Improved Permeability-Prediction Relations for in Low Permeability Sands


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Presentation Outline:

● Objectives
● Concepts:
  ■ Gas Slippage Phenomenon.
  ■ Correlations of the Gas Slippage Factor and Petrophysical Data.
● Existing correlations:
  ■ Heid et al, Jones-Owens
  ■ Sampath-Keighin, Square-Root
● Klinkenberg revisited:
  ■ Concept
  ■ Unified Flow Model
  ■ Correlation of the Knudsen Number
  ■ Application using Field Data
● Summary, Issues and Challenges
● Future Work
Objectives:

- Overall objectives of this work:
  - *(Initial)* To validate/enhance the correlation proposed by Jones and Owens for estimating the gas slippage factor.
    - Use correlation to "correct" single-point, steady-state permeability measurements
    - Also, consider other correlations/models.
  - *(Final)* To consider/develop other models or correlations to estimate the absolute (liquid) permeability directly from gas permeability measurements. That is, to create a rigorous flow model to relate gas and liquid permeabilities.
Gas Slippage Phenomenon:

- "Gas slippage" occurs at laboratory conditions when the mean free path ($\lambda$) of the gas molecules is "not negligible" compared to effective pore throat radius: the gas molecules "slip" on the surfaces of the porous media.

\[ v_x(y = 0) = v_0 \text{, with } v_0 = c\lambda \left[ \frac{dv}{dy} \right]_{y=0} \]


- Ignoring the gas slippage effect lead to an overestimation of the gas flowrate and the core permeability (Darcy's law).
Gas Slippage Phenomenon:

- The "gas slippage" effect for porous media was first documented by Klinkenberg. The gas permeability tends to a limiting value at an infinite mean pressure — referred to as equivalent liquid permeability or Klinkenberg-corrected permeability.

\[
k_a = k_\infty \left[ 1 + \frac{b_K}{p_m} \right]
\]

Objective:
Rapidly estimate $k_\infty$ using a pair of $(k_a, 1/p_m)$ data.

Principle:
- Use a correlation which relates $b_K$, $k_\infty$, and other petrophysical data with the form:
  $$b_K = f(k_\infty, \phi...)$$
- Substituting this form in the Klinkenberg equation:
  $$k_a = k_\infty \left[ 1 + \frac{f(k_\infty, \phi...)}{p_m} \right] \Rightarrow k_a - k_\infty \left[ 1 + \frac{f(k_\infty, \phi...)}{p_m} \right] = 0$$
- Two type of correlations are reported in the literature:
  - $b_K$ vs. $k_\infty$ (Heid et al, Jones and Owens)
  - $b_K$ vs. $k_\infty/d$ (Sampath-Keighin, square-root (this work))
Correlations: $b_K$ vs. $k_\infty$

- Using about 120 core samples — in 1950, Heid, et al. developed the following equation to relate the Klinkenberg gas slippage factor ($b_K$) and the equivalent liquid permeability ($k_\infty$):

$$b_K = 11.419 (k_\infty)^{-0.39}$$


- Prior to 1970, most of the experimental studies to address the "Klinkenberg" effects were conducted with relatively high permeability cores (>10md). Jones and Owens based later their study on low permeability cores and came out with the following equation:

$$b_K = 12.639 (k_\infty)^{-0.33}$$

Correlations: $b_K$ vs. $k_\infty / \phi$

Based on 10 tight sand core samples, the Sampath and Keighin correlation relates $b_K$ to $k_\infty / \phi$:

$$b_K = 13.851 \left[ \frac{k_\infty}{\phi} \right]^{-0.53}$$


**Discussion:** Sampath-Keighin

- The correlation is acceptable.
- Relatively small database.
The later formulation is interesting as it is close to the theoretical definition of a capillary radius \( r \) as a function of the square-root of \( (k_\infty/\phi) \):

\[
r = 8.886 \times 10^{-6} \sqrt{k_\infty / \phi}
\]

Klinkenberg defines \( b_K \) by:

\[
\frac{b_K}{p_m} \approx \frac{4 \lambda_m}{r} \quad \lambda_m = \text{mean free path of the gas molecules at the mean pressure } p_m
\]

It is possible to derive rigorously a correlation between \( b_K \) and the square-root of \( (k_\infty/\phi) \) using the previous definitions and the classical definition of the mean free path of the gas molecules:

\[
\lambda(p,T) = \sqrt{\pi / 2} \frac{1}{p} \mu \sqrt{\frac{RT}{M}} \quad [\mu \equiv \mu(p,T)]
\]

This "square-root" correlation has the following form:

\[
b_K = \beta \left[ \frac{k}{\phi} \right]^{-0.5} \quad \beta = \text{parameter (intercept) depending on the type of gas used for the core flow experiment — usually nitrogen, for which } \beta = 43.345
\]
**Correlations: Comparison on Example Case**

**Comparison of Steady-State Data — $b_K$ Versus $k_\infty$**

Heid *et al.* model:

$$b_K = 11.419(k_\infty)^{-0.39}$$

Jones-Owen Model:

$$b_K = 12.639(k_\infty)^{-0.33}$$

**Discussion: "$b_K$ vs. $k_\infty$" Correlations — Lower Cotton Valley Formation**

- The Heid *et al.* correlation gives better results for these datasets, whereas the Jones-Owen model generally underestimates the gas slippage factor.

