A Modified Purcell/Burdine Model for Estimating Absolute Permeability from Mercury-Injection Capillary Pressure Data

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Executive Summary — $k = f(Hg\ p_c)$

- Correlations relating permeability and capillary pressure data do exist in the literature — however, these correlations can yield inconsistent and/or inaccurate results.
- We propose a new permeability model based on the Brooks/Corey/Purcell/Burdine concept for $k$ (i.e., a "bundle of capillary tubes" model).
- In this work, we obtained correlations of:
  - $k = f(\phi,S_{wi},p_d,\lambda)$ (excellent correlation)
  - $p_d = f(k,\phi,S_{wi})$ (excellent correlation)
  - $\lambda = f(k,\phi,S_{wi},p_d)$ (very good correlation)

where these correlations are based on 89 sets of Hg $p_c$ data obtained from literature and industry sources.
Executive Summary — $k, \rho_d, \lambda = f(\text{Hg} \ \rho_c)$ (2/4)

a. $k = f(\phi, S_w, \rho_d, \lambda)$ — simple power law model.

b. $\rho_d = f(k, \phi, S_w)$ — simple power law model.

c. $\lambda = f(k, \phi, S_w, \rho_d)$ — simple power law model.

● Orientation:
  
  (simple power law models)

  - $k = f(\phi, S_w, \rho_d, \lambda)$
    
    → Excellent correlation.

  - $\rho_d = f(k, \phi, S_w)$
    
    → Excellent correlation.

  - $\lambda = f(k, \phi, S_w, \rho_d)$
    
    → Very good correlation.
Executive Summary — $k, p_d, \lambda = f(Hg \ p_c)$ (3/4)

a. $k = f(\phi, S_{wi}, p_d, \lambda)$ — ARE = 17.3 percent.

b. $p_d = f(k, \phi, S_{wi})$ — ARE = 14.9 percent.

c. $\lambda = f(k, \phi, S_{wi}, p_d)$ — ARE = 13.7 percent.

● Orientation:

(best fit models)

- $k = f(\phi, S_{wi}, p_d, \lambda)$
  → Slightly "overfit."

- $p_d = f(k, \phi, S_{wi})$
  → Excellent regression.

- $\lambda = f(k, \phi, S_{wi}, p_d)$
  → Good regression.
a. \( k = f(\phi, S_{wi}, p_d, \lambda) \) — ARE = 19.0 percent.

b. \( p_d = f(k, \phi, S_{wi}) \) — ARE = 14.0 percent.

c. \( \lambda = f(k, \phi, S_{wi}, p_d) \) — ARE = 15.3 percent.

- **Orientation:**
  
  (non-parametric regression)
  
  - \( k = f(\phi, S_{wi}, p_d, \lambda) \)  
    → near-perfect regression.
  
  - \( p_d = f(k, \phi, S_{wi}) \)  
    → near-perfect regression.
  
  - \( \lambda = f(k, \phi, S_{wi}, p_d) \)  
    → dispersion of fit indicates other variables are needed.
Outline — IPTC 10994

- Executive Summary
- Introduction
- Data Collection and Analysis
  - Permeability Models
  - Data collection
- Correlations (Hg $p_c$ data)
  - $k$ Correlation
  - $p_d$ Correlation
  - $\lambda$ Correlation
- Summary
- Recommendations for Future Work
Introduction — IPTC 10994

- Characterization of porous media through the interpretation of capillary pressure:
  - Cores, drill cuttings and fragmented cores.
- Empirical permeability estimations:
  - Not robust (can yield different answers...)
- Theoretical permeability estimations:
  - "Adjustment" factors:
    - Tortuosity factor,
    - Lithology factor, and
    - Pore geometric factor.
- To improve $k$ estimates — we require characteristic capillary pressure behavior:
  - $S_{wi}$: Represents influence of minimum pore distribution.
  - $p_d$: Represents maximum pore size.
  - $\lambda$: Represents pore throat size distribution.
Permeability Models:
- Review of different models

Data Collection:
- Review of capillary pressure measurement
- Data sources
- Data characteristics
• **Mercury injection:**
  - Reasonably accurate.
  - Difficult to relate capillary pressure data to oil/water or gas/oil systems.

