Please provide a self-evaluation of the course competencies by addressing the questions given below.

<table>
<thead>
<tr>
<th>Competency</th>
<th>Not At All</th>
<th>Not Well</th>
<th>Adequate</th>
<th>Well with Effort</th>
<th>Easily</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. I can explain the relationships between porosity and permeability; and how these properties influence the flow of fluids in reservoir rocks.</td>
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<td>2. I can use correlations and laboratory data to estimate the properties of reservoir fluids which are relevant for reservoir engineering — analysis and modeling.</td>
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<td>3. I can sketch a plot of pressure versus logarithm of radius and identify the primary flow regimes (i.e., transient radial flow, pseudosteady-state flow, and steady-state flow behavior)</td>
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<td>4. I can derive and apply the material balance relation for a slightly compressible liquid (oil) system and the material balance relation for a dry gas system.</td>
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<td>5. I can derive and apply the steady-state flow equations for horizontal linear and radial flow of liquids and gases, including the pseudopressure and pressure-squared forms.</td>
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<td>6. I can derive and apply the pseudosteady-state flow equations for the &quot;black oil&quot; and &quot;dry gas&quot; reservoir systems (&quot;black oil&quot; → pressure form, &quot;dry gas&quot; → pseudopressure form).</td>
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<td>7. I can derive and apply the &quot;skin factor&quot; concept derived from steady-state flow to represent damage or stimulation (including the apparent wellbore radius concept).</td>
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<td>8. I am familiar with and can derive the diffusivity equations for liquids and gases — and I am aware of the assumptions, limitations, and applications of these relations.</td>
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<td>9. I am familiar with and can use of dimensionless variables and dimensionless solutions to provide a generic mathematical representation for a particular reservoir model.</td>
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<td>10. I am familiar with and can use the concepts of temporal (time) and spatial superposition — time superposition is used for variable rate/pressure problems; spatial superposition is used to generate reservoir boundary configurations (faults, closed boundaries, etc.).</td>
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<tr>
<td>11. Well Test Analysis — Conventional Plots</td>
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<tr>
<td>For well test data, I can construct, interpret, and analyze &quot;conventional plots&quot; as follows:</td>
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<tr>
<td>a. Pressure versus time to establish the parameters related to wellbore storage (domination behavior) (i.e., the &quot;early time&quot; plot).</td>
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<td>b. Pressure versus the logarithm of time (pressure drawdown case) or versus the logarithm of superposition time (e.g., Horner Time for the pressure buildup case) to establish the parameters related to radial flow behavior (i.e., the &quot;semilog&quot; plot).</td>
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<td>c. The logarithm of pressure drop and pressure drop derivative versus the logarithm of time (or an appropriate superposition time function) to establish the parameters for wellbore storage, radial flow, and vertical fracture behavior (i.e., the &quot;log-log&quot; plot).</td>
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<td>12. Well Test Analysis — Type Curve/Model-Based Analysis</td>
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<td>For well test data, I can use a static type curve solution or a graphical model presentation from a software package to analyze well test data obtained from:</td>
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<td>a. An unfractured well which includes wellbore storage distortion and radial flow behavior (including damage/stimulation (i.e., skin effects)).</td>
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<td>b. A vertically fractured well (finite or infinite fracture conductivity cases) which includes wellbore storage distortion, fracture flow regimes, and radial flow behavior.</td>
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<td>c. A well test performed in a reservoir with closed boundaries or sealing faults.</td>
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<td>d. A well test performed in a &quot;dual porosity&quot; or &quot;naturally fractured&quot; reservoir system.</td>
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<td>13. Production Data Analysis</td>
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<td>I can analyze, interpret, model, and forecast well production performance as follows:</td>
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<tr>
<td>a. Estimate the &quot;absolute open flow&quot; from a gas well &quot;deliverability&quot; test.</td>
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<td>b. Develop and use an Inflow Performance Relation (IPR) which uses flowrate, wellbore pressure, and aver-age reservoir pressure data to create an interpretative/predictive relation.</td>
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<td>c. Estimate the &quot;reserves&quot; for an oil or gas well using plots of rate versus time (semilog rate format) and rate versus cumulative production.</td>
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<td>d. Use decline type curves (or an equivalent software-based tool) to analyze production data from an unfrac-tured or hydraulically fractured oil or gas well.</td>
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<td>e. Provide a forecast of future rate or pressure performance of an oil or gas well using empirical methods (hand/software) and analytical/numerical models (software).</td>
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</tbody>
</table>
Well Deliverability:

- The first efforts to analyze well performance were an attempt to quantify well potential — not to estimate reservoir properties.

