A NEW METHOD TO ACCOUNT FOR PRODUCING TIME EFFECTS WHEN DRAWDOWN TYPE CURVES ARE USED TO ANALYZE PRESSURE BUILDUP AND OTHER TEST DATA

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INTRODUCTION

Type curves have appeared in the petroleum literature since 1970 to analyze pressure transient (pressure drawdown and pressure buildup) tests taken on both unfractured and fractured wells. The majority of type curves which have been developed and published to date were generated using data obtained from pressure drawdown solutions and obviously are most suited to analyze pressure drawdown tests. These drawdown type curves are also commonly used to analyze pressure buildup data. The application of drawdown type curves in analyzing pressure buildup data is not as bad as it may first appear. As long as the producing time, \( t_p \), prior to shut-in is sufficiently long compared to the shut-in time, \( \Delta t \) (that is \( (t_p + \Delta t)/t \approx 1 \)), for liquid systems, it is reasonable to analyze pressure buildup data using drawdown type curves. However, for cases where producing times prior to pressure buildup tests are of the same magnitude or only slightly larger than the shut-in times (that is, \((t_p + \Delta t)/t \gg 1\)), the drawdown type curves may not be used to analyze data from pressure buildup tests. The above requirement on the duration of producing times is the same for the conventional semi-log analysis. If pressure buildup data obtained after short producing times are to be analyzed, the Horner method is recommended over the MDH (Miller-Dyes-Hutchinson) method. The MDH method is generally used to analyze buildup data collected after long producing times, whereas the Horner method is used for those obtained after relatively short producing times. Although pressure buildup tests with short producing times may occur often under any situation, they are rather more common in the case of drill stem tests and pre-fracturing tests on low permeability gas wells.

Thus, there is a need for generating buildup type curves, which account for the effects of producing time. Some limited work has been done in this regard. McKinley has published type curves for analyzing buildup data for a radial flow system. However, his buildup type curves were generated on the assumption of long producing times; and these type curves are therefore very similar to drawdown
type curves and are obviously unsuitable for cases where producing times prior to shut-in are relatively short. Crawford, et al.\textsuperscript{12} pointed out the above limitations for McKinley type curves in analyzing pressure buildup data from the DST tests. They also presented buildup curves for short producing times. Since their curves deal with specific values of real producing times prior to shut-in, they are limited in scope and utility. Recently, the effect of producing time on analysis of pressure buildup data using drawdown type curves has been discussed by Raghavan.\textsuperscript{13} His study clearly points out the limitations of drawdown type curves for analyzing buildup data collected after small producing times. A family of buildup type curves is presented both for unfractured and fractured wells with producing time as a parameter. Although these type curves offer a definite advantage over the existing drawdown type curves, they are difficult to use because of the multiplicity of type curves. In a recent paper, Agarwal\textsuperscript{14} also discussed the limitations of using drawdown type curves for analyzing buildup data obtained after small producing times but no details were given. These limitations are discussed here in this paper. Recently Gringarten, et al.\textsuperscript{15}, presented drawdown type curves, plotted in a slightly different form, and suggested some guidelines regarding the portions of buildup data which may be analyzed by drawdown type curves. Although these guidelines may be useful in certain cases, the basic problem still remains.

To overcome the above-mentioned difficulties and to eliminate dependence on producing time, a new method has been developed. This method should provide a significant improvement over the current methods because (1) this permits us to account for the effects of producing time, and (2) data are normalized in such a fashion that instead of utilizing a family of type curves with producing time as a parameter, the existing drawdown type curves may be used. This concept appears to work for both unfractured and fractured wells. Wellbore storage effects with or without damage may also be taken into account provided that producing time prior to shut-in is long enough to be out of such wellbore effects.

This method has been extended to include analysis of data from two-rate tests\textsuperscript{8,16,17} and multiple rate tests\textsuperscript{8,17,18} by type curve methods. Although not shown, it appears to have a potential for applying type curve methods to other kinds of testing.

This new method, although originally conceived for type curve analysis of buildup data, is quite suitable for the conventional semi-log analysis. It is similar to the Horner method because it includes the effects of producing time, and may be used to determine formation flow capacity, skin factor and the initial reservoir pressure. However, it has an added advantage. It allows the plotting of pressure buildup data, with and without producing time effects, on the same time scale as the graph paper. This enables a better comparison of data using the MDH and Horner type graphs.

Although the new method will be developed using the solutions for liquid systems, its applicability to gas wells will also be indicated.

**Basis of Drawdown and Buildup Type Curves**

A type curve is a graphical representation of a mathematical solution (obtained analytically or numerically) for a specific flow type. The solution is normally plotted, in terms of dimensionless variables, on log-log graph paper. The graph thus prepared becomes the type curve for the specific flow problem with given inner and outer boundary conditions. Depending on the type of solution (drawdown or buildup), drawdown and buildup type curves are generated.

**Drawdown Type Curves**

As the name implies, these type curves are based on the drawdown solutions. The pressure drawdown solution for a well producing at a constant rate as a function of flowing time, \( t \), may be written as

\[
\frac{kh[p_i-p_{wf} \circ(t)]}{141.2 \, qB_u} = p_{BD} \circ (t_D)
\]

where,

\[
t_D = 2.634 \times 10^{-4} \, \frac{t}{\phi \mu \bar{C_i} \, \bar{r}_w^2}
\]

Eq. (1) is a general solution and is not meant to be restricted to any particular drainage shape or well location. The majority of the published type curves\textsuperscript{17,18} for both unfractured and fractured wells are based on pressure drawdown solutions for liquid systems. Examples of pressure drawdown type curves for unfractured wells are those presented by Agarwal, et al.\textsuperscript{1,} Earlougher and Kersch\textsuperscript{6} and Gringarten, et al.\textsuperscript{15}. In another publication Gringarten, et al.\textsuperscript{15} presented type curves for vertically fractured wells with infinite flow capacity and uniform flux fractures. Type curves for finite flow capacity fractures were provided by Cinco, et al.\textsuperscript{8} and Agarwal, et al.\textsuperscript{7}. Note regarding the use of above type curves for analyzing buildup data will be said later.

