Objectives: (things you should know and/or be able to do)

- Be familiar with and be able to develop flow relations for gases and compressible liquids, in terms of pressure, pressure-squared, and pseudopressure using the non-laminar Forchheimer flow relation, which is quadratic in terms of velocity.

Lecture Outline:

- Developments using the Forchheimer, non-laminar flow relation.
  - Liquid flow relations
  - Gas flow relations
  - Plotting functions

- Various applications of the non-laminar flow equations (discussion)
  - Transient flow (isochronal tests)
  - Pseudosteady-state flow (flow-after-flow tests)
Orientation to Non-Laminar Flow in Porous Media
Estimating the Coefficient of Inertial Resistance in Fluid Flow Through Porous Media

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RIJSWIJK, THE NETHERLANDS

\[
\frac{\beta}{\sqrt{\alpha}} \phi^{5.5} = 0.005
\]

FIG. 2 — CORRELATION BETWEEN POROSITY AND VISCOS AND INERTIAL RESISTANCE COEFFICIENT.
An Analysis of High-Velocity Gas Flow Through Porous Media
Abbas Firoozabadi, Abadan Institute of Technology
Donald L. Katz, SPE-AIME, U. of Michigan

<table>
<thead>
<tr>
<th>Reference</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fancher et al.¹ (1933)</td>
<td>&quot;... the flow of fluids through these porous materials closely resembles that through pipes; that there is a condition of flow in porous systems which resembles viscous flow, another which corresponds to turbulence.&quot;</td>
</tr>
<tr>
<td>Elenbaas and Katz² (1948)</td>
<td>&quot;A Radial Turbulent Flow Formula&quot;</td>
</tr>
<tr>
<td>Green and Duwez³ (1951)</td>
<td>&quot;The inertial coefficient $\beta$ ... may be interpreted as a measure of the tortuosity of the flow channels, perhaps as an average curvature of the streamline determining the acceleration experienced by the fluid.&quot;</td>
</tr>
<tr>
<td>Hubbert⁴ (1956)</td>
<td>&quot;... we have seen that the cause of the failure of Darcy's Law is the distortion that results in the flow lines when the velocity is great enough that the inertial force becomes significant.&quot;</td>
</tr>
<tr>
<td>Tek⁵ (1957)</td>
<td>&quot;The generalized Darcy equation may be referred to as the 'non-Darcy flow' regime. The transition from Darcy to non-Darcy flow is a gradual one.&quot;</td>
</tr>
<tr>
<td>Katz et al.⁶ (1959)</td>
<td>&quot;If one includes extra motion of the fluid to consume the extra pressure loss, then the term 'turbulent flow' here is justified.&quot;</td>
</tr>
<tr>
<td>Houpeurt⁷ (1959)</td>
<td>&quot;... we do not think, the flow can be really turbulent ...; we consider the kinetic energy losses are responsible for the deviation... from Darcy's Law.&quot;</td>
</tr>
<tr>
<td>Tek et al.²³ (1962)</td>
<td>&quot;The Effect of Turbulence on Flow of Natural Gas...&quot;</td>
</tr>
<tr>
<td>Swift and Kiel¹⁸ (1962)</td>
<td>&quot;Prediction of Gas-Well Performance Including the Effect of Non-Darcy Flow&quot; “Analysis of data ... to give direct &quot;in situ&quot; information for ... turbulence or non-Darcy coefficients.&quot;</td>
</tr>
<tr>
<td>Wright¹⁰ (1968)</td>
<td>&quot;... Four regimes of flow for water in an unconsolidated bed. 1) a laminar regime 2) a steady inertia regime 3) a turbulent transition regime 4) a fully turbulent regime.&quot;</td>
</tr>
<tr>
<td>Gewers and Nichol¹² (1969)</td>
<td>&quot;Gas Turbulence Factor in a Microporous Carbonate.&quot;</td>
</tr>
<tr>
<td>Geertsma¹⁴ (1974)</td>
<td>&quot;Coefficient of Inertial Resistance&quot; &quot;The flow regime of concern is usually fully laminar. The observed departure from Darcy's Law is the result of convective accelerations and decelerations of the fluid particles on their way through the pore space.&quot;</td>
</tr>
</tbody>
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An Analysis of High-Velocity Gas Flow Through Porous Media

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Fig. 1—Flow in a cylindrical conduit.

Fig. 2—Flow in an idealized pore.

(a) Low velocity
(b) Velocity higher
(c) Intermediate, transition
(d) High velocity, turbulent

Fig. 3—Idealized flow through alternating cross sections.
An Analysis of High-Velocity Gas Flow Through Porous Media

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Fig. 5—Correlation of $\beta$ from Eq. 9.

