THE APPLICATION OF THE LAPLACE TRANSFORMATION TO FLOW PROBLEMS IN RESERVOIRS

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Fig. 8 - Constant rate of production in the stock tank, adjusting for the unloading of fluid in the annulus, $P(t)$ versus $t$ (where $C = c/[2\pi\phi c_t(R_b)^2]$, and $c$ is the volume of fluid unloaded from the annulus), corrected to reservoir conditions, per atmosphere bottom-hole pressure drop, per unit sand thickness.
Short-Time Well Test Data Interpretation in the Presence of Skin Effect and Wellbore Storage

H. J. Ramey, Jr., SPE-AIME, Stanford U.

The log-log type-curve described here shows clearly the presence and duration of wellbore storage as well as the presence of linear flow due to fracturing. It can be used to obtain quantitatively the information normally obtained from pressure buildup analyses and to identify the proper straight line in pseudo-radial flow for a fractured well.

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Fig. 1 - $p_D$ vs $t_D$ for infinite radial system with storage and skin effect. Wellbore storage coefficient in constructing type curves for short-time transient behavior.
Fig. 3 - Field data plot for Well A.

Fig. 4 - Type curve matching for Well A.

Fig. 5 - $p_D$ vs $t_D$ for vertically fractured wells ($A/r_w = 10^6$).
Short-Time Well Test Data Interpretation in the Presence of Skin Effect and Wellbore Storage

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Introduction

Specifications for modern well testing (drawdown or buildup) are usually written in such a way that a well will be tested for a period of time long enough to reach and define a proper "straight line" when test data are plotted in conventional manners. Pressure data obtained before the straight line is reached are not often analyzed, despite the fact that a number of publications have advanced methods for doing so.1-3

One reason for this situation is that many factors are known to affect the short-time data. "Short-time data" specify data obtained before the conventional straight line is reached. Some of the factors are the effects of wellbore storage, perforations, partial penetration, and well stimulation such as fracturing or acidizing.

Although the effects of such factors are generally known, the duration and importance have not been clearly defined in all cases — particularly when these effects are combined in a well test. However, recent studies have revealed a great deal of potentially useful information concerning the analysis of short-time well test data.4 The purpose here is to illustrate the interpretation of short-time well test data through presentation of field examples. Factors to be considered will include wellbore storage, well damage, and fractured wells.

Wellbore Storage and Skin Effect

The effect of wellbore storage or unloading was originally considered by van Everdingen and Hurst.5

The log-log type-curve described here shows clearly the presence and duration of wellbore storage as well as the presence of linear flow due to fracturing. It can be used to obtain quantitatively the information normally obtained from pressure buildup analyses and to identify the proper straight line in pseudo-radial flow for a fractured well.

\[
\log p_a = \log \left( \frac{k h}{[141.4 q_i B]} \right) + \log(p_i - p_w) .
\]

Thus the only difference between a log-log plot of dimensionless pressures and times and real \( \Delta p \) and real time is a translation of both coordinates by appropriate constants. Type-curve matching permits determination of the constants, i.e., the first group in brackets on the right-hand side in both Eqs. 5 and 6.

One important result of the Agarwall et al. study was the realization that wellbore storage (if evident at all) was the controlling effect at times immediately after start of a test, and that during the storage period as indicated by a unit slope on a log-log type-curve, absolutely nothing could be learned concerning formation flow capacity, diffusivity, or well skin effect. Furthermore, the type-curve shows clearly whether or not storage is controlling well behavior, and thus it sets a lower time limit on utility of well-test data for normal purposes.

Another important result was the realization that the physical nature of the skin effect could influence interpretation of the short-time data for times after complete storage control. That is, the depth of formation damage could change the shape of the transition from a line of unit slope to the beginning of the usual straight line. These effects were studied by Waynemberg and Ramey,6 but will not be reviewed here.

Field Example of Damaged Well with Storage

Fig. 2 presents a pressure buildup curve for a well with a prominent storage effect. This set of data was presented in detail by Russell7 (his Well A) to illustrate a method for analyzing the data in the curvature portion of the plot indicated by the arrow on Fig. 2. Fig. 3 presents a type-curve plot of the same data, i.e., the logarithm of \( (p_{up} - p_{wp}) \) vs the logarithm of the buildup time, \( \Delta t \). This plot is valid for pressure buildup if the producing time, \( t_i \), is long compared with the buildup time. For pressure drawdown, \( (p_i - p_w) \) would be plotted vs log time. Either pressure buildup or pressure drawdown can be plotted on a
type-curve and compared with dimensionless type-curves such as Fig. 1.