**Buildup Type Curves**

To obtain pressure buildup solutions, superposition may be applied in the normal manner to pressure drawdown solutions. This provides buildup pressures at shut-in times, \( \Delta t \) after a producing time, \( t_p \). Fig. 1 shows a schematic of pressure buildup behavior obtained following a constant rate drawdown for a production period, \( t_p \). Flowing pressures \( p_{WF}(t) \) are shown as a function of flowing time, \( t \) up to a production period, \( t_p \), when a buildup test is initiated. Buildup pressures, \( p_{BF} (t_p + \Delta t) \), are shown as a function of shut-in time, \( \Delta t \).\textsuperscript{7} Instead of taking a buildup test, if the well was allowed to produce beyond time, \( t_p \), flowing pressures as shown by \( p_{BF} (t_p + \Delta t) \) would have been obtained. Note that the flowing pressure at the end of the production period which is denoted by \( p_{WF}(t_p) \) is same as the buildup pressure at the instant of \( p_{BF} (t_p + \Delta t) \).
shut-in which is shown as $p_{ws}(\Delta t=0)$. Superposition when applied to drawdown solutions provides the following.

$$\frac{kh[p_{i}-p_{ws}(t+p+\Delta t)]}{141.2 \text{ qBu}} = \frac{p_{wd}[(t+\Delta t)_{D}]-p_{wd}[(\Delta t)_{D}]}{141.2 \text{ qBu}}$$

The flowing pressure, $p_{wf}(t)$ at the end of producing period, $t_{p}$ is given by

$$\frac{kh[p_{i}-p_{wf}(t)]}{141.2 \text{ qBu}} = \frac{p_{wd}[(t)_{D}]}{141.2 \text{ qBu}}$$

(4)

Subtracting Eq. (3) from Eq. (4) and substituting $p_{ws}(\Delta t=0)$ for $p_{wf}(t)$, we obtain

$$\frac{kh[p_{ws}(t+\Delta t)-p_{ws}(\Delta t=0)]}{141.2 \text{ qBu}} = \frac{p_{wd}[(t)_{D}]}{141.2 \text{ qBu}}$$

(5)

Eq. (5) provides a basis for buildup type curves and has been utilized in this paper.

Before discussing the new method, let us review the simplified version of Eq. (5) which has been commonly used in the past and has provided the basis of utilizing drawdown type curves for analyzing pressure buildup data.

If producing time, $t_{p}$, is significantly larger than the shut-in time, $\Delta t$, it is reasonable to assume that $[(t+\Delta t)/t_{p}] = 1$. Although approximate, this also implies that $t_{p} = t$ or $t_{p} = [(t+\Delta t)/t_{p}] = p_{wd}[(t)_{D}]$.

Thus, Eq. (5) can be simplified as a pressure buildup equation as shown below:

$$\frac{kh[p_{i}-p_{ws}(t+\Delta t)]}{141.2 \text{ qBu}} = \frac{p_{wd}[(\Delta t)_{D}]}{141.2 \text{ qBu}}$$

(6)

A comparison of pressure buildup equation (6) and the pressure drawdown equation (1) indicates that they are similar at least for cases where producing period, $t_{p}$ is significantly longer than the shut-in time, $\Delta t$. It also implies that $(\Delta p)$ drawdown vs. flowing time, $t$ is equivalent to $(\Delta p)$ buildup vs. shut-in time, $\Delta t$, where

$$\frac{(\Delta p)_{\text{drawdown}}}{p_{i}-p_{wf}(t)}$$

$$\frac{(\Delta p)_{\text{buildup}}}{p_{ws}(t+\Delta t) - p_{ws}(\Delta t=0)}$$

Since Eq. (6) has been derived from Eq. (5) based on the assumption of long producing period, $t_{p}$, the difference

$$p_{wd}[(t)_{D}]-p_{wd}[(t+\Delta t)_{D}] = 0$$

(9)

On Fig. 1, the above difference has been shown as the cross-hatched area and may be defined as

$$(\Delta p)_{\text{difference}} = p_{wf}(t) - p_{wf}(t+\Delta t)$$

(10)

or

$$(\Delta p)_{\text{difference}} = p_{ws}(\Delta t=0) - p_{wf}(t+\Delta t)$$

(11)

As producing period, $t_{p}$ gets smaller or $\Delta t$ gets larger, the difference shown by Eqs. (9) through (11) can no longer be ignored and the use of drawdown type curves to analyze pressure buildup data becomes invalid. The impact of the assumption shown by Eq. (9) will be discussed first in a generalized fashion followed by its impact on type curves for specific flow regimes. Finally, the new method will be discussed which accounts for producing time effects for analyzing pressure buildup data.

Fig. 2 schematically shows pressure buildup behavior obtained following a constant rate drawdown but at the end of three successively increasing producing periods, $t_{p}$ such that $p_{3} > p_{2} > p_{1}$. The cross-hatched area shown at the end of each production period denotes the difference between $(\Delta p)$ drawdown and $(\Delta p)$ buildup represented by Eq. (10) or (11). Note that $(\Delta p)$ difference gets smaller as the length of the producing period increases. This can be better shown by means of Fig. 3 where $(\Delta p)$ drawdown vs. flowing time, $t$ has been compared with $(\Delta p)$ buildup vs. shut-in time, $\Delta t$ with producing period $t_{p}$ as a parameter. Although schematic, Fig. 3 clearly indicates that there is a significant difference between $(\Delta p)$ drawdown and $(\Delta p)$ buildup for small producing periods. However, this difference gets smaller as the length of the producing period increases. Also note that for a given producing period, the difference between the two $(\Delta p)$s is small at early shut-in times but it gets bigger as shut-in time, $\Delta t$, increases. Fig. 3 clearly indicates the limitations of using drawdown type curves for analyzing pressure buildup data where producing period, $t_{p}$, prior to shut-in is relatively small.

Next we will examine the impact of this difference on type curve analysis for the specific flow regimes (radial flow, linear flow, etc.) and discuss the new method which accounts for producing time effects.

UNFRACKED WELL

Infinite Radial System ($a=0$; $C_{D}=0$)

Let us first consider the pressure drawdown solution for a well producing at a constant rate in a radial system.
\[
P_{wD}(t_p) = \frac{1}{2} \ln(t_p) + 0.80907 \quad (12)
\]

Eq. (12) is based on the assumption that wellbore effects (storage and skin) are negligible and the dimensionless time, \( t_D \geq 100 \) such that the log approximation applies to the \( t_p \) solution. Substitution of Eq. (12) in Eq. (1) provides

\[
\frac{kh[p_i-p_{wf}(t)]}{141.2 \ \text{qBu}} = \frac{1}{2} \ln(t_p) + 0.80907 \quad (13)
\]

Eq. (13) is a pressure drawdown solution for a radial system which also forms the basis for semi-log straight line on a semi-log graph paper. If Eq. (12) is substituted in Eq. (3), the well known Horner pressure buildup equation is obtained.

\[
\frac{kh[p_i-p_{ws}(t + \Delta t)]}{141.2 \ \text{qBu}} = \frac{1}{2} \ln(t_p + \Delta t) \quad (14)
\]

In Eq. (14) the subscript \( D \) may be dropped if desired. The above equation also takes into account producing time effects. Unfortunately \( \Delta p = [p_i - p_{wf}(t + \Delta t)] \) on the left hand side of Eq. (14) requires \( \frac{1}{2} \) knowledge of initial reservoir pressure, \( p_i \) which is generally not known. Consequently, Eq. (14) is not suitable for the purposes of type curve matching. However, \( \Delta p \) defined by Eq. (8) is generally known and is normally used for type curve analysis. If Eq. (12) is substituted in Eq. (6), the simplified pressure buildup equation is obtained.

\[
\frac{kh[p_i-p_{ws}(t + \Delta t) - p_{ws}(\Delta t=0)]}{141.2 \ \text{qBu}} = \frac{1}{2} \ln(\Delta t_D) + 0.80907 \quad (15)
\]