Fig. 7—Plot according to Eq. 7 of USBM data$^2$ for a limestone core.
Using the Inertial Coefficient, $\beta$, To Characterize Heterogeneity in Reservoir Rock

by S.C. Jones, Core Research, Div. of Western Atlas
SPE Member

**TABLE 1**

**CORRELATIONS FOR $\beta^*$**

<table>
<thead>
<tr>
<th>Equation resulting from correlation</th>
<th>'Goodness of fit', R²</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 6.15 \times 10^{10} , k^{-1.55}$</td>
<td>0.9220</td>
<td>Fig. 1, this paper</td>
</tr>
<tr>
<td>$\beta = 3.13 \times 10^{12} , k^{-1.78} , \phi^{-1.78}$</td>
<td>0.9189</td>
<td>$n = 1$, Eq. 7, this paper</td>
</tr>
<tr>
<td>$\beta = 1.88 \times 10^{10} , k^{-1.47} , \phi^{-0.53}$</td>
<td>0.8899</td>
<td>Fig. 2, this paper</td>
</tr>
<tr>
<td>$\beta = 1.27 \times 10^{5} , k^{-0.52} , \phi^{-5.68}$</td>
<td>0.8058</td>
<td>$n = 6$, Eq. 7, this paper</td>
</tr>
<tr>
<td>$\beta = 4.85 \times 10^{4} , k^{-0.50} , \phi^{-5.50}$</td>
<td>--</td>
<td>Geertsma$^7$</td>
</tr>
<tr>
<td>$\beta = 4.2 \times 10^{10} , k^{-1.35}$</td>
<td>--</td>
<td>Katz, et. al$^{17}$</td>
</tr>
<tr>
<td>$\beta = 1.12 \times 10^{11} , k^{-1.06}$</td>
<td>--</td>
<td>Eq. 14 (gas well tests), Noman, et. al$^{6}$</td>
</tr>
<tr>
<td>$\beta = 2.48 \times 10^9 , (k/(\phi S_g))^{-1.22}$</td>
<td>--</td>
<td>Eq. 13 (Best fit for gas well tests), Noman, et. al</td>
</tr>
<tr>
<td>$\beta = 1.9 \times 10^{11} , k^{-1.73}$</td>
<td>--</td>
<td>Fig. 8 (lab data), Noman, et. al</td>
</tr>
<tr>
<td>$\beta = 5.5 \times 10^9 , k^{-1.25} , \phi^{-0.75}$</td>
<td>--</td>
<td>Tek, et. al$^{4}$</td>
</tr>
</tbody>
</table>

*For all equations: $\beta = ft^{-1}$, $k = \text{md}$, $\phi$ and $S_g$ = fraction.

The reader should refer to Firoozabadi and Katz$^5$ for additional correlations.
Using the Inertial Coefficient, $\beta$, To Characterize Heterogeneity in Reservoir Rock

by S.C. Jones, Core Research, Div. of Western Atlas

SPE Member

Fig. 1—Correlation of inertial coefficient, $\beta$, vs. Klinkenberg permeability. Pluses are sandstones and diamonds are limestones.
Forchheimer Equation for Non-Laminar Flow in Porous Media
Forschheiner Equation for Non-Laminar Flow in Porous Media

The Forschheiner equation for non-laminar flow in porous media is given by:

\[- \frac{dp}{dx} = \frac{C}{k} \frac{\rho v + \rho \beta \frac{\partial \rho}{\partial x}}{v^2} \]  (1)

where,

\[C = \frac{1}{(1.0325 \times 10^6 \text{ dyn cm}^{-2})(12 \text{ in ft}) (2.54 \text{ cm in})} \]

or

\[C = 3.2379 \times 10^{-8} \frac{\text{ dyn cm}^{-2}}{\text{ cm}^{-2} \text{ ft}^{-1}} \]

\[\beta = \text{ "inertial flow coefficient," ft}^{-1} \]

By inspection we note that the first term on the right-hand-side (RHS) of Eq. 1 is simply Darcy's law (for laminar flow). The \(C \rho v^2\) term is the "add-on" term used to account for non-laminar flow.

The average velocity, \(v\), is given as

\[v = \frac{1}{A} q_{res} = \frac{1}{A} q_{sc} B \]  (2)

where,

\[v = \text{average velocity}, \quad A = \text{cross-sectional area}, \quad q_{res} = \text{volumetric flowrate (reservoir volume)}, \quad q_{sc} = \text{volumetric flowrate ("standard" volume)}, \quad B = \text{formation volume factor, res vol/std vol} \]

Substituting Eq. 2 into Eq. 1, we obtain

\[- \frac{dp}{dx} = \frac{\mu B}{k A} q_{sc} + \frac{\rho \beta B^2}{A^2} q_{sc} ^2 \]  (3)

For a dry gas, we have

\[\rho g = \frac{1}{B} q_{sc} \]

Substituting Eq. 4 into Eq. 3, we obtain

\[- \frac{dp}{dx} = \frac{\mu B}{k A} q_{sc} + \frac{\rho \beta B^2}{A^2} q_{sc} ^2 \]