The first buildup points on Fig. 2 do fall on a line of unit slope, indicating that buildup behavior was completely controlled by storage until a buildup time of about 0.67 hours. There is no possibility of obtaining information about formation flow capacity or skin effect from buildup data obtained before 0.67 hours of buildup. But this buildup information is important on the type-curve since it graphically displays that storage is the controlling factor to 0.67 hours. This casts a light of new importance upon measuring pressures immediately after shut-in.

Any single point of corresponding buildup time and pressure difference from the line of unit slope can be used to compute the unit storage factor \( C \) (in reservoir bbl/psi). From Fig. 2, such a point is 560 psi at 0.67 hours. Thus,

\[
C = \frac{q_{2} \Delta \tau}{(p_{w} - p_{m})} = \frac{(157 \text{ STB/D})(1.6 \text{ res. bbl/STB})(0.67 \text{ hr})}{(560 \text{ psi})(24 \text{ hr/D})} = 0.125 \text{ res. bbl/psi.}
\]

Several comments are pertinent here. First, the value of the storage constant can be determined accurately regardless of the mode of fluid storage, i.e., whether due to compression of a single-phase fluid, or due to rise of a liquid level in the wellbore. Second, the effective volume of the wellbore can be found from the storage constant by considerations reviewed in an earlier work. Normally, one would expect a reasonably close check with the known volume obtained from well-completion data. But there is the interesting possibility of estimating the volume of reservoir voids that communicate with the wellbore when the measured volume is larger than the known wellbore volume. An example of this situation will be shown later. Third, it should be clear that the mode of storage of fluid in the wellbore could change during a test, and thus the storage constant could change abruptly as well. An example would be a damaged well producing at a pressure below the bubble point, whose mean reservoir pressure is considerably above the bubble point. Upon shut-in, the liquid level might rise until the wellbore is filled with liquid and all gas has gone into solution, then storage would continue due to compression of the undersaturated liquid. The same effect would be common in pressure fall-off in water injection wells. Peculiar effects that could be attributed to this factor have been noticed in short-time well test data.

Once the storage constant, \( C \), has been determined, it is possible to compute the dimensionless storage constant, \( C_{0} \), from Eq. 3. For Well A,

\[
C_{0} = C = \frac{q_{2} \Delta \tau}{(p_{w} - p_{m})} = \frac{(157 \text{ STB/D})(1.6 \text{ res. bbl/STB})(0.67 \text{ hr})}{(560 \text{ psi})(24 \text{ hr/D})} = 0.125 \text{ res. bbl/psi.}
\]

The formation constants used were obtained from Ref. 3. Since the equation of any unit-slope straight line on Fig. 1 is known to be

\[
p_{s} = p_{a} + C_{0} \cdot t_{o} = \frac{1}{\rho_d} = \frac{1}{\rho_d} \cdot t_{o} \quad \quad \quad \quad \quad (5)
\]

dimensionless storage constant can be used to locate the unit-slope line and thus the approximate position of match on the type-curve as shown on Fig. 1. The field data plot is moved along the \( C \) unit slope line of 5.68 x 10^2 until the field and analytical curves appear to be a reasonable match. Once a match has been obtained, the field matching point from the two type-curves can be used to compute the formation flow capacity, \( k \), and the hydraulic diffusivity, \( k/(\phi \mu c) \).

Such a match is shown on Fig. 4 for Well A to

\[
\Delta t = 10 \text{ hrs, } p_{w} - p_{m} = 10^4 \text{ psi}, \quad t_{o} = 3.17 \times 10^4 \text{ psi}.
\]

From Eq. 2:

\[
p_{o} = 68.3 = \frac{k(h(p_{w} - p_{m}))}{141.4 q_{2} \Delta \tau}.
\]

The value of the skin effect, \( s \), can be read directly from Fig. 4 from the master type-curves and the position of the match. For Well A, the value estimated is \( s = 2.3 \). The value of the skin effect obtained by conventional analysis for Well A was 2.94. The good comparison found here can be more accidental than real. It should be clear from inspection of either Fig. 1 or Fig. 4, that the curvature away from the unit slope storage line is fairly similar for lines representing different skin effects. Thus it is possible to obtain a match with the type-curve of different skin effect, \( k \), and the hydraulic diffusivity, \( k/(\phi \mu c) \).

