. The \( n \) parameter is not especially important because we will "jump" \( n \) and other parameters into a physical constant to be obtained from data analysis.

Equating Eqs. 4 and 5, and solving for the matrix pressure derivative, \( \frac{\partial P_m}{\partial t} \), we have

\[ \frac{\partial P_m}{\partial t} = \frac{k_m}{k_f} \left( \frac{P_f}{n} - \frac{P_m}{n} \right) \]

Interface Condition (6)

Substituting Eq. 4 into Eq. 1 gives us a fracture pressure relation that includes the matrix pressure change with respect to time. This result is

\[ \frac{\partial^2 P_f}{\partial t^2} = \frac{k_f}{k_f} \left[ \frac{\partial^2 P_f}{\partial x^2} + \left( \frac{\partial P_m}{\partial t} \right) \frac{\partial P_m}{\partial x} \right] \]

Fracture Pressure (7)

Using the following definitions for dimensionless time and dimensionless pressure:

\[ t_0 = \frac{k_f}{m} \left( \frac{1}{3} \frac{\partial P_f}{\partial x} + \frac{\partial P_m}{\partial x} \right) \]

Dimensionless Time (8)

\[ P_0 = \frac{2 \pi k_f h}{gh} (p_f - p_m) \]

Dimensionless Pressure (9)
Solving Eq. 8 for time, we have
\[ t = \frac{\mu \left( \frac{d\psi_1}{d\eta_{x1}} + \frac{d\psi_2}{d\eta_{x2}} \right)}{k_f} \]

Substituting Eqs. 10-12 into Eq. 6 gives
\[ \frac{d^2 p_m}{d t^2} = \frac{\alpha k_m}{m} \left( \frac{\rho_m - \rho_b}{k_f} \right) \left( \frac{\psi_1 - \rho_m}{\psi_2 - \psi_1} \right) \]

Defining the following "lumped" parameters we have
\[ \lambda = \frac{\alpha k_m R^2}{k_f} \] (Dimensionless Flow Coefficient)

and
\[ \omega = \frac{[\psi_1]_f}{[\psi_1]_f + [\psi_2]_m} \]

or rewriting
\[ 1 - \omega = \frac{[\psi_2]_m}{[\psi_1]_f + [\psi_2]_m} \]

Substituting Eqs. 14 and 16 into Eq. 13 gives
\[ \frac{d\psi_m}{d t} = \frac{\lambda}{1 - \omega} \left( \frac{\psi_f - \psi_m}{\psi_1 - \psi_m} \right) \] (Dimensionless Interface Condition)

Expanding
\[ \frac{d^2 \psi_f}{d t^2} = \frac{1}{1 - \omega} \left( \frac{k_f}{k_f} \right) \frac{d\psi_f}{d t} \]

Substituting Eq. 22 into Eq. 18 and cancelling the \( R^2 \) gives
\[ \frac{1}{R_0} \left( \frac{d\psi_f}{d t} \right) = \omega \frac{d\psi_f}{d t} + (1 - \omega) \frac{d\psi_m}{d t} \] (Pressure Equation)
Summarizing our major results so far

\[ \frac{d L (w)}{d \phi} \left( \frac{\phi_m - \phi_f}{\phi_m - \phi_f} \right) \]  \hspace{1cm} \text{(Dimensionless Interface Condition)} \tag{17}

\[ \frac{w}{w} \left[ \frac{\phi_m - \phi_f}{\phi_m - \phi_f} \right] = \frac{1 - w}{1 - w} \left[ \frac{\phi_m - \phi_f}{\phi_m - \phi_f} \right] \]  \hspace{1cm} \text{(Dimensionless Fracture Pressure)} \tag{22}

Taking the Laplace transform of Eq. 17 we have

\[ \frac{\mu (\phi_m - \phi_f)}{\phi_m} = \frac{\lambda}{1 - w} \left[ \frac{\phi_m - \phi_f}{\phi_m - \phi_f} \right] \]

or

\[ \frac{\lambda}{1 - w} \left[ \frac{\phi_m - \phi_f}{\phi_m - \phi_f} \right] = \frac{\phi_m - \phi_f}{\phi_m} \]

and finally

\[ \frac{\phi_m}{\phi_m} = \frac{\lambda}{\phi_m + (1 - w) m} \] \hspace{1cm} \text{(24)}