• **Porous plate technique:**
  - Very accurate.
  - Can use actual reservoir fluids.
  - Pressure is limited by displacement pressure of porous plate (usually less than 300 psia).

• **Centrifuge technique:**
  - Reasonably accurate.
  - Can use actual reservoir fluids.
  - Limitation on the maximum measurable pressures.
Data Collection/Analysis — $k$ Models

- Leverett J-Function ($J(S_w)$) (1941)
- Purcell Permeability Relation (1949)
- Burdine, et al. Permeability Relation (1950)
- Wyllie/Spangler Permeability Relation (1952)
- Thomeer Permeability Relation (1983)
- Wells/Amaefule Permeability Relation (1985)
- Winland (1980)/Pittman (1992)
Selection of a large variety of samples provides a representative data set which ranges over several orders of magnitude in permeability. Most consistent data are cases of mercury injection capillary pressure data.

Data Sources:
- Literature data
- Industry data

Petrophysical Characteristics:
- $0.0041 < k < 8340$ md
- $0.003 < \phi < 0.34$ (fraction)
- $0.007 < S_{wi} < 0.33$ (fraction)
- $2.32 < p_d < 2176$ psia
2-40 HS1
Capillary Pressure Versus Wetting Phase Saturation

Brooks and Corey $p_c$ Model:

$$p_c = p_d \left[ \frac{S_w - S_{wi}}{1 - S_{wi}} \right]^{-\frac{1}{\lambda}}$$

where:

- $S_{wi} = 0.13$ fraction
- $p_d = 400$ psia
- $\lambda = 2.1$ dimensionless

2-40 HS1

Permeability 0.087 md
Porosity 0.076
$p_d$ 400 psi
$S_{wi}$ 0.13
$\lambda$ 2.1

**a. Capillary Pressure vs. Wetting Phase Saturation — Cartesian Capillary Pressure Format**

**b. Table of Brooks-Corey Parameters — Mercury $p_c$ data**
a. Capillary Pressure vs. Wetting Phase Saturation — Logarithmic Capillary Pressure Format

b. Capillary Pressure vs. Normalized Wetting Phase Saturation — Logarithmic Capillary Pressure Format
Data Collection/Analysis — Good Example (3/3)

a. Dimensionless Capillary Pressure vs. Normalized Wetting Phase Saturation — Log-Log "Type Curve" for Capillary Pressure

b. Dimensionless Capillary Pressure vs. Dimensionless Wetting Phase Saturation — Log-Log "Type Curve" for Capillary Pressure

21 Nov 2005  IPTC 10994 — Correlation of Hg-\(p_c\) and Permeability  Slide — 14/37
Data Collection/Analysis — Bad Example

Stevens A1-R 13240
Logarithm of Capillary Pressure Versus Wetting Phase Saturation

Brooks and Corey $p_c$ Model:

$$p_c = p_d \frac{([S_w - S_{sw}]/[1 - S_{sw}])^{1/3}}{\lambda}$$

where:

$S_{sw} = 0.3$ fraction

$p_d = 152.25$ psia

$\lambda = 1.2$ dimensionless

Capillary Pressure ($p_c$, psia) vs. Wetting Phase Saturation ($S_w$, fraction of pore volume)

Match of low pressure $p_c$ data only!