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A semilog plot represents only a long-time solution to the flow problem. This important difference means that it is possible to extract more information from type-curve analysis (when it is possible to find a match) than from conventional analysis. The example presented previously does not point out sufficiently the generality of the type-curve method. For example, the match shown on Fig. 4 could have been found without calculation of the dimensionless storage constant, \( C \). Once the match was identified, both \( C \) and \( s \) could have been read as parameters from the type-curve. Recall that it was assumed that the porosity and compressibility were known in calculation of \( C \) in the example. This is not necessary. Finally, inspection of usual expressions for calculation of the skin effect (see Ref. 4) reveals that conventional analysis provides a summation of terms involving the skin effect and hydraulic diffusivity. It is not possible to separate this summation by conventional analysis, unless one or the other is known. This can be done by type-curve matching. But it should be obvious that type-curve matching is a graphical procedure with certain limitations. It does not replace conventional analysis, but is, instead, an additional tool of great utility.

Stimulated Wells

In the case of a stimulated well (skin effect less than zero), the cause of the negative skin effect can have an important effect upon a type-curve. If a negative skin effect is caused by acidizing at injection pressures too low to cause fracturing, it is reasonable to postulate an annular region of high permeability near the well. If the formation is limestones, both this effect and a physically-increased wellbore radius could result. On the other hand, hydraulic fracturing could result in a high-conductivity fracture of any natural orientation extending from the well into the formation.

For the case of improved permeability near the wellbore, we might expect that type-curves similar to Fig. 1 but including negative skin effects would be satisfactory for test interpretation. Such curves have been presented. But these curves would not be valid for hydraulically fractured wells. It is well known that flow from a fracture plane intersecting a well should initially be similar to flow in a linear system. A
Field Example of Stimulated Well with Storage

Normally, one would not expect to see a significant storage effect with stimulated well test data, i.e., a well exhibiting a large, negative skin effect. However, it is possible, as the next set of field data will demonstrate. Fig. 6 presents a conventional pressure buildup plot for a gas well—one that produces natural steam from the Geysers Steam field in California. The well (here called Well B) was drilled with air and completed open-hole. During initial production clean-up, rock and debris were blown from the well. A conventional buildup plot such as Fig. 6 usually shows the characteristics of a stimulated well for other wells in this field; however, Fig. 6 has the character of a slightly damaged well with possible wellbore storage. The type-curve plot for a damaged straight line was established during the later portion of the buildup since the extrapolation yields a *p* value reasonably close to the static pressure prior to the test. The start of the straight line occurs at a buildup time, *Δt*, of about 35 hours.

Fig. 7 presents a type-curve plot for the pressure buildup data shown in Fig. 6. Data taken during the first 0.1 hour of buildup usually form a line on the slope and indicate that storage is controlling. At about 0.25 hour, the buildup data form a line of slope 0.5, indicating a long period of linear flow, and the likelihood that this well communicated through a natural fracture. Linear flow persists for this test until a buildup time of about 4 hours. It is possible to estimate the time of beginning of pseudoradial flow by a criterion from Ref. 6: the pressure rise at the end of the line.

In regard to wellbore storage effects for fractured wells, it would be rare that wellbore storage would ever be apparent. This is because for a fractured well the wellbore pressure changes slowly. Thus the wellbore storage changes slowly. However, extreme cases have been observed in field data and will be presented later.

It should be emphasized that our main purpose here is to illustrate the application of short-time well test data analysis. Refs. 2, 4, 5, and 6 present detailed information concerning the origin of the type-curves, as well as background information.

The type-curve plot was not necessary to establish the proper straight line. However, it has been useful for this purpose in other buildup test analyses. In a number of cases, the type-curve plot has indicated that the proper straight line was not the apparent one when several straight lines appeared on a conventional plot. In one particularly complicated buildup test, the type-curve indicated that the first of three successive straight lines was actually the proper line and that the well was in the corner of intersecting, sealing flow barriers.