Eq. (15) is the familiar MDH (Miller-Dyes-Hutchinson) equation for pressure buildup and assumes that the producing period prior to shut-in is sufficiently long such that transients during the flow period do not affect the subsequent pressure buildup data. Thus, it should be obvious that Eq. (15) is not suitable for pressure buildup analysis (conventional or type curve) when producing times prior to shut-in are small. However, Eq. (5) may be used, as was done by Raghavan, to generate a family of pressure buildup curves with dimensionless producing time, \( t_{Dp} \), as a parameter. For the sake of simplicity and brevity let us define the new time group as an equivalent drawdown time, \( \Delta t_e \), or further abbreviated as \( \Delta t_e^* \), where

\[
\Delta t_e = \frac{t_p x \Delta t}{p} \quad (17)
\]

In a dimensionless form, Eq. (17) may be expressed as
The dimensionless pressure change during buildup or a rate change may be defined as

$$\Delta p_{eD} = \frac{(t_{PD} \times \Delta t_{D})}{(t_{P} + \Delta t)_{D}}$$

(18)

The dimensionless pressure change during buildup or a rate change may be defined as

$$\Delta p_{eD} = \frac{kh[p_{w} (t + \Delta t) - p_{w} (\Delta t=0)]}{141.2 \ qBu}$$

(19)

In establishing the new method, it was previously assumed that wellbore effects (such as storage and skin) are negligible. It appears that skin effect, s, may be considered in the development of this method.

Infinite Radial System ($s=0$; $C_D=0$)

If skin effect, s, is introduced in the pressure drawdown solution given by Eq. (12), we obtain

$$P_{wD}(t_{D}) = \frac{1}{2} [\ln(t_{D}) + 0.80907] + s$$

(20)

If we go through the same steps as we did for an infinite radial system ($s=0$; $C_D=0$) and instead of utilizing Eq. (12) we use Eq. (20), it will be obvious that the new method is equally valid when $s\neq 0$. Pressure drawdown Eq. (13) and the new pressure buildup Eq. (16) will respectively become Eqs. (21) and (22) as shown below:

$$\frac{kh[p_{p} - p_{w} (t)]}{141.2 \ qBu} = \frac{1}{2} [\ln(t_{D}) + 0.80907] + s$$

(21)

$$\frac{kh[p_{w} (t + \Delta t) - p_{w} (\Delta t=0)]}{141.2 \ qBu} = \frac{(t_{PD} \times \Delta t_{D})}{(t_{P} + \Delta t)_{D}} + 0.80907] + s$$

(22)

The above equations establish the validity of using pressure drawdown type curves for pressure buildup analysis even when skin is present.

Based on the encouraging results obtained thus far, we wanted to apply this concept of the equivalent drawdown time, $\Delta t$, to other wellbore effects and also to other flow regimes. The attempt was made to establish the validity of this concept for the above situations by graphical means rather than the mathematical solutions. Let us first consider the infinite radial system with wellbore storage effects.

Infinite Radial System ($s=0$; $C_D\neq 0$)

To study the effect of storage on buildup type curves, data presented by Agarwal, et al.,1 were utilized. Pressure drawdown data $P_{wD}$ vs. $t_{D}$ data for $s=0$ and $C_D=1000$ were taken from Table 3 of the above paper. Eq. (5) was used to generate the pressure buildup data for a number of producing times as was done by Raghavan.13 Both pressure drawdown data and pressure buildup data are plotted on Fig. 6 (semi-log graph paper) as a function of $t_{D}$ and $\Delta t_{D}$ respectively. Note that a family of buildup curves is obtained with producing time, $t_{PD}$ (10^3 to 10^6) as a parameter. These data are also plotted on log-log graph paper as shown by Fig. 7. These figures further emphasize the limitations of using pressure drawdown curves to analyze pressure buildup data obtained after short producing periods. Fig. 7 shows that unit slope lines for buildup data are shifted to the right of the drawdown curves. If dimensionless storage, $C_D'$ is computed using the buildup data, the computed value of $C_D$ will be erroneously high. Moreover, if pressure buildup data are forced to match the pressure drawdown type curve, the computed value of formation flow capacity $(kh)$ will be erroneously optimistic. The magnitude of error will increase with decreasing producing period.

Figs. 8 and 9 are the replots of pressure buildup solution on semi-log and log-log graph papers utilizing the new time group. Fig. 8 indicates that almost all pressure buildup curves are normalized except two which correspond to dimensionless producing period, $t_{PD}$ equal to 10^3 and 10^4. Although these curves do not seem to appear bad on the semi-log paper, they look rather poor on the log-log graph in Fig. 9. The reason for this may be obvious if we inspect the following equation 19 which provides the time for storage effects to become negligible.

$$t_{PD} > 60 C_D$$

(23)

If Eq. (23) is used for the subject problem, $C_D=1000$, the minimum producing time required for the storage effects to become negligible will be equal to 6 x 10^4. Since the producing periods in the two cases were only 10^3 and 10^4 respectively, pressure buildup data could not be normalized. Based on a number of cases studied, it appears that it is possible to normalize the pressure buildup curves provided that the producing time, $t_{PD}$, is at least equal to or greater than that given by Eq. (23).

Infinite Radial System ($s\neq 0$; $C_D\neq 0$)

Agarwal, et al.,1 data were taken for a number of cases for non-zero values of $C_D$ and $s$. Although not shown in this paper, results indicate that pressure buildup curves are normalized when the new method is used. The lower limit of the producing time for $s\neq 0$ is determined by the following equation.20
The preceding discussion establishes the validity of using Agarwal et al.'s pressure drawdown type curves (radial flow with storage and skin effect) for analyzing pressure buildup data provided that the new method is used.

**APPLICABILITY OF NEW METHOD TO OTHER TYPE CURVES**

It appears that it is possible to extend this method to other drawdown type curves which have appeared in the petroleum literature. Two sets of type curves will be considered: (1) Earlougher and Kersch, and (2) Gringarten et al.

**Earlougher and Kersch Type Curves**

These type curves are based on pressure drawdown solution and are applicable to an infinite radial system with wellbore storage and skin. They are basically the same type curves as that of Agarwal et al., because both use the same solution. However, they are distinctly different in appearance because data are plotted differently. A schematic of their drawdown type curves is shown in Fig. 10. Here, p/(p(t)) has been plotted as a function of t/tw with Ce as a parameter. Where Ce is the dimensionless storage coefficient, and is defined as

\[ C_o = \frac{5.615}{2 \pi \phi \delta C_w^2} \]  

(25)

Although no proof is demonstrated, their type curves may be converted for analyzing pressure buildup data if dimensionless time, tdp appearing both in y-axis and x-axis is replaced by atp, and p(t) on the y-axis is replaced by p(t) as shown in Fig. 11. In performing type curve analysis, the basic steps as outlined by Earlougher and Kersch remain the same except for some minor changes in the preparation of the data plot (APbuildup)/At vs. At, should be plotted instead of plotting Ap/t vs. t. Limitations on the lower limits of producing time, tp, given by Eqs. (23) and (24) should also apply in this case.