- The original well deliverability relation was completely empirical (derived from observations), and is given as:

\[ q = C\left(\bar{p}^2 - p_{wf}^2\right)^n \]

- This relationship is rigorous for low pressure gas reservoirs, \( n=1 \) for laminar flow.
Discussion: Well Deliverability (4-point test)

- Probably oldest "reservoir engineering" technique.
- Assumption of pseudosteady-state flow is the weakest link in analysis.
- Does not directly relate time, rate, and pressure performance.

\[ q = C (\bar{p}^2 - p_{wf}^2)^n \]
Discussion: Unfractured oil well (SPE 12777)

- This result is an excellent match of all functions.
- $\beta$-derivative function is an excellent diagnostic for the wellbore storage and transition flow regimes.
A Field Case — Hydraulically Fractured Wells

Type Curve Analysis — SPE 9975 Well 5 (Buildup Case)
(Well with Infinite Conductivity Hydraulic Fractured)

Reservoir and Fluid Properties:
- \( r_w = 0.33 \text{ ft}, h = 30 \text{ ft}, \)
- \( c_t = 6.37 \times 10^{-5} \text{ psi}^{-1}, \phi = 0.05 \text{ (fraction)} \)
- \( \mu_{gi} = 0.0297 \text{ cp}, B_{gi} = 0.5755 \text{ RB/Mscf} \)

Production Parameters:
- \( q_{ref} = 1500 \text{ Mscf/D} \)

Match Results and Parameter Estimates:
- \[ \frac{p_D/\Delta p}_{\text{match}} = 0.000021 \psi^{-1}, C_{Dr} = 0.01 \text{ (dim-less)} \]
- \[ \frac{(t_{Dxf}/C_{Dxf})_{\text{match}}}{t} = 0.15 \text{ hours}^{-1}, k = 0.0253 \text{ md} \]
- \( C_{rd} = 1000 \text{ (dim-less)}, x_f = 279.96 \text{ ft} \)

Discussion: Fractured gas well: buildup test (SPE 9975 — Well 5)
- Wellbore storage effects (\( p_{D\beta_d} = 1 \)).
- Linear flow regime could be diagnosed clearly (\( p_{D\beta_d} = 1/2 \)) — very good match.
**Pressure Transient Analysis (Tight Gas)**

**Production Data Analysis Plot for East Texas Gas Well**
"Summary" History Plot — Rate and Pressure Functions

- Good rate and pressure histories.
- Unique case — bottomhole and "production" pressure data.
"Blasingame" Plot: Production Analysis — East TX Gas Well

- Used surface pressure measurements converted to $p_{wf}$
- Minor issues with early-time analysis, good match of performance.
Discussion: "Log-Log" Plot (Well Test Analysis) — East TX Gas Well

- Used high frequency bottomhole pressure measurements ($p_{ws}$).
- Consistent match of bottomhole and "production" pressure data.
The unexamined life is not worth living.
— Socrates (399 B.C.)

**Topic:** Solutions to the Radial Flow Diffusivity Equation

**Note:** The 2nd Edition of the Lee text, *Well Testing*, are available at the Copy Center in the Reed McDonald Building. Approximate cost is $50—this text is optional.