Stevens A1-R 13240
Logarithm of Dimensionless Capillary Pressure Versus Logarithm of Normalized Wetting Phase Saturation

Brooks and Corey $p_c$ Model:

$$p_c = p_d \frac{([S_w - S_{sw}]/[1 - S_{sw}])^{1/3}}{\lambda}$$

where:

$S_{sw} = 0.3$ fraction

$p_d = 152.25$ psia

$\lambda = 1.2$ dimensionless

Dimensionless Capillary Pressure ($p_c/p_d$, dimensionless) vs. Normalized Wetting Phase Saturation ($S_w^* = (S_w - S_{sw})/[1 - S_{sw}]$)

a. Capillary Pressure vs. Wetting Phase Saturation — Logarithmic Capillary Pressure Format

b. Dimensionless Capillary Pressure vs. Normalized Wetting Phase Saturation — Log-Log "Type Curve" for Capillary Pressure

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Motivation:
- Correlation of permeability based on the "bundle of capillary tubes" model proposed by Purcell and Burdine — and later extended by Wyllie and Gardner (based on the work of Brooks and Corey).
- Such a correlation could yield permeability from capillary pressure — and vice-versa. Possible extensions to relative permeability.

Results: (Hg $p_c$ data only)
- Correlation of $k = f(\phi, S_{wi}, p_d, \lambda)$
- Correlation of $p_d = f(k, \phi, S_{wi})$
- Correlation of $\lambda = f(k, \phi, S_{wi}, p_d)$
Nakornthap and Evans Permeability Relation:

\[ k = 10.66 \omega \frac{1}{n} \frac{1}{2} (\sigma_{Hg-air \cos \theta})^2 (1 - S_{wi})^3 \phi^3 \int_0^{1} \frac{1}{p^2_c} dS^*_w \]

Normalized Saturation:

\[ S^*_w = \frac{S_w - S_{wi}}{1 - S_{wi}} \]

Capillary Pressure:

\[ p_c = p_d \left( \frac{S_w - S_{wi}}{1 - S_{wi}} \right) \frac{1}{\lambda} \]

Brooks/Corey/Purcell/Burdine Permeability Relation:

\[ k = 10.66 \alpha (\sigma_{Hg-air \cos \theta})^2 (1 - S_{wi})^4 \phi^2 \frac{1}{p^2_d} \left( \frac{\lambda}{\lambda + 2} \right) \]

(Simple) Regression Relation: (mercury \( p_c \) only)

\[ k = a_1 \frac{1}{(p_d)^{a_2}} \left( \frac{\lambda}{\lambda + 2} \right)^{a_3} (1 - S_{wi})^{a_4} \phi^{a_5} \]
Summary: $k=f(\phi, S_{wi}, p_d, \lambda)$

a. Model 1 — Power law.  
b. Model 2 — Modified power law.  
c. Model 3 — Base exponential.  
d. Model 4 — Exponential polynomial.  
e. Model 5 — Exponential rational.  
f. Model 8 — Non-parametric regression.
Results: $k = f(\phi, S_{wi}, p_{d}, \lambda) \quad \text{— Model 1}$

Model: 1

$$k = \frac{123356.512}{(p_d)^{1.8139} \left[ \frac{\lambda}{\lambda + 2} \right]^{1.4385} (1 - S_{wi})^{2.2761} \phi^{1.7296}}$$

Comment:

**Simple, compact, and accurate.**

<table>
<thead>
<tr>
<th>Statistical Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>$2.3865 \ln(\text{md})^2$</td>
</tr>
<tr>
<td>Variance</td>
<td>$369278.5839 \text{ md}^2$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>$607.6830 \text{ md}$</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>$26.4580 \text{ percent}$</td>
</tr>
</tbody>
</table>
Results: \( k = f(\phi, S_{wi}, p_d, \lambda) \) — Model 5