**Gringarten et al. Type Curves**

Recently, Gringarten et al. presented another set of type curves in a form different than that of Agarwal et al. and Earlougher and Kersch. Pressure drawdown data have been plotted as p/D vs. (t/p)/Cw with Ce as a parameter. Their drawdown type curves are schematically shown in Fig. 12. To convert their type curves for pressure buildup data, p(t) is changed to p(t+dt) and the parameter t/Cw on the abscissa should be replaced by (t+dt)/Cw. In using their type curves for buildup, (APbuildup)/At vs. At should be plotted on the data plot. Steps for type curve matching remain the same. The limitations on the lower limit of producing time as discussed earlier should also apply in this case.

**CONVENTIONAL ANALYSIS USING THE NEW METHOD**

Although the new method was originally conceived as to be used for type curve analysis purposes, it also appears useful for analyzing pressure buildup data by the conventional semi-log analysis methods. This can be seen by rewriting Eq. (22) in the following familiar form:

\[
\frac{p_u (t, +dt) - p_{uw} (dt=0)}{p_u (t, x \Delta t) + \log \left( \frac{k}{(t_p + \Delta t)} \right) - \log \left( \frac{k}{(t_p)} \right) - \frac{2}{\phi \mu C_w^2} - 3.23 + 0.87 s} = m \log (\Delta t_e)
\]  

(26)

or

\[
\frac{(\Delta p)_{buildup}}{\Delta t_e} = m \log (\Delta t_e) + \log \left( \frac{k}{(t_p + \Delta t)} \right) - \log \left( \frac{k}{(t_p)} \right) - \frac{2}{\phi \mu C_w^2} - 3.23 + 0.87 s
\]  

(27)

where, m is the slope per log cycle, s is the usual skin factor and \( \Delta t_e \) was defined earlier by Eq. (17).

\[
m = \frac{162.6 \, gB}{\phi \mu C_w^2} \]  

(28)

\( (\Delta p)_{buildup} \) is the left hand side of Eq. (22) and was defined earlier by Eq. (8). The form of Eq. (26) or Eq. (27) suggests that a graph of buildup pressure, Pws or \( (\Delta p)_{buildup} \) vs. \( \Delta t_e \), should be linear on a semi-log graph paper. This will be shown later by means of Fig. 16. The slope of the line should provide the value of formation flow capacity, k. Note that the graph utilizing \( \Delta t_e \) is similar to the Horner graph because it also takes into account the effect of producing time, t. Moreover, this graph appears more general than the Horner graph because the value of \( \Delta t \) increases with the increasing value of shut-in time, \( \Delta t \) as opposed to the Horner time group \((t_p + \Delta t)/t_p = 1\). Eq. (17) reverts back as

\[
\Delta t_e = \Delta t
\]  

(29)

Eq. (29) also provides the basis for making a MDH plot for long producing times. Eq. (26) may be solved for the skin effect, s as
Field Example

Pressure Buildup Analysis Using New Method

A field example taken from Gringarten et al.'s\textsuperscript{15} paper will be utilized to illustrate the application of the new method to analyze pressure buildup data taken on an acidized well. Both conventional and type curve methods will be used to analyze the data. Results will be compared with those of Gringarten et al.\textsuperscript{15} To maintain the continuity, part of the information appearing in their paper will be reproduced here.

Table 1 lists the pertinent reservoir and well data, along with pressure-time data both during the drawdown and buildup periods. Fig. 13 is a graph showing well pressures both during the constant rate drawdown vs. flowing time, \( t \), and during the subsequent buildup vs. \( t \) or \( t + \Delta t \). Drawdown data were replotted using the new time group, \( (t + \Delta t)/t \) or \( t/\Delta t \). On Fig. 13, they are plotted as a function of \( t + \Delta t \). Note that there is a significant difference between the two buildup curves. Data plotted simply as a function of shut-in time, \( t \), (shown by open circles) appear flatter compared to the second buildup curve plotted vs. \( t + \Delta t \) and shown by solid circles. This is to be expected because the first curve does not take into account the effect of producing time. More will be said about this later.

Although initial reservoir pressure was not known for this problem, it is possible to estimate it by means of an expanded plot (not shown here) of early time buildup and drawdown data and recognizing that a graph of \( (\Delta P)_{\text{buildup}} \) vs. \( \Delta t \) is equivalent to \( (\Delta P)_{\text{drawdown}} \) vs. \( \text{flowing time, } t \). Using the known value of \( (\Delta P)_{\text{buildup}} \) at a given \( \Delta t \), the corresponding \( (\Delta P)_{\text{drawdown}} \) at the same value of flowing time, \( t \), was estimated. This provided \( p_i = 3251 \text{ psi, which was used to compute } (\Delta P)_{\text{drawdown}} \text{vs. flowing time, } t \).

Fig. 14 shows a comparison of drawdown data [plotted as \( (\Delta P)_{\text{drawdown}} \) vs. \( t \) and shown by triangles] with the pressure buildup data [(\( \Delta P \))_{buildup} vs. \( \Delta t \)] (shown by solid circles). The comparison between the two plots is excellent. \( (\Delta P)_{\text{buildup}} \) data plotted as a function of conventional shut-in time, \( \Delta t \), are also shown by a dotted line with open circles. Note that between 200 and 250 minutes there is a departure between the conventional buildup curve and the other two curves. Gringarten et al.\textsuperscript{15} observed a similar departure between the conventional buildup curve and the drawdown curve at about 250 minutes and concluded that the buildup data beyond this time should not be analyzed by drawdown type curves. However, the modified buildup plot does not suffer from the above restriction. If the new method is used, the majority of data may be type curve matched. Fig. 16 also suggests that in this case, the real shut-in time, \( \Delta t \), of 4281 minutes is only equal to \((1347 \times 4281)/(1347 + 4281) = 1025 \) minutes in terms of equivalent drawdown time, \( \Delta t' \).

Next \( (\Delta P)_{\text{buildup}} \) vs. \( \Delta t \) data were type curve matched using Gringarten et al.'s\textsuperscript{15} type curve as shown in Fig. 15. Note that a very satisfactory match has been obtained. Computations for type curve analysis are shown in Table 2 and results summarized in Table 3. It is also possible to read the initial pressure directly from the log-log plot. To accomplish this, read \( (\Delta P)_{\text{buildup}} \) at \( \Delta t = t' \). This provides \( p_i = p(\Delta t = 0) + (\Delta P)_{\text{buildup}} \). In this case \( p_i = 3251 \text{ psi, as shown on Fig. 15.}

To demonstrate applicability of the new method to conventional semi-log analysis, buildup pressures, \( p_{w,s} \), were plotted on a semi-log graph paper both as a function of conventional shut-in time, \( \Delta t \) (shown by open circles) and the equivalent time, \( \Delta t' \) (shown by solid circles). This is shown in Fig. 16. As expected, there is a significant difference between the plots. In a way it is similar to comparing a MDH plot with a Horner plot. However, the new method is better because data can be compared on an equivalent time scale. It also provides a reasonable straight line, whose slope was used to compute formation flow capacity, \( kh \), as given by Eq. (29) or Eq. (28). Eq. (30) is used to compute the skin effect, \( s \). It is also possible to directly read the initial pressure from the semi-log straight line or its extension where \( \Delta t \) = \( t' \). Results of both conventional semi-log and type curve analyses are listed in Table 3. For comparison purposes, analysis results obtained by Gringarten et al.\textsuperscript{15} are also shown in Table 3.