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**Correlations: Comparison on Example Case**

Comparison of Legacy and Modern Data — $b_K$ Versus $k_\infty/\phi$

(Sampath-Keighin and Square-Root Correlations are Superimposed for Emphasis)

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Discussion: "$b_K$ vs. $k_\infty/\phi$" Correlations — Lower Cotton Valley Formation

- **Square-root correlations:** CO$_2$ trend is better than N$_2$ (best-fit: $\beta_{fit} = 34.17$).
- **The Sampath-Keighin underestimates** $b_K$.

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Discussion: Lower Cotton Valley Formation No. 1

- The Jones-Owens and Sampath-Keighin models overestimate the permeability; the square-root model underestimates $k_\infty$. 
Discussion: Lower Cotton Valley Formation No. 2

- The Jones-Owens/Sampath-Keighin correlations — good for $k_\infty > 0.001$ md.
- The square-root model gives good results for low permeabilities.
In both cases (Cotton Valley samples 1 and 2), the theoretical square-root model (nitrogen curve) underestimates the Klinkenberg-corrected permeability — but the overall performance of the square-root relation is comparable to the other methods.

While it is difficult to discriminate the "best" correlations, based on our results, we believe that the Sampath-Keighin correlation and the generalized square-root correlations are more "consistent" with the theory — and as such, should be favored for this application.
Microflow model:

**Concept:**

- The gas slippage phenomenon occurs as a subset of a larger area of study known as the *theory of rarefied gases* — this theory experienced a substantial growth in the last 50 years (aeronautics, aerospace, MEMS...)

- The first order gas slippage correction proposed by Klinkenberg is based on the results of this theory available *in 1941*:

  \[
  k_a = k_\infty \left[ 1 + \frac{b_K}{p_m} \right] = k_\infty \left[ 1 + \frac{4c\lambda_m}{r} \right], \text{ with } c \approx 1
  \]

- The theory of rarefied gases describes the flow of a gas at very low pressures, or in small micro-channels. The flow regime for a gas flowing in a micro-channel is typically determined by the value of the Knudsen number \((Kn)\):

  \[
  Kn = \frac{\lambda}{l_{\text{char}}}
  \]

  Where \(\lambda\) is the mean free path of the gas molecules (i.e., the average distance (length) between 2 consecutive molecular interactions), and \(l_{\text{char}}\) is the characteristic length of the flow geometry (e.g., channel height, pipe radius).
Limits of the different flow regimes, as a function of the length characteristic of the geometry, \( I_{\text{char}} \), and the reciprocal mean free path normalized at atmospheric conditions and 300 K. The lines defining the various Knudsen number regimes are based on air at isothermal conditions (Modified from Karniadakis and Beskok, 2002).
Microflow model:

■ Unified Flow Model:

- In 2002, Karniadakis and Beskok developed a "unified flow model" for gas flows in micro-pipes, valid over the entire range of flow regimes: (for a micro-pipe of radius \( r \) and length \( L \))

\[
q = \frac{\pi}{8} \frac{r^4}{\mu} \left[ 1 + \alpha(Kn)Kn \right] \left[ 1 + \frac{4Kn}{1 + Kn} \right] \frac{\Delta p}{L}, \text{ where } \alpha(Kn) = \frac{128}{15\pi^2} \tan^{-1}\left[ 4Kn^{0.4} \right]
\]


- By analogy with Klinkenberg's derivation using Poiseuille equation (corrected for slippage), we derive a microflow model:

\[
k_a = k_\infty \left[ 1 + \frac{128}{15\pi^2} \tan^{-1}\left[ 4Kn^{0.4} \right] Kn \right] \left[ 1 + \frac{4Kn}{1 + Kn} \right]
\]

Where \([1 + \alpha(Kn)Kn]\) is defined as the rarefaction coefficient, accounting for the change of "density" of the gas.
Correlation of the Knudsen Number ($Kn$):

- The Knudsen number cannot be measured by direct laboratory methods — we need to define a "pseudo"-Knudsen number which is a function of the measurable parameters ($p_m$, $\phi$, $k_a$...)