**Model: 5**

\[
\begin{align*}
1.1602 \quad -17.4834 & \quad \ln p_d + 3.250 \quad .9900 & \quad \ln \frac{\lambda}{\lambda + 2} - 237.7238 & \quad \ln (1 - S_{\omega}) \\
214.9225 & \quad \ln \phi - 559.9114 & \quad \ln p_d & \quad \ln \frac{\lambda}{\lambda + 2} - 42.6880 & \quad \ln p_d \ln (1 - S_{\omega}) \\
-257.582 & \quad \ln p_d \ln \phi + 1189.6335 & \quad \ln \frac{\lambda}{\lambda + 2} \ln \phi + 1084.4966 & \quad \ln \frac{\lambda}{\lambda + 2} \ln (1 - S_{\omega}) \\
-0.6519 & \quad \ln \phi \ln (1 - S_{\omega}) + 0.0246 & \quad \ln \phi \ln (1 - S_{\omega}) & \quad \ln \frac{\lambda}{\lambda + 2} \\
-141.3981 & \quad \ln \phi \ln (1 - S_{\omega}) & \quad \ln p_d + 1.2541 & \quad \ln \phi \ln \frac{\lambda}{\lambda + 2} \\
-0.2865 & \quad \ln (1 - S_{\omega}) & \quad \ln \frac{\lambda}{\lambda + 2} \ln p_d + 140.9318 & \quad \ln (1 - S_{\omega}) & \quad \ln \frac{\lambda}{\lambda + 2} \ln p_d \ln \phi \\
1 - 5.2919 & \quad \ln p_d + 162.7202 & \quad \ln \frac{\lambda}{\lambda + 2} + 6.8005 & \quad \ln (1 - S_{\omega}) + 39.7913 & \quad \ln \phi \\
6.5984 & \quad \ln p_d \ln \frac{\lambda}{\lambda + 2} + 9.5643 & \quad \ln p_d \ln (1 - S_{\omega}) - 13.7159 & \quad \ln p_d \ln \phi \\
-7.4197 & \quad \ln \frac{\lambda}{\lambda + 2} \ln \phi + 68.3802 & \quad \ln \frac{\lambda}{\lambda + 2} \ln (1 - S_{\omega}) + 35.0810 & \quad \ln \phi \ln (1 - S_{\omega}) \\
+62.4315 & \quad \ln \phi \ln (1 - S_{\omega}) \ln \frac{\lambda}{\lambda + 2} - 19.9561 & \quad \ln \phi \ln (1 - S_{\omega}) \ln p_d \\
-0.5488 & \quad \ln \phi \ln \frac{\lambda}{\lambda + 2} \ln p_d - 5.6490 & \quad \ln (1 - S_{\omega}) \ln \frac{\lambda}{\lambda + 2} \ln p_d \\
-0.0019 & \quad \ln (1 - S_{\omega}) \ln \frac{\lambda}{\lambda + 2} \ln p_d \ln \phi 
\end{align*}
\]

**Comment:**

EXTREMELY complicated "exponential" model.

<table>
<thead>
<tr>
<th>Statistical Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>0.9779 ln(md)²</td>
</tr>
<tr>
<td>Variance</td>
<td>1145308 md²</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1070.1908 md</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>17.2631 percent</td>
</tr>
</tbody>
</table>

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Model: 8 (Non-Parametric Regression for $k$)

There is no model for non-parametric regression

Comment:
The non-parametric correlation is the optimal fit of the data -- this is not a "model-based" regression, but rather, a statistical correlation of all variables use a non-parametric algorithm. Any (model-based) regression that achieves a better regression than the non-parametric regression algorithm has fitted the errors in the data.
The variables for this case were regressed using the natural logarithm of each variable.

<table>
<thead>
<tr>
<th>Statistical Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>1.1395 ln(md)$^2$</td>
</tr>
<tr>
<td>Variance</td>
<td>1188906.807 md$^2$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1090.3700 md</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>18.9947 percent</td>
</tr>
</tbody>
</table>
Summary:  \( p_d = f(k, \phi, S_{wi}) \)

a. Model 1 — Power law.

b. Model 2 — Modified power law.

c. Model 3 — Base exponential.

d. Model 4 — Exponential polynomial.

e. Model 5 — Exponential rational.

f. Model 6 — Non-parametric regression.
Results: \( p_d = f(k, \phi, S_{wi}) \) — Model 1 (2/4)

Model: 1

\[
p_d = 751.3360\phi^{0.8460} k^{-0.5166} (1 - S_{wi})^{0.0489}
\]

Comment:
Simple, compact, and accurate.