And taking the Laplace transform of Eq. 23 gives

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \omega \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} + (1 - w) \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] \]

or

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \omega \phi_m (\phi_m - \phi_f) + (1 - w) \phi_m (\phi_m - \phi_f) \] \hspace{1cm} \text{(25)}

Substituting Eq. 24 into Eq. 25 gives

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \omega \phi_m (\phi_m - \phi_f) + (1 - w) \phi_m (\phi_m - \phi_f) \]

Collecting

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \omega \phi_m (\phi_m - \phi_f) + \frac{(1 - w) \phi_m (\phi_m - \phi_f)}{\phi_m + (1 - w) m} \]

or finally

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \frac{(1 - w) \phi_m (\phi_m - \phi_f)}{\phi_m + (1 - w) m} \] \hspace{1cm} \text{(26)}

Using shorthand notation we define

\[ f(w) = \frac{(1 - w) \phi_m (\phi_m - \phi_f)}{\phi_m + (1 - w) m} \] \hspace{1cm} \text{(27)}

Substituting Eq. 27 into Eq. 26 we have

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \frac{(1 - w) \phi_m (\phi_m - \phi_f)}{\phi_m + (1 - w) m} \] \hspace{1cm} \text{(28)}

Multiplying through Eq. 28 by \( \phi_m \) gives

\[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \]

Defining a variable of substitution, \( \tilde{z} \), as follows

\[ \tilde{z} = \sqrt{\omega \phi_m} \]

or

\[ \phi_m = \frac{1}{\sqrt{\omega \phi_m}} \] \hspace{1cm} \text{(31)}

where

\[ \sqrt{\omega \phi_m} = \sqrt{\omega \phi_m} \]

Applying the chain rule on the \( d(1)/d \phi \) terms in Eq. 29 gives

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] \]

or

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] \]

Substituting Eqs. 31 and 34 into Eq. 33 we obtain

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] \]

Cancelling the \( \lambda \) terms we have

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] \]

Expanding the left-hand-side of Eq. 35 gives

\[ \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] = \frac{d}{d \phi} \left[ \frac{\phi_m (\phi_m - \phi_f)}{\phi_m - \phi_f} \right] \]

From Abramowitz and Stegun, "Handbook of Mathematical Functions" (p. 744, Eq. 9.1.1), the modified Bessel differential equation is given by

\[ \frac{d^2 w}{d \phi^2} + \frac{2 \phi}{d \phi} - \left( \frac{\lambda^2}{\phi^2 + \theta^2} \right) w = 0 \] \hspace{1cm} \text{(57)}

The general solution of Eq. 37 is given by

\[ w = A J_\lambda (\phi) + B Y_\lambda (\phi) \] \hspace{1cm} \text{(58)}

(Derivation of the Warren and Root Approach: Pseudosteady-State Interporosity Flow)
where the functions $I_0(z)$ and $k_0(z)$ are the modified Bessel functions of the first and second kind, respectively. This implies that the general solution of Eq. 37 is
\[ f_b(r, \omega) = A I_0(\omega \sqrt{r}) + B k_0(\omega \sqrt{r}) \]
Substituting Eq. 30 into Eq. 39 we have
\[ f_b(r, \omega) = A I_0(\sqrt{\omega^2 r \mu}) + B k_0(\sqrt{\omega^2 r \mu}) \]
where
\[ \sqrt{\omega^2} = \sqrt{\omega \mu \frac{d}{dt}} \]
Substituting Eq. 32 into Eq. 40 gives
\[ f_b(r, \omega) = A I_0(\sqrt{\omega^2 r \mu}) + B k_0(\sqrt{\omega^2 r \mu}) \]
Comparing to the homogeneous case we have
\[ f_h(r, \omega) = A I_0(\sqrt{\omega^2 r \mu}) + B k_0(\sqrt{\omega^2 r \mu}) \]
The governing identity that relates the homogeneous reservoir solution in the real domain, $f_h(r, \omega)$, with the Laplace transform solution for the naturally fractured/dual porosity reservoir case, $f_b(r, \omega)$, is given by
\[ f_b(r, \omega) = \int \left[ f_h(r, \omega) \exp(-\omega \mu \frac{d}{dt}) \right] \omega^{\alpha} d\omega \]

where Eq. 43 was proposed by Thompson, Manrique, and Tejeda, "Efficient Algorithms for Computing the Bounded Reservoir Horizontal Well Pressure Response," paper SPE 21827, Denver 1991.