Note that excellent agreement has been obtained between the conventional semi-log and type curve methods when the new method is used. Moreover, results also agree very well with that of Gringarten et al.\textsuperscript{15} when they used the Horner method for the semi-log analysis and the desuperposed data for type curve matching purposes. Obviously, their MDH type results shown in Table 3 obtained by ignoring the effect of producing time (using semi-log or type curve method) will be wrong, as expected. This was also pointed out by Gringarten et al.\textsuperscript{15} In regard to the desuperposition principle, it should be pointed out that it is not always possible to desuperpose the buildup data because it requires a knowledge of pressure vs. time data from the preceding flow period. If the new method is used,
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Desuperposition of data may not be necessary. Although not shown, the new method may be used to desuperpose pressure buildup data.

EXTENSION OF NEW METHOD TO OTHER KINDS OF TESTING

It appears that the new method may be extended to analyze other types of testing such as two-rate and multiple rate tests in an infinite radial system. Once this method is used, data may be analyzed by both the conventional semilog method and type curve matching techniques.

Two Rate Testing

A schematic of two rate testing with rate and pressure history is shown in Fig. 17. This type of testing consists of flowing a well at a constant rate, \( q_1 \), for time, \( t_1 \), when the rate is changed to \( q_2 \) during the incremental time, \( \Delta t \). The flowing pressure, \( p_{wf1}(t_1) \) at the end of the first flow rate, \( q_1 \), can be obtained from Eq. (21) as

\[
\frac{kh[p_1 - p_{wf1}(t_1)]}{141.2 \mu B} = \frac{1}{2} q_1 \left[ \ln(t_1) + 0.80907 + 2s \right]
\]

Note that the above equation assumes the log approximation. By applying the superposition principle, we obtain the following equation for the flowing pressure, \( p_{wf2}(t_1 + \Delta t) \) during the second flow rate, \( q_2 \),

\[
\frac{kh[p_1 - p_{wf2}(t_1 + \Delta t)]}{141.2 \mu B} = \frac{1}{2} q_1 \left[ \ln(t_1 + \Delta t) + 0.80907 + 2s \right]
\]

Subtracting Eq. (32) from Eq. 31, we obtain

\[
\frac{kh[p_{wf2}(t_1 + \Delta t) - p_{wf1}(t_1)]}{141.2(q_1 - q_2)\mu B} = \frac{1}{2} \left( \ln \left( \frac{t_1}{t_1 + \Delta t} \right) \right) \frac{q_1 - q_2}{q_1 - q_2} \cdot (\Delta t) + 0.80907 + 2s
\]

Comparison of the above equation with Eq. (21) indicates that a plot of \( [p_1 - p_{wf}(t)] \) vs. \( t \) during a pressure drawdown test is equivalent to plotting of data as

\[
[p_{wf2}(t_1 + \Delta t) - p_{wf1}(t_1)] \text{ vs. } \left( \frac{t_1}{t_1 + \Delta t} \right) \frac{q_1 - q_2}{q_1 - q_2} \cdot \Delta t
\]

obtained during the second rate testing. Also note that Eq. (33) reverts back to the pressure buildup Eq. (22) when \( q_2 = 0 \). This is to be expected in view of the fact that a pressure buildup test is a special case of two rate tests with \( q_2 = 0 \).

For a two rate testing, let us define equivalent drawdown time, \( t_{e2} \) as

\[
\Delta t_{e2} = \left( \frac{q_1}{t_1 + \Delta t} \right) \frac{q_1 - q_2}{q_1 - q_2} \cdot \Delta t
\]

Substituting Eq. (34) in Eq. (33) and expressing it for conventional semilog analysis, we obtain

\[
\frac{kh}{\phi \mu c t_w} \log \frac{k}{\phi \mu c t_w} - 3.23 + 0.87s
\]

Eq. (35) suggests that a graph of \( [p_{wf2}(t_1 + \Delta t) - p_{wf1}(t_1)] \) or \( p_{wf}(t_1 + \Delta t) \) vs. \( t_{e2} \) should be linear with slope, \( m \). Formation flow capacity is computed as

\[
kh = \frac{162.6(q_1 - q_2)\mu B}{md-ft}
\]

and skin effect, \( s \), is computed using

\[
s = 1.151 \left[ \frac{p_{wf2}(\Delta t_{e2} = 1 \text{ hr}) - p_{wf1}(t_1)}{m} - \log \frac{k}{\phi \mu c t_w} + 3.23 \right]
\]

An example will be shown later for multiple rate testing.
Multiple Rate Testing

A schematic of multiple rate testing is shown in Fig. 18. This type of testing consists of flowing a well at a constant rate \( q_1 \) for time \( t_1 \), at rate \( q_2 \) for time \( t_2 \), to \( t_3 \) and so on. Say the final rate is \( q_n \) for time \( t_{n-1} \) to any incremental time, \( t_n \). Although not shown, pressures are denoted as \( P_{wf1}(t_1), P_{wf2}(t_2), ..., P_{wfn-1}(t_{n-1}) \) at the end of first, second and \( t_{n-2} \) time periods. \( P_{wf0}(At) \) are the pressures during the final (nth) period. If the steps similar to those shown for two rate testing are followed for multiple rate testing, the following equation is obtained.

\[
\frac{kh[P_{wfn}(At) - P_{wfn-1}(t_{n-1})]}{141.2 (q_{n-1} - q_n)Bu} = \frac{1}{2} \ln \left[ \frac{1}{n-1} \sum_{j=1}^{n-1} \left( \frac{t_{n-1} - t_j}{At + t_{n-1} - t_j} \right) \right] \quad (38)
\]

where \( t_1 = 0; \ q_1 = 0 \) and \( n > 2 \). Eq. (38) is very general and should apply to any number of flow and buildup periods, provided that the system is behaving like an infinite radial system and log approximation is valid. Eq. (38) also suggests that multiple rate test data during any flow or buildup period may be analyzed using drawdown type curves. For multiple rate testing, the equivalent drawdown time may be defined as

\[
\Delta t_{en} = \frac{1}{2} \ln \left[ \frac{1}{n-1} \sum_{j=1}^{n-1} \left( \frac{t_{n-1} - t_j}{At + t_{n-1} - t_j} \right) \right] \quad (39)
\]

For conventional semi-log analysis, Eq. (38) may be written as

\[
\frac{[P_{wfn}(At) - P_{wfn-1}(t_{n-1})]}{162.6(q_{n-1} - q_n)Bu} \ log \left( \frac{k}{\phi \mu c_r w^2} \right) = \frac{1}{2} \left( \Delta t_{en} \right) \log \left( \frac{k}{\phi \mu c_r w^2} \right) + 3.23 + 0.87s \quad (40)
\]

where \( \Delta t_{en} \) is the equivalent drawdown time. Eq. (40) is expressed for the skin effect, s, as follows:

\[
s = \frac{1.151}{m} \left( P_{wfn}(t_n) - P_{wfn-1}(t_{n-1}) \right) - \log \left( \frac{k}{\phi \mu c_r w^2} \right) + 3.23 \quad (42)
\]

Next a computer simulated example will be considered to illustrate the application to multiple rate testing.