The test of Well B indicated in Figs. 6 and 7, where the type-curve did indicate useful additional data. Recall that Fig. 6 had the appearance of a slightly damaged well with possible storage. The type-curve plot for a damaged straight line was established during the later portion of the buildup since the extrapolation yields a *p* value reasonably close to the static pressure prior to the test. The start of the straight line occurs at a buildup time, *Δt*, of about 35 hours.

Since the well produces a single-phase fluid (gas), storage volume can be estimated readily.

\[ V = \frac{C_r T}{L} \left( \frac{1}{1.14} \right) \left( \frac{0.928}{0.73} \right) \left( \frac{815}{R} \right) \frac{M}{C_{p}} \left( \frac{18}{(0.0069 \text{ psi}^{-1})} \right) = 74,500 \text{ cu ft} \]

Gas compressibility was evaluated at a mean storage pressure.

The astonishing fact in this case is that the actual wellbore volume is approximately 1,550 cu ft. This means a volume of 74,500 cu ft is considerably larger than the estimated wellbore volume. Part of the difference could be due to lagging of rock that was blown from the well during clean-up. In this case, the explanation is supported by natural formation cavelike that communicate directly with the wellbore. Driller reported numerous instances of bit dropping, indicating natural cavities. The large storage volumes for the long period of linear flow, suggests that this well communicates with one or more natural cavities of large total volume and that each cavity is probably large in area but not very thick.

Although the behavior of Well B is not unusual for wells in the Geysers Steam field, it would not be expected that such field data would be encountered often in conventional buildup analysis. The long periods of linear flow in tests on hydraulically fractured gas wells, the unit slope storage effect has not been apparent on type-curves. Clearly, however, storage should be a predominant factor in wells completed by nuclear fracturing. The analysis presented in this study should find excellent application in this case.

In regard to the foregoing estimate of storage volume, one must approximate — the compressibility of the gas is a function of pressure. For more precise work, well-test data identified as being under storage control can be analyzed with a rigorous material balance for a volumetric gas container (storage volume).

As for the linear flow period, the presence of damaged wellbore storage (or a great deal of stor- age) near a well could prevent or distort a line of slope 0.5 on a type-curve. (See Ref. 6 for a discussion of this point.) In one case with a notch-fractured well, the well was a low one (252 ft) and the linear flow, but the type-curve data approached the line of slope 0.5 from above the line. There was no indication of storage for the well, and it was concluded that the well was damaged at the wellbore level.

The type-curve was the only clue to this information.

Linear flow will also result in a straight line on cartesian coordinates when the pressure is plotted vs the square root of time. The importance of wellbore storage will often yield a straight line when there is no line of slope 0.5 on a type-curve plot, or when data are not properly selected. It is recommended that both type-curve and cartesian plots be used in analyzing data when making any decision concerning the interpretation of data. This remark also applies to wellbore test analysis. The type-curve does not replace conventional buildup or drawdown plotting. The graphs complement each other. An interpretation that explains detail on both the conventional plot and the type-curve plot can be viewed with some confidence.

A few concluding remarks are in order concerning the interpretation of well tests. First, linear flow has been encountered many cases for fractured gas wells in which linear flow controlled behavior to 20 or 30 hours of either flow time or buildup time. Common based linear flow analysis is a good approximation of wellbore storage in the center of a square. There is need for further investigation of static pressure extrapolation for fractured wells. It is significant that Russell and Truitt recommended that Muskat-type semilog plots be used to correct buildup pressures to the static pressure for fractured wells. Their work was based upon the wellbore storage concept that wellbore storage in the center of a square. There is a need for further investigation of static pressure extrapolation for fractured wells. It is significant that Russell and Truitt recommended that Muskat-type semilog plots be used to correct buildup pressures to the static pressure for fractured wells. Their work was based upon the wellbore storage concept that wellbore storage in the center of a square.

Discussion and Conclusions

The preceding examples are only a few of many field cases in which the log-log type-curve reveals a great deal of information not available from conventional buildup analysis. The importance of wellbore storage will usually be shown clearly on the type-curve if enough information has been taken immediately after shut-in. This has been observed for both oil and gas wells.