<table>
<thead>
<tr>
<th>Statistical Variable</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>1.2239 ln(psia)^2</td>
</tr>
<tr>
<td>Variance</td>
<td>113392.3297 psia^2</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>336.7378 psia</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>22.2482 percent</td>
</tr>
</tbody>
</table>
Results: $p_d = f(k, \phi, S_{wi})$ — Model 5

$\text{Model: 5}$

$$p_d = \exp\left[\frac{6.5325 + 0.8603 \ln \phi - 0.2141 \ln k}{1 - 0.0002 \ln \phi + 0.0519 \ln k} + 1.1769 \ln(1 - S_{wi}) + 0.1737 \ln \phi \ln k + 0.0029 \ln \phi \ln(1 - S_{wi}) - 0.0305 \ln k \ln(1 - S_{wi}) + 0.1971 \ln k \ln \phi \ln(1 - S_{wi}) - 0.8402 \ln(1 - S_{wi}) + 0.02647 \ln \phi \ln(1 - S_{wi}) - 0.4981 \ln \phi \ln(1 - S_{wi}) + 0.0027 \ln k \ln(1 - S_{wi}) + 0.0084 \ln k \ln \phi \ln(1 - S_{wi})}{1 - 0.0002 \ln \phi + 0.0519 \ln k - 0.8402 \ln(1 - S_{wi}) + 0.02647 \ln \phi \ln(1 - S_{wi}) - 0.4981 \ln \phi \ln(1 - S_{wi}) + 0.0027 \ln k \ln(1 - S_{wi}) + 0.0084 \ln k \ln \phi \ln(1 - S_{wi})}\right]$$

$\text{Comment:}$

"Rational-Linear" exponential polynomial form — highest accuracy.

<table>
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<tr>
<th>Statistical Variable</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>0.5978 ln(psia)$^2$</td>
</tr>
<tr>
<td>Variance</td>
<td>115533.5473 psia$^2$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>339.9023 psia</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>14.9016 percent</td>
</tr>
</tbody>
</table>
Results: $p_d = f(k, \phi, S_{wi})$ — Model 6

Model: 6 (Non-Parametric Regression for $p_d$)

There is no model for non-parametric regression

Comment:

The non-parametric correlation is the optimal fit of the data -- this is not a "model-based" regression, but rather, a statistical correlation of all variables use a non-parametric algorithm. Any (model-based) regression that achieves a better regression than the non-parametric regression algorithm has fitted the errors in the data.

The variables for this case were regressed using the natural logarithm of each variable.

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<th>Statistical Variable</th>
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</thead>
<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>0.5401 ln(psia)$^2$</td>
</tr>
<tr>
<td>Variance</td>
<td>124402.6310 psia$^2$</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>352.7076 psia</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>13.9832 percent</td>
</tr>
</tbody>
</table>
Summary: $\lambda = f(k, \phi, S_{wi}, p_d)$

1. Model 1 — Power law.
2. Model 2 — Modified power law.
3. Model 3 — Base exponential.
4. Model 4 — Exponential polynomial.
5. Model 5 — Exponential rational.

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Results: $\lambda = f(k, \phi, S_{wi}, p_d) — Model 1$ (2/4)

Model: 1

$$\lambda = 0.00084k^{0.5498}\phi^{-1.0485}(1-S_{wi})^{-2.2790}p_d^{0.9939}$$

Comment:

Simple, compact, and accurate.