As with previous efforts, we require the $\frac{d}{dr} [f_b(r, \omega)]$ term. Using our "$a" notation, the derivative of Eq. 40 is
\[ \frac{d}{dr} [f_b(r, \omega)] = A \sqrt{\omega} I_1(\omega \sqrt{r}) - B \sqrt{\omega} k_1(\omega \sqrt{r}) \]
Summarizing the relevant results (where $\omega = \sqrt{\omega \mu \frac{d}{dt}}$)
\[ f_b(r, \omega) = A I_0(\sqrt{\omega^2 r \mu}) + B k_0(\sqrt{\omega^2 r \mu}) \]
and
\[ \frac{d}{dr} [f_b(r, \omega)] = A \sqrt{\omega} I_1(\omega \sqrt{r}) - B \sqrt{\omega} k_1(\omega \sqrt{r}) \]
where Eqs. 40 and 44 are identical to the homogeneous reservoir case where $f_h(\omega) = 1$. This result suggests that substituting $\sqrt{\omega \mu \frac{d}{dt}}$ for $\omega$ in the homogeneous solution will yield the dual porosity/naturally fractured reservoir solution.}

We note that Eq. 43 also verifies this concept.

Solution for an Infinite-Acting Reservoir:
The solution for a vertical well producing at a constant flowrate in an infinite-acting homogeneous reservoir was derived previously. The "like source" solution is given as
\[ f_b(r, \omega) = \frac{1}{\mu} k_0(\omega \sqrt{r \mu}) \]
Replacing $\sqrt{\omega}$ by $\sqrt{\mu \omega \frac{d}{dt}}$ gives us the dual porosity/naturally fractured reservoir solution. This result is
\[ f_b(r, \omega) = \frac{1}{\mu} k_0(\sqrt{\mu \omega \frac{d}{dt}} \sqrt{r \mu}) \]
where
\[ f_h(\omega) = \frac{\lambda + \omega(1-\omega)\mu}{\lambda + (1-\omega)\mu} \]
Obviously Eq. 44 cannot be analytically inverted due to the algebraic form of the function, $f_h(\omega)$. However, following the work of Warren and Root, we will use a logarithmic approximation of $k_0(\omega)$. Our previous developments show that
\[ k_0(\omega) = \frac{1}{2} \ln \left( \frac{4 \omega}{(\omega^2 + 1) \left( e^{\omega^2} - 1 \right)} \right) \] as $\omega \to 0$
Substituting Eq. 47 into Eq. 46 gives
\[ f_b(r, \omega) = \frac{1}{2\mu} \ln \left( \frac{4 \omega}{(\omega^2 + 1) \left( e^{\omega^2} - 1 \right)} \right) \]
Expanding the logarithmic term gives
\[ f_b(r, \omega) = \frac{1}{2\mu} \ln \left( \frac{4 \omega}{(\omega^2 + 1) \left( e^{\omega^2} - 1 \right)} \right) \]
Isolating the $f_h(\omega)$ term
\[ \ln[f_h(\omega)] = \ln \left( \frac{\lambda + \omega(1-\omega)\mu}{\lambda + (1-\omega)\mu} \right) \]
Dividing thru the logarithmic argument by $\lambda$ gives
\[ \ln[f_h(\omega)] = \ln \left( \frac{1 + \omega(1-\omega)\mu}{1 + (1-\omega)\mu} \right) \]