Again it should be emphasized that type-curve plotting is not intended to replace conventional buildup plotting. The plots are supplementary; each can reveal data that may not be apparent in the other.

The examples presented illustrate some of the simplest applications of the methods. Significant variations in flow rate on drawdown, or production per-
Analysis of Short-Time Transient Test Data
By Type-Curve Matching

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A new type-curve matching technique significantly easier than those previously published allows the estimation of permeability, skin factor, and wellbore storage coefficient from short-time transient test data. The method is explained here and is illustrated with several examples.

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Fig. 1 - Master type curve (reprinted by permission of Marathon Oil Co.).
Fig. 2 - Example 1 — Drawdown test on a new oil well.
Conclusions: (Earlougher and Kersch)

1. The curve-matching technique presented in this paper can be used to estimate formation transmissibility, skin factor, and wellbore storage coefficient from short-time transient test data. The method should not be used if well known semi-logarithmic methods can be applied. (why do they say this?)

2. To use this technique it is necessary that formation porosity, formation thickness, system compressibility, and wellbore radius be known or estimated. The test must involve a known constant flow rate.

3. The type-curve matching technique presented here applies to pressure drawdown, pressure buildup, injectivity, and pressure falloff testing.

4. Results tend to be more accurate if the wellbore storage coefficient, $C$, can be determined by means independent of the transient test. However, if that is not possible, acceptable results can still be obtained.

5. During this study, we demonstrated numerically that the skin factor can be included with the wellbore storage coefficient in constructing type curves for short-time transient behavior.
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Introduction

Occasionally, insufficient transient test data are available for analysis using semilogarithmic plotting methods. This usually happens when data collection stops before wellbore storage (afterflow) becomes negligible. Under these circumstances, the semilogarithmic straight line does not develop, and common semilogarithmic analysis methods cannot be used. When such methods cannot be used, the engineer either obtains no information from the test or must use available, short-time data to estimate reservoir characteristics. This paper presents a technique for the approximate analysis of such short-time transient test data. The method applies to buildup, falloff, drawdown, and injectivity tests when wellbore storage effects are important. It should not be used if data can be analyzed by more conventional, semilogarithmic plotting methods.

It has long been recognized that wellbore storage (afterflow) can provide transient test analysis. Several ways have been suggested for determining when wellbore storage exists. For example, semilogarithmic techniques can be used for transient test analysis. Gladfelter et al. and Russell present calculation methods for analyzing the portion of transient test data influenced by wellbore storage. Curve matching techniques have also been proposed for accomplishing such analyses.

All these methods have disadvantages. The techniques presented by Gladfelter et al. and Russell utilize either trial-and-error analysis or require that the afterflow schedule be calculated, or both. These approaches are tedious and not always successful.

In spite of its disadvantages, curve matching seems to be the most promising of the methods, particularly for the engineer who does not have a computer available. Cooper et al. present type curves and an analysis technique for specific flow and injection tests with the well shut in before testing. At the start of the test, the pressure instantaneously changes to some new value. Then both pressure and flow rate vary during the test. The Cooper-Bredoe-Posadopulos type curves are useful for analyzing data taken during the flow period of a depletion test. Agarwal et al. point out that neglect of the skin effect makes the Cooper-Bredoe-Posadopulos type curves of dubious value. In any case, these curves do not apply to the more common transient testing situations: buildup, falloff, injectivity, and drawdown. Ramsey and Agarwal suggest matching these type curves for obtaining skin factors and wellbore storage coefficient as parameters. Ramsey’s curve-matching method requires that the data plot be slid both horizontally and vertically to obtain a match. This feature and the fact that the curves have very similar shapes make the matching technique difficult to use unless there are data at least onto the start of the semilog straight line.