Statistical Variable | Value
--- | ---
Sum of Squared Residuals | 1.0262
Variance | 0.1943
Standard Deviation | 0.4408
Average Absolute Error | 18.9111 percent
Results: \( \lambda = f(k, \phi, S_{wi}, p_d) \) — Model 5

\[ \lambda = \exp \left( 0.1510 - 0.0184 \ln p_d - 0.0009 \ln k + 0.0285 \ln(1 - S_{wi}) + 0.0301 \ln \phi - 0.0011 \ln p_d \ln k + 0.0573 \ln p_d \ln(1 - S_{wi}) + 0.0003 \ln p_d \ln \phi + 0.0312 \ln k \ln \phi + 0.0074 \ln k \ln(1 - S_{wi}) + 0.1425 \ln \phi \ln(1 - S_{wi}) + 0.01620 \ln \phi \ln(1 - S_{wi}) \ln k - 0.0007 \ln \phi \ln(1 - S_{wi}) \ln p_d + 0.0005 \ln \phi \ln k \ln p_d + 0.0848 \ln(1 - S_{wi}) \ln k \ln p_d - 0.0038 \ln(1 - S_{wi}) \ln k \ln p_d \ln \phi - 0.1925 \ln p_d - 0.0656 \ln k - 0.0934 \ln(1 - S_{wi}) + 0.4324 \ln \phi - 0.0012 \ln p_d \ln k + 0.1501 \ln p_d \ln(1 - S_{wi}) - 0.0834 \ln p_d \ln \phi + 0.2989 \ln k \ln \phi - 0.0274 \ln k \ln(1 - S_{wi}) + 0.2394 \ln \phi \ln(1 - S_{wi}) - 0.0551 \ln \phi \ln(1 - S_{wi}) \ln k - 0.0585 \ln \phi \ln(1 - S_{wi}) \ln p_d - 0.0013 \ln \phi \ln k \ln p_d + 0.2336 \ln(1 - S_{wi}) \ln k \ln p_d - 0.0467 \ln(1 - S_{wi}) \ln k \ln p_d \ln \phi \right) \]

Comment:

VERY complex "exponential" model, significant improvement in accuracy (best case).

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<tr>
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</tr>
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<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>0.6350</td>
</tr>
<tr>
<td>Variance</td>
<td>0.2416</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.4916</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>13.6761 percent</td>
</tr>
</tbody>
</table>
Results: $\lambda = f(k, \phi, S_{wi}, p_d)$ — Model 6

Model: 6 (Non-Parametric Regression for $\lambda$)

There is no model for non-parametric regression

Comment:
The non-parametric correlation is the optimal fit of the data -- this is not a "model-based" regression, but rather, a statistical correlation of all variables use a non-parametric algorithm. Any (model-based) regression that achieves a better regression than the non-parametric regression algorithm has fitted the errors in the data.
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<tbody>
<tr>
<td>Sum of Squared Residuals</td>
<td>0.7305</td>
</tr>
<tr>
<td>Variance</td>
<td>0.1766</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.4202</td>
</tr>
<tr>
<td>Average Absolute Error</td>
<td>15.2642 percent</td>
</tr>
</tbody>
</table>
Comparison with Timur Permeability Model  (1/2)

Timur Permeability Relation:

\[ k_{Timur} = A \frac{\phi^B}{S_{wi}^C} \]

- \( k_{Timur} \) = permeability, md
- \( \phi \) = porosity, fraction of pore volume
- \( S_{wi} \) = irreducible wetting phase saturation, fraction of pore volume

- Timur correlation based on 155 sandstone samples from three different oil fields from North America.
- Used both the highest correlation coefficient and the lowest standard deviation to define correlation.
Comparison with Timur Permeability Model (2/2)

**Modified Model**

\[
\begin{align*}
    k &= a_1(p_d)^{a_2} \lambda^{a_3} (1 - S_{wi})^{a_4} \phi^{a_5} \\
    p_d &= b_1 \phi^{b_2} k^{b_3} (1 - S_{wi})^{b_4} \\
    \lambda &= c_1 \phi^{c_2} k^{c_3} (1 - S_{wi})^{c_4} p_d^{c_5}
\end{align*}
\]