SIMULATED EXAMPLE
Multiple Rate Analysis Using New Method

To demonstrate the application, a computer generated example will be utilized. Table 4 lists the reservoir and well data for a gas well, where \( \phi \mu c_r \) product is kept constant to eliminate the effect of \( \phi \mu c_r \) variations as a function of pressure. Rate and pressure histories are shown on Fig. 19. Note that the gas well was produced at three different flow rates (15, 10 and 5 MMCF/D) with intermediate buildup periods. Each flow period was simulated to be 1/2 day long whereas each of the first two buildup periods was 1 day long. The third and the final buildup period was 2 1/2 days long. Pressure vs. time data for each flow period are shown on Fig. 19 by means of open symbols, whereas for each subsequent buildup period by means of corresponding solid symbols. Multiple rate test Eq. (40) was expressed in terms of real gas pseudo pressure 19, \( m(p) \) and rearranged slightly for the purpose. In practical gas units, the following equation is obtained:

\[
\frac{m[P_{wfn}(\Delta t_{en}) - P_{wfn-1}(t_{n-1})]}{1637T} \log\left( \frac{k}{\phi \mu c_r w^2} \right) + 3.23 + 0.87s \quad (43)
\]

Eq. (43) was used to process the pressure vs. time data obtained for each test period. Results are shown on Fig. 20 using the same symbols as shown on Fig. 19. The left hand side of Eq. (43) denoted by \( [\Delta m(p)/\Delta t] \) vs. the equivalent drawdown time, \( \Delta t_{en} \) have been plotted on a semilog graph paper for each test. Note that it was possible to normalize all test data on the semi-log straight line obtained using single rate drawdown data. Although details are not shown, the slope of the semi-log straight line provided the value of formation flow capacity (kh) which was consistent with the kh value entered into the program, as expected. The preceding example was used to demonstrate the application and establish the validity of the new method for multiple rate testing data.
A NEW METHOD TO ACCOUNT FOR PRODUCING TIME EFFECTS WHEN DRAWDOWN TYPE CURVES ARE USED TO ANALYZE PRESSURE BUILDUP AND OTHER TEST DATA

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Other Kinds of Testing

Although not shown in this paper, it appears that the new method may be applied to other kinds of testing methods such as interference, constant pressure testing, etc.

VERTICALLY FRACTURED WELL

The new method was next applied to vertically fractured wells with both infinite and finite flow capacity fractures. Results are discussed below. Eq. (5) again forms the basis of this study. Dimensionless time, $\frac{t_{DF}}{x_f}$ for a fractured well is defined as follows:

$$t_{DF} = \frac{2.634 \times 10^{-4} k t}{\phi (\mu c)^{0.5} x_f}$$

Where $x_f$ is the fracture half-length in feet. Definitions of real and dimensionless drawdown and buildup pressures were kept the same.

Infinite Flow Capacity Fracture

Gringarten et al.'s pressure drawdown data for the infinite reservoir case were taken from their Table 1. Eq. (5) was used to generate a family of pressure buildup curves with producing time, $\Delta t$, as a parameter. Results similar to those of Raghavan are presented in Fig. 21. Since Raghavan adequately discussed the limitations of using pressure drawdown curves for analyzing pressure buildup data for fractured wells, only certain key points will be re-emphasized.

(i) Computed formation flow capacity will be optimistic.

(ii) Computed value of fracture length will be pessimistic.

(iii) The characteristic half slope line may not appear on the log-log paper.

Fig. 22 shows the replot of pressure buildup data utilizing the new method. Data are plotted as $\Delta t_{DF} / x_f$ vs. $\Delta t_{DF}$, or $(t_{DF} / x_f) / (t_{DF} + \Delta t_{DF})$. Note that $\Delta t_{DF}$ is the equivalent drawdown time, expressed in the dimensionless form, for a vertically fractured well. The majority of buildup data have been normalized on the drawdown curve. It was rather a surprising observation in view of the fact that a time group developed for the radial system should also be applicable for a fractured well which is normally associated with linear, elliptical and radial flow regimes.

Although not included in this paper, the pressure drawdown data of Gringarten, et al., for the uniform flux fracture case were also considered. Pressure buildup data were generated and plotted using the new method. Once again it was possible to normalize the majority of buildup data on the drawdown type curve. The plot was very similar to that shown in Fig. 22.

Finite Flow Capacity Fracture

The new method was next applied to data for a vertically fractured with finite flow capacity fracture. Constant rate pressure drawdown data of Agarwal, et al., were used to generate a family of buildup type curves with producing time as a parameter. This had to be done for each value of dimensionless fracture flow capacity. These results were replotted using the new method. Once again, it was possible to normalize the majority of buildup data on drawdown type curves. For the sake of brevity, results are not presented here. However, it should be suffice to say that constant rate pressure drawdown type curves of Agarwal, et al., and Cinco, et al., may be utilized to analyze pressure buildup data. Requirement is that $\Delta t_{DF}$ buildup data are plotted as a function of $\Delta t$ rather than the conventional shut-in time, $\Delta t$.

ANALYSIS OF GAS WELL BUILDUP DATA

The development of the new method, for analyzing pressure buildup data, has been discussed mainly utilizing solutions for liquid systems. However, it appears that the method may be extended to include the analysis of data from gas wells, if real gas pseudo-pressure $m(p)$ of Al-Hussainy, et al., is used and variations of $(\mu c)$ vs. pressure are accounted for. The latter may be accomplished if real times in the new time group are replaced by real gas pseudo-time, $t(p)$ of Agarwal. For example, if pressure buildup data collected after short producing time from an MHF gas well are to be analyzed by drawdown type curves, the following procedure is recommended.

Graph $[m(p_{ws})(t + \Delta t)] - m(p_{ws}(\Delta t = 0))$ vs. $\frac{(t_{ap} x \Delta t a)}{(t_{ap} + \Delta t a)}$

the data plot, utilizing the appropriate type curve. Steps outlined in Ref. 14 for type curve matching remain the same. In the above time group, $t$ and $\Delta t$ represent flowing time, $t$, and shut-in time, $\Delta t$, expressed in terms of real gas pseudo-time. If variations of $(\mu c)$ vs. pressure during the test period appear to be small, instead of using pseudo-time, real times may be used.