(Derivation of the Warren and Root Approach: Pseudosteady-State Interporosity Flow)
Expanding the logarithm we obtain
\[ \ln [hw] = \ln [1 + \omega \mu] - \ln [1 + \omega \mu] \]
\[ = \ln [1 + \omega \mu] - \ln [1 + \omega \mu] + 1 \ln [1 + (\omega \mu)] \]
Substituting Eq. 50 into Eq. 49 gives
\[ \dot{Q}_w(r_p, t_p) = \frac{1}{2} \left[ \ln \left( \frac{4}{e^{2t_p}} \right) - \ln \left( \frac{4}{e^{2t_p}} \right) \right] \]
The analytical inversion of Eq. 51 is possible using the
table of transforms given below
\[ \text{Reference(s)} \]
\[ \frac{1}{\mu} \left( \frac{1}{e^{2t_p}} \right) \] or
\[ \frac{1}{\mu} = \text{constant} \]
\[ -\frac{1}{\mu} \ln(\mu) \] or
\[ \ln(e^{t_p}) \]

Collecting terms in Eq. 52 gives
\[ \dot{Q}_w(r_p, t_p) = \frac{1}{2} \ln \left( \frac{4}{e^{2t_p}} \right) - \frac{1}{2} \ln \left( \frac{4}{e^{2t_p}} \right) + \frac{1}{2} \ln \left( \frac{4}{e^{2t_p}} \right) \]
We would also like to use Eq. 53 to determine the well testing
derivative. This derivative is given by
\[ \dot{p}_w(r_p, t_p) = t_0 \frac{d}{dt} \left[ p_w(r_p, t_p) \right] \]
The general forms of the time derivatives are given below
\[ \frac{d}{dt} \left[ f(t) \right] \]
\[ = \frac{1}{t} \frac{d}{dt} \left[ f(t) \right] \]
\[ = \frac{1}{t} \exp(-t) \]
\[ = \frac{1}{t} \exp(-at) \]
Using these relations, the well testing derivative is given by
\[ \dot{p}_w(r_p, t_p) = \frac{t_0}{2} \left[ \frac{1}{t_0} + \frac{1}{t_0} \exp(-\frac{\lambda}{t_0}) - \frac{1}{t_0} \exp(-\frac{\lambda}{t_0}) \right] \]
Expanding
\[ \dot{p}_w(r_p, t_p) = \frac{1}{2} \left[ \frac{1}{t_0} + \frac{1}{t_0} \exp\left(-\frac{\lambda}{t_0}\right) - \frac{1}{t_0} \exp\left(-\frac{\lambda}{t_0}\right) \right] \]
Summary of Results: Warren and Root Solution for a Well in an Infinite-Acting, Dual Porosity Reservoir

Warren and Root Interporosity Flow Function: \( f(w) \)

\[
f(w) = \frac{\lambda + w(w-1)u}{\lambda + (1-w)u} \quad (27)
\]

Thompson, Manrique, and Jakemt Transform Relation:

\[
f_{pf}(r_0, u) = f(w) \int_0^\infty f_{in}(r_0, z) \exp \left[-u(w-1)z\right] dz \quad (43)
\]

Where Eq. 43 is valid for any case, regardless of geometry, well type (vertical, fractured, or horizontal wells), or flow regime (transient, post-transient, and boundary-dominated flow conditions).

Radial Flow Solutions - Infinite-Acting Reservoirs:

The line source solution is given by

\[
f_{p0}(r, u) = \frac{1}{u} \ln \left( \frac{4}{\pi r^2} \right) \quad (45)
\]

And the "log approximation" is given by

\[
f_{p0}(r, u) = \frac{1}{2 u} \ln \left( \frac{4}{\pi r^2} \frac{1}{u} \right) \quad (48)
\]

Combining Eqs. 47 and 48 we obtain

\[
f_{p0}(r, u) = \frac{1}{\pi} \ln \left( \frac{4}{\pi r^2} \right) - \frac{1}{2 u} \ln \left( \frac{1 + u(w-1)u}{\lambda} \right) \quad (51)
\]

Where the inverse Laplace transform of Eq. 51 is given by

\[
f_{p0}(r, t) = \ln \left( \frac{4}{\pi r^2} \right) - \frac{1}{2} E_1 \left( \frac{\lambda}{t} \right) \quad (52)
\]

And the "well testing" derivative is given by

\[
f_{p0}^\prime(r, t) = \frac{1}{2} E_1 \left( \frac{\lambda}{t} \right) - \exp \left( -\frac{\lambda}{w-1} t \right) \quad (55)
\]
Well Test Analysis (Paper SPE 13054)  
An Infinite-Acting Dual Porosity  
(Naturally Fractured) Reservoir  

(History Matching (Simulation) Solutions)  

from Department of Petroleum Engineering Course Notes (1994)
Well A-17 (SPE 13054) **PDD** Transient Interporosity Flow.