- Assuming the isothermal flow of an ideal gas, $Kn$ is inversely proportional to $p_m$ and $l_{char}$ (typically, the pore throat radius, or a function of $k_\infty$ and $\phi$)

- When $k_a$ and $k_\infty$ are known, the "pseudo"-Knudsen number is computed by solving the following equation:

$$k_a - k_\infty \left[1 + \frac{128}{15\pi^2} \tan^{-1} \left[ 4Kn_p^{0.4} \right] Kn_p \right] \left[1 + \frac{4Kn_p}{1 + Kn_p} \right] = 0$$
Microflow model:

Application using Field Data:

- To demonstrate the application of the "pseudo"-Knudsen number \((Kn_p)\), we only use the Lower Cotton Valley data (No. 2): the pseudo-Knudsen numbers \((Kn_p)\) can be related to the available petrophysical data (\(k_a\), \(k_\infty\), \(p\) and \(\phi\)), yielding two relations:

  — Relation 1: \[
  Kn_p = 0.4 \frac{1}{p_m} k_a^{-0.598} \phi^{-0.352}
  \]

  — Relation 2: \[
  Kn_p = 2.62 \frac{1}{p_m} k_\infty^{-0.553} \phi^{0.3897}
  \]

- These two relation can be applied to the microflow model, yielding to an explicit equation (with Relation 1) or an implicit equation (with Relation 2) that can be solved for \(k_\infty\):

  \[
  k_a - k_\infty \left[ 1 + \frac{128}{15\pi^2} \tan^{-1} \left[ 4Kn_p^{0.4} \right] Kn_p \right] \left[ 1 + \frac{4Kn_p}{1 + Kn_p} \right] = 0
  \]
**Discussion: Implicit and Explicit models**

- The two models achieve a fairly good correlation — the implicit model seems to be more accurate.
"Fully implicit" formulation:

- The Klinkenberg-corrected permeability may not be available in practice. The process of solving simultaneously for $Kn_p$ and $k_\infty$ requires an implicit formulation with the form:

$$Kn_p = a_0 \frac{1}{p_m} k_\infty^{a_1} \phi^{a_2}$$

- The coefficients $a_0$, $a_1$ and $a_2$ have to be determined simultaneously with the estimation of $k_\infty$. This can be handled (with sufficient quantity of data) by using non-linear regression methods on the following equation:

$$\frac{k_a}{k_\infty} = \left[1 + \frac{128}{15\pi^2} \tan^{-1}\left[4 \left[a_0 \frac{1}{p_m} k_\infty^{a_1} \phi^{a_2}\right]^{0.4}\right]\right]\left[1 + \frac{4}{1 + 1/\left[a_0 \frac{1}{p_m} k_\infty^{a_1} \phi^{a_2}\right]}\right]$$

- The coefficients ($a_i$) computed are not "universal" and must be computed for each datasets.
Microflow model: Field Data — Results

Computed Equivalent Liquid Permeability versus Klinkenberg-Corrected Permeability — Lower Cotton Valley No. 2

Legend:
- "Fully Implicit" Model

$k_{\infty, \text{computed}} = k_{\infty, \text{measured}}$

"Fully Implicit" Model:

$$Kn_p = 0.93 \frac{1}{p_m} k_a^{-0.490} \phi^{0.130}$$

Discussion: "Fully implicit" formulation

- The fully implicit formulation achieves a good match for this data set.

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Summary, Issues, and Challenges:

**Summary:**
— The Jones-Owens, Sampath-Keighin and square-root correlations can be used to estimate the gas slippage factor \( b_K \) for the typical "Klinkenberg" correction.
— Our new theoretical model (i.e., the "microflow" model) has been developed and partially validated for the direct estimation of permeability. We note that the microflow model includes a second order correction for gas slippage.

**Issues and Challenges:**
— Continued validation of the new microflow model.
— Concern regarding the Klinkenberg model is reiterated — particular as the focus shifts to low/ultra low permeabilities.
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Comparison with the "classical" correlations:

Absolute Relative Error on Computed Equivalent Liquid Permeability versus Mean Pressure.
Lower Cotton Valley No.2. Sample 1-5. Reference Klinkenberg-corrected permeability ($k_r$) = 0.000044 md

Legend:
- Correlations —
  - Heid et al (1950)
  - Jones-Owens (1979)
  - Sampath-Keighin (1981)
  - Square-root (— this work)
- Microflow Models —
  - Calibration on dataset No.1 only:
    - Explicit Model
    - Implicit Model
  - Calibration on dataset No.2 only:
    - Explicit Model
    - Implicit Model
  - Calibration on datasets Nos.1 and 2:
    - Explicit Model
    - Implicit Model

**Discussion:** Comparison — $k_\infty = 0.000044$ md

Better results achieved with the microflow model

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