CONCLUDING REMARKS

1. A new method has been developed to analyze pressure buildup data by pressure drawdown type curves. It provides a significant improvement over the current methods because

(i) the effects of producing time are accounted for;

(ii) data are normalized in such a way that instead of using a family of buildup curves, the existing drawdown type curves may be used;

(iii) wellbore storage and damage effects may be considered except under certain conditions.
2. This method can also be used to perform conventional semi-log analysis to estimate formation flow capacity, $k_h$, skin effect, $s$, and initial pressure, $p_i$. It appears similar to the Horner method because both methods take into account producing time effects. However, this method is more general and has the advantage that

(i) both the MDH plot and the plot using the new method utilize the common time scale, which permits comparing the two plots and determining the effects of including or excluding the producing time;

(ii) it provides a relationship between the flowing time, $t_f$, during a drawdown test to an equivalent time, $\Delta t_e$, during a buildup test.

3. For long producing periods, the new method reverts back to the MDH method.

4. A field example is included to demonstrate the application of the new method and point out the utility.

5. This method has been extended to include the analysis of two rate and multiple rate test data by both type curve and conventional methods. This was shown both theoretically and also by means of example problems.

6. Although originally developed for radial systems, this method appears to work well for vertically fractured wells with infinite and finite flow capacity fractures.

7. The method, although developed using liquid solutions, should be applicable to data from gas wells, as shown in the paper.

8. Finally, it appears that the new method may be applied to a variety of testing methods such as interference testing and constant pressure testing to name a few.

**NOMENCLATURE**

- $B$ = formation volume factor, RB/STB (res. m$^3$/stock tank m$^3$)
- $c_g$ = gas compressibility, psi$^{-1}$ (kPa$^{-1}$)
- $C_t$ = total system compressibility, psi$^{-1}$ (kPa$^{-1}$)
- $C$ = storage coefficient, RB/psi (res. m$^3$/kPa)
- $C_D$ = dimensionless storage coefficient [see Eq. (25)]
- $F_{CD}$ = dimensionless fracture flow capacity (see Ref. 7)
- $h$ = formation thickness, ft (m)
- $k$ = formation permeability, md
- $\log$ = logarithm to base 10
- $\ln$ = natural logarithm
- $m$ = slope of the semilog straight line, psi/log cycle (kPa/log cycle)
- $m(p)$ = real gas pseudo pressure, psi$^2$/cp (kPa$^2$/Pa·s)
- $p_i$ = initial pressure, psi (kPa)
- $p_{WD}$ = dimensionless pressure drop [see Eq. (1)]
- $p_{WDn}$ = dimensionless pressure rise or change [see Eq. (19)]
- $p_{wf}$ = wellbore flowing pressure, psi (kPa)
- $p_{wn-1}$ = pressure at the end of test period, $t_{n-1}$, psi (kPa)
- $p_{wn}$ = pressures during nth test period, psi (kPa)
- $p_{ws}$ = shut-in pressure, psi (kPa)
- $(\Delta p)_{buildup}$ = pressure change during buildup, psi (kPa) [see Eq. (8)]
- $(\Delta p)_{difference}$ = pressure difference, psi (kPa) [see Eq. (10) and (11)]
- $(\Delta p)_{drawdown}$ = pressure change during drawdown, psi (kPa) [see Eq. (7)]
- $q$ = flow rate, STB/D or MCF/D ("standard" m$^3$/D)
- $q_n$ = flow rate during the final flow period, STB/D ("standard" m$^3$/D)
- $r_w$ = wellbore radius, ft (m)
- $s$ = skin effect
- $t$ = flow time, hours
- $t_{n-1}$ = total time up to (n-1) test period, hours
- $t_D$ = dimensionless time based on wellbore radius (see Eq. 2)
- $t_{a(p)}$ = real gas pseudo time (see Ref. 14)
- $t_{ap}$ = real gas pseudo-producing time
- $t_{Dx_f}$ = dimensionless time based on half fracture length (see Eq. 44)
- $t_p$ = producing period, hours
- $t_{p1}$ = producing period during the first test, hours.
- $t_{PD}$ = dimensionless producing period
- $\Delta t$ = shut-in time or incremental time during the final flow test period, hours
- $\Delta t_a$ = real gas pseudo shut-in time
A NEW METHOD TO ACCOUNT FOR PRODUCING TIME EFFECTS WHEN DRAWDOWN TYPE CURVES ARE USED TO ANALYZE PRESSURE BUILDUP AND OTHER TEST DATA  

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{ap}$</td>
<td>real gas pseudo producing period</td>
</tr>
<tr>
<td>$\Delta t_e$</td>
<td>equivalent drawdown time, hours (see Eq. 17)</td>
</tr>
<tr>
<td>$\Delta t_{e_f}$</td>
<td>dimensionless shut-in time based on fracture half length</td>
</tr>
<tr>
<td>$\Delta t_{e2}$</td>
<td>equivalent drawdown time for two rate test, hours (see Eq. 34)</td>
</tr>
<tr>
<td>$\Delta t_{en}$</td>
<td>equivalent drawdown time for multiple rate test, hours (see Eq. 39)</td>
</tr>
<tr>
<td>$T$</td>
<td>reservoir temperature, °R</td>
</tr>
<tr>
<td>$x_f$</td>
<td>fracture half length, ft (m)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity, cp (Pa·s)</td>
</tr>
<tr>
<td>$(\mu c_t)_{i}$</td>
<td>viscosity-compressibility product at initial condition, cp</td>
</tr>
<tr>
<td>$\phi$</td>
<td>formation porosity, fraction</td>
</tr>
<tr>
<td>$\pi$</td>
<td>constant, 3.14159</td>
</tr>
</tbody>
</table>

Subscripts

- $a$: adjusted or pseudo
- CD: dimensionless flow capacity
- D: dimensionless
- $Dx_f$: dimensionless, based on $x_f$
- e: equivalent
- eD: equivalent, dimensionless
- e2: equivalent for two rate test
- en: equivalent for multiple rate test
- f: fracture
g: gasi: initialj: indexn: indexp: producing	: totalw: wellbore

REFERENCES


TABLE 1
RESERVOIR AND WELL DATA
(Field Example - Ref. 15)

<table>
<thead>
<tr>
<th>Formation thickness, h</th>
<th>30 ft</th>
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<tbody>
<tr>
<td>Formation porosity, ( \phi )</td>
<td>0.15 fraction PV</td>
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<tr>
<td>Wellbore radius, ( r_w )</td>
<td>0.3 ft</td>
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<tr>
<td>Fluid viscosity, ( \mu )</td>
<td>1.0 cp</td>
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<tr>
<td>System compressibility, ( c_t )</td>
<td>( 10 \times 10^{-6} ) psi-l</td>
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<tr>
<td>Formation volume factor, B</td>
<td>1.25 RB/STB</td>
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<tr>
<td>Production rate, q</td>
<td>800 STB/D</td>
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<tr>
<td>Producing period, ( t_p )</td>
<td>1347 minutes</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Drawdown Match Point</th>
<th>Production flow capacity, ( kh )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta t )</td>
<td>( P_{wf} )</td>
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<tr>
<td>45</td>
<td>3198</td>
</tr>
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<td>85</td>
<td>3180</td>
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<td>4281</td>
<td>3246</td>
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TABLE 2
TYPE CURVE ANALYSIS
(Field Example - Ref. 15)