McKinley uses a similar approach, but with a different kind of type-curve plot. He plots his type curves so there is only one family of curves and so the data plot is slid only horizontally during the matching process, thus providing two advantages over Ramsey’s method. But McKinley assumes that \( \Delta P/\Delta t = 1.208 \times 10^{-2} \) psi per day and \( \Delta t = 0 \) in his type curves, thus reducing the accuracy of the data match. McKinley’s and Barbé and Boyd’s present data and several examples of the use of McKinley’s method. They imply that this analysis procedure can be used in lieu of the more common semilogarithmic techniques, thus allowing much shorter transient tests than normal. An MS thesis from the U. of Zulia compares the Ramsey and McKinley methods for well tests on Venezuelan wells. It concludes that the Ramsey curves gave good results in about two-thirds of the cases, and that the McKinley curve gave good results in about 40 percent of the cases—both that neither method was very good by itself. Our experience indicates that any type-curve matching technique is less accurate than semilogarithmic analysis methods. This applies to techniques already in the literature as well as the one presented here. We propose using a type-curve matching analysis only as a last resort, when it is clear that a test has been run for insufficient time to be analyzed in better, more accurate ways.

Changes in the wellbore storage coefficient have been shown to have a significant effect on the early-time pressure response during a well test. Type curves presented in this paper and elsewhere are valid only if the wellbore storage coefficient stays constant throughout the test.

We calculated the same kind of data presented by Ramsey, Agarwal et al., and McKinley to construct type curves that should allow better type-curve analysis of data presently available. The type curves included in this paper provide the following advantages:

1. There is only one family of curves.
2. The matching is essentially in one dimension. Most of the match is performed with horizontal sliding only, although slight vertical corrections can be made to improve match accuracy.
3. They allow for estimating skin factor.
4. They include the effects of porosity, compressibility, and wellbore radius known or estimated.

In spite of these advantages, the approach should be used only when semilog analysis techniques cannot be used because a transient test has been too brief for wellbore storage effects to become significant. The formation permeability or \( k/\mu \) calculated by this technique is essentially correct within a factor of 2 or 3; the skin factor calculation is qualitative, indicating the approximate degree of damage or improvement. The curve-matching method requires that formation porosity and thickness, fluid viscosity and compressibility, and wellbore radius be known or estimated.

Six examples illustrating the type-curve matching technique are included. They employ actual field data and computed data (taken from the literature) to show that the technique gives satisfactory results.

The Type-Curve Matching Method

Type Curves

The theoretical reasoning behind the type-curve plotting used here is explained in the Appendix. Fig. 1 shows the type curves used in the curve-matching method.

The dimensionless wellbore storage coefficient used in Fig. 1 is defined by

\[
C_w = \frac{5.6146}{\phi \beta \rho g \Delta \mu} \quad (1a)
\]

or

\[
C_w = \frac{0.89359}{\phi \beta \rho g \Delta \mu} \quad (1b)
\]

where standard SPE nomenclature and field units are used.

The parameter on the curve in Fig. 1 is

\[
C = \frac{5.6146}{\phi \beta \rho g \Delta \mu} \quad (2)
\]

Test Analysis Method

We recommend using this type-curve matching technique only if conventional transient test analysis cannot be used. For this reason, the Ramsey type curve (log \( \Delta P/\Delta t \) vs log \( \Delta t \)), where \( \Delta P \) is the pressure change during the test and \( \Delta t \) is the time since the beginning of the test) should be plotted to determine how long wellbore storage is important. If the type curve indicates the possibility of analyzing with a semilog plot, that should be done and the technique presented here should not be used. If the test is not long enough for semilogarithmic analysis, the following approach can be used.

1. Plot observed test data as \( \Delta P/\Delta t \) vs \( \Delta t \) on the axis of a log-log paper. The axes of the abscissa of log-log paper of the same scale. We normally plot tracing paper over Fig. 1. Trace the major grid lines for reference, and use the grid of Fig. 1 to plot actual data on the tracing paper. Thus, the data plot and the type curve have the same scale. The curves on Fig. 1 are ignored during this plotting—only the grid of Fig. 1 is used.

2. Estimate the wellbore storage coefficient expected from completion details by using

\[
C = \frac{V_e}{V_o} \quad (3)
\]

for a completely field-filled wellbore, or

\[
C = \frac{V_e}{V_o} \left( \frac{2}{\kappa + 2} \right) \quad (4)
\]

for a wellbore with a rising or falling liquid level.