**Objectives:** (things you should know and/or be able to do)

- Be able to recognize and apply the following solutions for an unfractured well produced at a constant flow rate in a homogeneous reservoir (with no skin or wellbore storage effects) for the following outer boundary conditions:
  - The dimensionless diffusivity equation for radial flow is given as
    \[
    \frac{\partial^2 P_D}{\partial r^2} + \frac{1}{r} \frac{\partial P_D}{\partial r} = \frac{\partial P_D}{\partial t_D} \tag{Fundamental identity}
    \]
  - "Infinite-Acting" Reservoir Case: (line source solution)
    \[
    P_D(t_D, r_D) = \frac{1}{2} \pi \left( \frac{r_D^2}{4t_D} \right) \text{ for } t_D \geq 10 \text{ (the Exponential Integral Solution)}
    \]
  - Note that $E_1(-x) = \text{Ei}(-x)$ — recall that the Lee texts use the -Ei(-x) form.
  - The "well testing" derivative function, $P_D' = d/dt [P_D(r_D, t_D)]$ is given by
    \[
    P_D'(r_D, t_D) = \frac{1}{2} \exp \left[ -\frac{r_D^2}{4t_D} \right]
    \]
  - The so-called "log approximation" of the Exponential Integral solution is:
    \[
    P_D(t_D, r_D) = \frac{1}{2} \ln \left( \frac{4t_D}{e^2 r_D^2} \right) \text{ for } t_D \geq 25 \text{ (} \approx 0.577216\ldots \text{ Euler's Constant)}
    \]
  - The "well testing" derivative function for the "log approximation" is given by
    \[
    P_D'(r_D, t_D) = \frac{1}{t_D} \exp \left[ -\frac{r_D^2}{4t_D} \right]
    \]
  - "No-Flow" Outer Boundary: ("no-flow" at the outer boundary)
    \[
    P_D(t_D, r_D, r_D) = \frac{1}{2} \pi \left[ \frac{r_D^2}{4t_D} \right] + 2t_D \exp \left[ -\frac{r_D^2}{2t_D} \right] + \frac{1}{2} \exp \left[ -\frac{r_D^2}{4t_D} \right]
    \]
  - The "well testing" derivative function, $P_D' = d/dt [P_D(r_D, t_D)]$ is given by
    \[
    P_D'(r_D, t_D, r_D) = \frac{1}{t_D} \exp \left[ -\frac{r_D^2}{4t_D} \right] + 2t_D \exp \left[ -\frac{r_D^2}{2t_D} \right] + \frac{1}{2t_D} \left( \frac{r_D^2}{4} + \frac{r_D^2}{8} \right) \exp \left[ -\frac{r_D^2}{4t_D} \right]
    \]
  - Constant Pressure Outer Boundary: "constant pressure" at the outer boundary
    \[
    P_D(t_D, r_D, r_D) = \frac{1}{2} \pi \left[ \frac{r_D^2}{4t_D} \right] + \frac{1}{8t_D} \left( r_D^2 - r_D^2 \right) \exp \left[ -\frac{r_D^2}{4t_D} \right]
    \]
  - The "well testing" derivative function, $P_D' = d/dt [P_D(r_D, t_D)]$ is given by
    \[
    P_D'(r_D, t_D, r_D) = \frac{1}{t_D} \exp \left[ -\frac{r_D^2}{4t_D} \right] + \frac{1}{8t_D} \left( r_D^2 - r_D^2 \right) \exp \left[ -\frac{r_D^2}{4t_D} \right]
    \]

- Using the conversion factors given below, you should be able to "convert" the dimensionless solutions to Darcy, SI, or "field" units as necessary. The dimensionless variables and the appropriate conversions are given below.

<table>
<thead>
<tr>
<th>Dimensionless Radius $r_D$</th>
<th>Dimensionless Time $t_D$</th>
<th>Dimensionless Pressure $p_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_D = r/r_w$</td>
<td>$t_D = t_0 - \frac{kt}{\phi \mu c r_w^2}$</td>
<td>$p_D = p_{De} - \frac{kh}{qB} (p_i - p_r)$</td>
</tr>
</tbody>
</table>

**Conversion Constants for the $t_D$ and $p_D$ Functions**

<table>
<thead>
<tr>
<th>Constant $t_D$</th>
<th>Darcy Units $t_D$</th>
<th>SI Units $t_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_D$</td>
<td>1</td>
<td>2.637x10^{-4}</td>
</tr>
<tr>
<td>$P_D$</td>
<td>2π</td>
<td>7.081x10^{-3}</td>
</tr>
<tr>
<td>$P_{De}$</td>
<td>1/(2π)</td>
<td>141.2</td>
</tr>
</tbody>
</table>

- Be able to develop the results given below for transient radial flow (infinite-acting homogeneous reservoir). Be able to show all steps and details.

**Exponential Integral solutions:**

- Reservoir Equation: (skin effects are NOT included)
  \[
  p_i - p_r = 7.06 \frac{qB}{kh} \ln \left( \frac{1}{1688} \frac{k}{\phi \mu c r_w^2} \right) + 2s
  \]
- Wellbore Equation: (skin effects are included)
  \[
  p_i - p_{wf} = 7.06 \frac{qB}{kh} \ln \left( \frac{1}{1688} \frac{k}{\phi \mu c r_w^2} \right) + 2s
  \]
- "Log approximation" solutions:
  - Reservoir Equation: (skin effects are NOT included)
    \[
    p_i - p_r = 7.06 \frac{qB}{kh} \ln \left( \frac{1}{1688} \frac{k}{\phi \mu c r_w^2} \right) + 2s
    \]
  - Wellbore Equation: (skin effects are included)
    \[
    p_i - p_{wf} = 7.06 \frac{qB}{kh} \ln \left( \frac{1}{1688} \frac{k}{\phi \mu c r_w^2} \right) + 2s
    \]
  - Wellbore Equation: (log₁₀ form [ln(x)=10log(x)], skin effects are included)
    \[
    p_i - p_{wf} = 162.6 \frac{qB}{kh} \ln \left( \frac{1}{1688} \frac{k}{\phi \mu c r_w^2} \right) + 0.8686s
    \]