Match Point

\[
\begin{align*}
[\Delta P]_M &= 10 \text{ psi} \\
[P_{wf}] &= 0.17 \\
[\Delta t]_M &= 10 \text{ min} \\
[C^t]_M &= 0.64 \\
[c_t] &= 1.0 \times 10^{-6} \text{ psi-l} \\
\end{align*}
\]

(i) Formation flow capacity, \( kh \)

\[
kh = \frac{[\Delta P]_M}{[\Delta t]_M} \text{ md-ft}
\]

(ii) Skin effect, \( s \)

\[
C = \frac{(0.000295) \kh}{\mu} \text{ Res Bbl/psi} \\
\mu \text{ Res Bbl/psi} = (0.000295)(2400)(10/60) \text{ Res Bbl/psi} \text{ (1)(0.64)}
\]

\[
C = 0.184 \text{ Res Bbl/psi}
\]

Eq. (25) is used to compute

\[
C_D = \frac{(0.184)(5.615)}{2n(1.5)(30)(1X10^{-5})(.3)^2} = 40600
\]

\[
(c_D)^{2n} = \frac{1}{2} \ln \left( \frac{1}{C_D} \right) = \frac{1}{2} \ln \left( \frac{1}{40821} \right)
\]

\[
s = -5.31
\]

TABLE 3
SUMMARY OF RESULTS
(Field Example - Ref. 15)

Analysis Using This Method

<table>
<thead>
<tr>
<th>Semilog</th>
<th>Type Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>2323</td>
<td>2400</td>
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<tr>
<td>-5.15</td>
<td>-5.31</td>
</tr>
<tr>
<td>0.184</td>
<td>3253</td>
</tr>
<tr>
<td>3251</td>
<td></td>
</tr>
</tbody>
</table>

Gringarten et al. \textsuperscript{15} Analysis

<table>
<thead>
<tr>
<th>Semilog</th>
<th>Desuperposition</th>
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</thead>
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<tr>
<td>2274</td>
<td>4279</td>
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<td>2259</td>
<td>4095</td>
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<td>-4.0</td>
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<tr>
<td>-5.1</td>
<td>-4.0</td>
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<tr>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td>3253</td>
<td>3230</td>
</tr>
</tbody>
</table>

TABLE 4
RESERVOIR AND WELL DATA
(Simulated Example)

| Initial reservoir pressure, \( P_i \) | 500 psi |
| Reservoir temperature, \( T \) | 720°F |
| Formation permeability, \( k \) | 5 md |
| Formation thickness, \( h \) | 40 feet |
| Hydrocarbon porosity, \( \phi \) | 5% |
| Viscosity-compressibility product, \( \mu c_t \) | \( 3.93 \times 10^{-6} \) cp/psi |
| Wellbore radius, \( r_w \) | 0.25 feet |
FIGURE 1: SCHEMATIC OF PRESSURE BUILDUP BEHAVIOR FOLLOWING A CONSTANT RATE DRAWDOWN FOR A PRODUCTION PERIOD, $t_p$

FIGURE 2: SCHEMATIC OF PRESSURE BUILDUP BEHAVIOR FOLLOWING CONSTANT RATE DRAWDOWN OF SUCCESSIVELY INCREASING FLOW PERIODS, $t_p$

FIGURE 3: SCHEMATIC SHOWING COMPARISON BETWEEN $(\Delta p)_{\text{drawdown}}$ vs $t$ AND $(\Delta p)_{\text{buildup}}$ vs $\Delta t$

FIGURE 4: BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES - SEMI-LOG GRAPH (INFINITE RADIAL SYSTEM, $c_p = 0$ & $S = 0$)

$\Delta t_D \cdot \frac{2.634 \times 10^{-4} \times \Delta t}{\theta \mu c_l r_w^2}$
BUILDUP DATA FOR VARIOUS PRODUCING TIMES, $t_{PD}$

$\frac{n}{\mu} - \ln \left( \frac{\phi_m}{\mu} + \Delta P - \mu \Delta t \right)$

DIMENSIONLESS EQUIVALENT TIME, $6t_{PD}$

FIGURE 5: NORMALIZED BUILDUP TYPE CURVE-SEMI-LOG GRAPH
(INFINITE RADIAL SYSTEM, $C_0 = 1000; S = 0$)

FIGURE 6: BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES-SEMI-LOG GRAPH
(INFINITE RADIAL SYSTEM, $C_0 = 10000; S = 0$)

FIGURE 7: LOG-LOG BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES
(INFINITE RADIAL SYSTEM, $C_0 = 1000; S = 0$)

FIGURE 8: NORMALIZED BUILDUP TYPE CURVE (SEMI-LOG GRAPH)
(INFINITE RADIAL SYSTEM, $C_0 = 10000; S = 0$)
FIGURE 9: NORMALIZED BUILDUP TYPE CURVES - LOG-LOG GRAPH  
(INFINITE RADIAL SYSTEM, C_D=1000, S=0)

FIGURE 10: A GENERALIZED SCHEMATIC OF EARLOUGHER AND KERSCH 
DRAWDOWN TYPE CURVES

FIGURE 11: A SCHEMATIC OF EARLOUGHER AND KERSCH DRAWDOWN TYPE CURVES FOR A WELL WITH STORAGE AND SKIN 
(INFINITE RADIAL SYSTEM)-BY PERMISSION OF MARATHON OIL COMPANY
FIGURE 12: A SCHEMATIC OF GRINGARTEN & AL 15 DRAWDOWN TYPE CURVES FOR A WELL WITH STORAGE AND SKIN (BY PERMISSION OF FLOPETROL)

FIGURE 13: WELL PRESSURES VS TIME DURING PRESSURE DRAWDOWN AND BUILDUP PERIODS (FIELD EXAMPLE - REF. 15)

FIGURE 14: COMPARISON BETWEEN DRAWDOWN AND BUILDUP DATA USING NEW METHOD (FIELD EXAMPLE - REF. 15)

FIGURE 15: APPLICATION OF NEW METHOD USING GRINGARTEN & AL TYPE CURVES (FIELD EXAMPLE - REF. 15)
**Figure 16:** Application of New Method to Conventional Semilog-Analysis (Field Example - Ref. 15)

**Figure 17:** Schematic Rate and Pressure History for a Two-Rate Testing, $q > 0$

**Figure 18:** Schematic of Multiple Rate Testing

**Figure 19:** Rate and Pressure History for Simulated Example
FIGURE 20: NORMALIZED MULTIPLE RATE TEST DATA (SIMULATED EXAMPLE)

FIGURE 21: BUILDUP TYPE CURVES FOR VARIOUS PRODUCING TIMES-LOG-LOG GRAPH
(VERTICALLY FRACURED WELL WITH INFINITE FLOW CAPACITY FRACURE)

FIGURE 22: NORMALIZED BUILDUP TYPE CURVES (LOG-LOG GRAPH)
(VERTICALLY FRACURED WELL WITH INFINITE FLOW CAPACITY FRACURE)