3. Calculate the location of the horizontal asymptote on the data plot.

\[
\frac{\Delta P}{\Delta t} = \left( \frac{\Delta t}{\delta t} \right) = 1.0 \quad (5)
\]

The quantity on the left-hand side of Eq. 5 is the value of \( \Delta P/\Delta t \) observed on the data plot when

\[
\frac{\Delta P}{\Delta t} = \left( \frac{\Delta t}{\delta t} \right) \quad (6)
\]

*Typical example of type curve is available from the authors on request. JOURNAL OF PETROLEUM TECHNOLOGY

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on the type curve of Fig. 1.

4. Place the type curve data plot over Fig. 1 so that the asymptote calculated in Eq. 5 overlays the value of 1.0 on the ordinate of Fig. 1; that is, so

\[
\frac{3\Delta P}{24C} \frac{m}{Q \Delta t_P} = 1.0.
\]

5. Slide the data plot horizontally until the best match is obtained with one of the curves on Fig. 1.

To get a good match it may be necessary to add a slight amount of vertical movement to the data plot. In any case, it is important that the grids of the two curves be kept parallel to each other.

6. Sketch the matched curve onto the data plot. From Fig. 1 read the value of

\[
C (\frac{m}{Q \Delta t_P})_{fig.1,W}
\]

Pick any convenient match point and read the values of

\[
\left( \frac{\Delta P}{\Delta t} \right)_{u}, (\Delta t)_{u}
\]

from the data plot and the values lying directly under this point from Fig. 1.

\[
C = \frac{q \Delta P}{24C} \frac{m}{Q \Delta t_P} \left( \frac{\Delta t}{\mu} \right)_{u}, \ldots \ldots (6)
\]

These values are used in estimating transmissibility, skin factor, and wellbore storage coefficient.

7. If any vertical movement was necessary during the curve-matching process, recalculate the wellbore storage coefficient.

8. Estimate formation transmissibility:

\[
\frac{kh}{\mu} = C \left( \frac{m}{Q \Delta t_P} \right)_{fig.1,W}, \ldots \ldots \ldots (7)
\]

where \( C \) is from Eq. 6.

9. Estimate the skin factor from

\[
s = \frac{1}{3} \ln \left( \frac{4khC}{\mu \Delta t_P \left( \frac{m}{Q \Delta t_P} \right)_{fig.1,W}} \right)
\]

Although we feel this method is an improvement over other type-curve matching methods presented in the literature, it is still not exact. Values of \( kh/\mu \) may be uncertain within about a factor of three, as illustrated in the Examples Section. This uncertainty occurs because of the similarity in shape of the type curves and the possibility of matching two or three curves to the same data. The calculated value of the skin factor will probably be incorrect, also. However, the skin factor calculated from Eq. 8 should indicate the relative amount of damage or improvement.

This curve-matching approach will give a much more accurate value of formation transmissibility if the wellbore storage coefficient and skin factor are known independently. Then, in the matching process, one has the horizontal alignment from the wellbore storage coefficient and can pick the curve in Fig. 1 that should be matched.

**Examples**

**Example 1**

A pressure drawdown test on a new oil well appears to be strongly influenced by wellbore storage. Nevertheless, enough data exist to determine formation properties from the semilog plot. We analyze this test with the techniques presented in this paper, and compare the results with results from the semilog analysis. Table 1 gives pressure time data. Other known data are:

\[
q_p = 179 \text{ STB/D}, \quad c_1 = 8.2 \times 10^{-4} \text{ psi}^{-1},
\]

\[
B_o = 1.2 \text{ RB/STB}, \quad r_m = 0.276 \text{ ft},
\]

\[
A = 35 \text{ ft}, \quad \phi = 18 \text{ percent}.
\]

Since we do not know completion details, we cannot use Eq. 3 or 4 to estimate the wellbore storage coefficient. Thus, we must match without this aid. We plot \( \Delta P/\Delta t \) vs \( \Delta t \) on tracing paper laid over the Fig. 1 grid. Then we slide the tracing paper data plot on Fig. 1, as described in Step 5 of the section on Test Analysis Method, until a good match results. Fig. 2 schematically shows the data plot matched to Fig. 1. (For clarity in printing, the grid is omitted.) Match point data are shown in Fig. 2. This completes Steps 1, 5, and 6 of the analysis procedure.