Dividing through by \(\mu q_{sc}\)

\[- \frac{1}{\mu q_{sc}} \frac{dp}{dx} = \frac{1}{k A} q_{sc} + \frac{\rho \beta B^2}{\mu q_{sc}} q_{sc} ^2 \]  (5)

Multiplying through by \(\mu q_{sc} B\) \((\mu q_{sc} B) \); \(q_{sc} = \frac{q_{sc}}{B}\) \(q_{sc} = (q)\), we obtain

\[- \frac{\mu B}{k A} \frac{dp}{dx} = \frac{\mu}{\mu q_{sc} B} q_{sc} + \frac{\rho \beta B^2}{A^2} q_{sc} q_{sc} \]

Separating

\[- \frac{\mu B}{\mu q_{sc} B} \frac{dp}{dx} = \left[ \frac{\mu B}{k A} q_{sc} + \frac{\rho \beta B^2}{A^2} q_{sc} \right] \frac{d}{dx} \]

Integrating

\[- \mu B \frac{q_{sc}}{k A} \frac{1}{\mu q_{sc} B} dp = \left[ \frac{\mu B}{k A} q_{sc} + \frac{\rho \beta B^2}{A^2} q_{sc} \right] \frac{d}{dx} \]

The gas formation volume factor, \(B_g\), is defined as

\[B_g = \frac{q_{sc}}{\frac{T}{\gamma} \frac{p}{T}} \]  (7)

(Forschheiner Equation for Non-Laminar Flow in Porous Media)
Substituting Eq. 7 into the integral on the left-hand-side (LHS) of Eq. 6, we have

\[
I = -\frac{m_{3}B_{3}}{P_{i}} \int_{P_{2}}^{P_{1}} \frac{P}{m_{2}} \, dp = -\frac{m_{3}B_{3}}{m_{2}} \int_{P_{2}}^{P_{1}} \frac{P}{m_{2}} \, dp
\]

(assuming \( T_{n} = T \))

Using the initial reservoir pressure, \( P_{i} \), as the "normalizing" pressure, \( P_{n} \), we have

\[
I = \frac{m_{3}B_{3}}{P_{i}} \int_{P_{2}}^{P_{1}} \frac{P}{m_{2}} \, dp \quad \text{(reversing limits)}
\]

Expanding the integral

\[
I = \frac{m_{3}B_{3}}{P_{i}} \int_{P_{2}}^{P_{1}} \frac{P}{m_{2}} \, dp - \frac{m_{3}B_{3}}{P_{i}} \int_{P_{2}}^{P_{1}} \frac{P}{m_{2}} \, dp
\]

(8)

or

\[
I = P_{i}P_{1} - P_{i}P_{2}
\]

(9)

where

\[
P_{i}P_{1} = \frac{m_{3}B_{3}}{P_{i}} \int_{P_{2}}^{P_{1}} \frac{P}{m_{2}} \, dp
\]

(10)

Substituting Eq. 9 into Eq. 6, we obtain

\[
P_{1}P_{1} - P_{2}P_{2} = \left[ \frac{m_{3}B_{3}}{kA} q_{sc} + \frac{cE}{m_{q}A^{2}} \int q_{sc} m_{3}B_{3} q_{sc} \, dx \right] \int_{0}^{L} \, dx
\]

Completing the integration and rearranging

\[
\frac{P_{1}P_{1} - P_{2}P_{2}}{L} = \frac{m_{3}B_{3}}{kA} q_{sc} + \frac{cE}{m_{q}A^{2}} \int q_{sc} m_{3}B_{3} q_{sc} \, dx
\]

Dividing through by \( \frac{m_{3}B_{3}}{kA} q_{sc} \)

\[
\frac{A}{k} \frac{1}{P_{i}} \left( \frac{P_{1}P_{1} - P_{2}P_{2}}{L} \right) = 1 + \frac{cE q_{sc}}{m_{q}A}
\]

(11)

From Darcy's Law (i.e., steady-state laminar flow) we have,

\[
q_{sc} = \frac{k_{DL} A}{m_{3}B_{3}} \frac{(P_{1}P_{1} - P_{2}P_{2})}{L}
\]

or

\[
\frac{1}{k_{DL}} = \frac{1}{k} \left( \frac{P_{1}P_{1} - P_{2}P_{2}}{L} \right)
\]

(12)

Substituting Eq. 12 into Eq. 11

\[
\frac{1}{k_{DL}} = \frac{1}{k} + \frac{cE q_{sc}}{m_{q}A}
\]

(13)

where Eq. 13 is of the form,

\[
y = b + mx
\]

where

\[
y = \frac{L}{k_{DL}} \quad ; \quad x = q_{sc} \quad ; \quad m = \frac{cE q_{sc}}{m_{q}A} \quad \text{and} \quad b = \frac{1}{k}
\]

and

\(
m_{q} \) is taken at \( p = \frac{1}{2}(p_{1} + p_{2}) \)

A plot of Eq. 13 gives

\[\text{(Forchheimer Equation for Non-Laminar Flow in Porous Media)}\]