**Algebra!!**

\[
k = \alpha \phi^{\beta} (1 - S_{wi})^{\delta}
\]

\[
\begin{align*}
    \alpha &= \left[ a_1 b_1^{a_2} + a_3 c_5 c_1^{a_3} \right]^{-1} \\
    \beta &= \frac{(a_2 b_4 + a_3 c_4 + a_3 b_4 c_5 + a_4)}{1 - (a_2 b_3 + a_3 c_3 + a_3 b_3 c_5)} \\
    \delta &= \frac{(a_2 b_4 + a_3 c_4 + a_3 b_4 c_5 + a_4)}{1 - (a_2 b_3 + a_3 c_3 + a_3 b_3 c_5)}
\end{align*}
\]
Brooks/Corey/Purcell/Burdine Permeability Relation:

\[ k = 10.66 \alpha (\sigma_{Hg-air} \cos \theta)^2 (1 - S_{wi})^4 \phi^2 \frac{1}{p_d^2} \left[ \frac{\lambda}{\lambda + 2} \right] \]

The \( S_{wi}, p_d, \lambda \) parameters are derived from the *empirical* "Brooks/Corey" \( p_c \)-relation:

\[ p_c = p_d \left[ \frac{S_w - S_{wi}}{1-S_{wi}} \right]^{-\frac{1}{\lambda}} \]
Power Law Models: (general form)

\[ k = a_1 \frac{1}{(p_d)^{a_2}} \left[ \frac{\lambda}{\lambda + 2} \right]^{a_3} (1 - S_{wi})^{a_4} \phi^{a_5} \]

\[ p_d = b_1 \phi^{b_2} k^{b_3} (1 - S_{wi})^{b_4} \]

\[ \lambda = c_1 k^{c_2} \phi^{c_3} (1 - S_{wi})^{c_4} p_d^{c_5} \]

Excellent correlation of permeability results for different lithologies, pore systems, and pore structures.
More Complex Models:

- Better correlation of permeability, displacement pressure, and index of pore-size distribution results.
  - Modified power-law models
  - Exponential models
  - Complex (rational) exponential models

- Be careful about "over-fitting" data with excessively complex data models.
Advantages and Limitations — IPTC 10994

- **Advantages of this Work:**
  - Valid for different lithologies.
  - Can be used for cores, cuttings, etc. …
  - Easy to use these correlations.

- **Limitations of this Work:**
  - Sensitive to methods used to obtain the capillary pressure data — this work only uses Hg $p_c$ data.
  - Bimodal pore throat distributions are not fit by Brooks and Corey $p_c$ model (or at least the current form of the Brooks and Corey $p_c$ model).
Future Work — IPTC 10994

Recommendations for future work:

- Include more data samples.
- Include more data from carbonate systems.
- Extension of the results of this work to liquid-liquid and gas-liquid systems.
- A generalized correlation for flow in porous materials — including soils, filters, sintered metals, bead packs, and porous rocks?
A Modified Purcell/Burdine Model for Estimating Absolute Permeability from Mercury-Injection Capillary Pressure Data

End of Presentation

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FPQ: (Frequently Pondered Questions)

Q1. What is the most important problem in reservoir characterization?
A1. \( k = f(\text{position}) \).

Q2. Use of \( k = f(H_g \ p_c) \)?
A2. \( k \) from cuttings,
\[ p_c = f(k, \phi, S_{wi}) \],
"universal" J-function ...

Q3. Can reservoir scaling be quantified?
A3. ?

Q4. Is \( k = f(\phi, S_{wi}, \text{etc}) \) general?
A4. Probably ... Possibly ... Hopefully ... Maybe ... Maybe Not ... Not.

"All strangers are relations to each other."
Arabian Proverb
References: Hg $p_c$ Data and Permeability Models

Mercury Capillary Pressure: Data

Mercury Capillary Pressure: Permeability Models