**Log-Log Plot**

- **Legend:** Well A-17 Drawdown (SPE 13054)
  - $\Delta p$ data
  - $\Delta p'$ data
  - Simulation (Optimized Match)

- **Results for A-17 Well:**
  - $k=14.4$ md
  - $s=5.75$
  - $C_p=27,000$
  - $\lambda=9.11 \times 10^{-6}$
  - $\alpha=1.05 \times 10^{-3}$

- **Well Model:** Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Transient Interporosity Flow

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Well A-17 (SPE 13054) **PBU** Transient Interporosity Flow.

**Log-Log Plot (Full Test History—No Rate History)**

- **Legend:** Well A-17 Buildup (SPE 13054)
  - $\Delta p$ data
  - $\Delta p'$ data
  - Simulation (Optimized Match)

- **Results for A-17 Well:**
  - $k=27.5$ md
  - $s=5.4$
  - $C_p=16,100$
  - $\lambda=2.02 \times 10^{-5}$
  - $\alpha=1.001 \times 10^{-3}$

- **Well Model:** Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Transient Interporosity Flow

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Well A-17 (SPE 13054) **PDD** Pseudosteady-State Interporosity Flow.

**Log-Log Plot**

- **Legend:** Well A-17 Drawdown (SPE 13054)
  - $\Delta p$ data
  - $\Delta p'$ data
  - Simulation (Optimized Match)

- **Results for A-17 Well:**
  - $k=14.4$ md
  - $s=5.76$
  - $C_p=28,000$
  - $\lambda=8.91 \times 10^{-6}$
  - $\alpha=4.8 \times 10^{-3}$

- **Well Model:** Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

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Well A-17 (SPE 13054) **PBU** Pseudosteady-State Interporosity Flow.

**Log-Log Plot (Full Test History—No Rate History)**

- **Legend:** Well A-17 Buildup (SPE 13054)
  - $\Delta p$ data
  - $\Delta p'$ data
  - Simulation (Optimized Match)

- **Results for A-17 Well:**
  - $k=27.5$ md
  - $s=5.31$
  - $C_p=15,500$
  - $\lambda=3.83 \times 10^{-5}$
  - $\alpha=4.96 \times 10^{-2}$

- **Well Model:** Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

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(Well Test Analysis (Paper SPE 13054) An Infinite-Acting Dual Porosity (Naturally Fractured) Reservoir — History Matching (Simulation) Solutions)
Well Test Analysis (Paper SPE 13054) An Infinite-Acting Dual Porosity (Naturally Fractured) Reservoir — History Matching (Simulation) Solutions

Well Mach-3X (SPE 13054) **PDD** Transient Interporosity Flow.

![Log-Log Plot](image1)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

![Results for Mach-3X Well](image2)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

Well 14RN-2X (SPE 13054) **PDD** Pseudosteady-State Interporosity Flow.

![Log-Log Plot](image3)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

![Results for Well 14RN-2X](image4)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

Well Mach-3X (SPE 13054) **PBU** Transient Interporosity Flow.

![Log-Log Plot (Full Test History--No Rate History)](image5)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

![Results for Mach-3X Well](image6)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

Well 14RN-2X (SPE 13054) **PBU** Pseudosteady-State Interporosity Flow.

![Log-Log Plot (Full Test History--Includes Rate History)](image7)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

![Results for Well 14RN-2X](image8)

Well Model:
Unfractured Well in an Infinite-Acting Dual Porosity Reservoir, Pseudosteady-State Interporosity Flow

Petroleum Engineering 620 — Fluid Flow in Petroleum Reservoirs
Reservoir Flow Solutions Lecture 5 — Dual Porosity Reservoirs: Warren and Root Approach