We calculate the wellbore storage coefficient (Step 7) using Eq. 6 and the match data from Fig. 2:

\[
C = \frac{q \Delta P}{24C} \frac{m}{Q \Delta t_P} \left( \frac{\Delta t}{\mu} \right)_{u}, \ldots \ldots \ldots (6)
\]

\[
= \frac{(179)(1.2)(0.1053)}{24(10)} = 0.0942 \text{ RB/psi}.
\]

**TABLE 1 — PRESSURE DATA FOR EXAMPLE 1**

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Pressure Change (psi)</th>
<th>( \Delta P/\Delta t ) (psi/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.0</td>
<td>19.7</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0</td>
<td>28.1</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0</td>
<td>43.1</td>
</tr>
<tr>
<td>0.7</td>
<td>0.0</td>
<td>88.3</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>75.1</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0</td>
<td>114.5</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0</td>
<td>135.5</td>
</tr>
<tr>
<td>5.0</td>
<td>0.0</td>
<td>182.2</td>
</tr>
<tr>
<td>7.0</td>
<td>0.0</td>
<td>163.2</td>
</tr>
<tr>
<td>10.0</td>
<td>0.0</td>
<td>166.7</td>
</tr>
<tr>
<td>20.0</td>
<td>0.0</td>
<td>171.2</td>
</tr>
<tr>
<td>30.0</td>
<td>0.0</td>
<td>179.9</td>
</tr>
<tr>
<td>50.0</td>
<td>0.0</td>
<td>178.2</td>
</tr>
<tr>
<td>70.0</td>
<td>0.0</td>
<td>177.1</td>
</tr>
</tbody>
</table>

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We use Eq. 7 to calculate $kh/\mu$ (Step 8):

$$\frac{kh}{\mu} = \frac{C}{(\frac{\Delta p}{\Delta t})_{\text{Type-I, IJK}}}$$

$$= \frac{(0.942)(49.000)}{(1.0)} = 4620 \text{ md ft/cp}.$$  

The skin factor is estimated from Eq. 8 (Step 9):

$$s = \frac{2}{2} \ln \left[ \frac{\phi_{\text{bore}}/C_v}{C_v} \right]_{\text{Type-I, IJK}}$$

$$= \frac{2}{2} \ln \left[ \frac{0.188(2.2 \times 10^{-3})/35}{0.276(10^4)} \right] = 18.$$  

These results are approximate; the technique normally should be used only when another analysis method fails. In this example, however, we wish to illustrate the analysis method and give an indication of its accuracy. Thus, we chose a test with sufficient data for a conventional, semilog straight-line section analysis that gives

$$\frac{kh}{\mu} = 3500 \text{ md ft/cp}$$

and $s = 12$.

The transmissibility we calculate is within 32 percent of the value from the semilog plot; but the skin factor is off by 50 percent. In spite of the approximate nature of our analysis technique, fairly good results were obtained.

The wellbores storage coefficient, $C = 0.0942 \text{ RB/psi}$, appears reasonable for this reason. If we assume an oil gravity of 30° API (\(\rho_{\text{o}} = 54.7 \text{ lbm/cu ft}\)) and that the fluid level is dropping, then we compute $V_f = 0.0358 \text{ bbl/ft}$ from Eq. 5. This pressure drop in a 6-ft. ID pipe (\(r = 0.25 \text{ ft}\)) is not out of line with what little we know about the completion.

**Example 2**

Occasionally, early-time transient test data show considerable scatter. This happens because normal measurement errors in $p$ and $t$ represent a higher percentage of $\Delta p$ and $\Delta t$ at early times than at later times. The scatter may be further amplified by forming the quotient, $\Delta p/\Delta t$. This example shows that the type-curve-matching analysis technique can be applied to data with an early-time scatter. The data are from a pressure buildup test in a pumping well in Illinois: $h = 25 \text{ ft}$, $c = 1 \times 10^{-4} \text{ psii}^{-1}$, $q = 20 \text{ percent}$, $r_i = 4.5 \text{ in}$ (shot hole), $V_f = 0.0411 \text{ bbl/ft}$. $B = 1 \text{ RB/STB}$.

Test data are plotted as $\Delta p/\Delta t$ vs $\Delta t$ in Fig. 3. In this case we know that rising fluid level storage is important. Using Eq. 4, we estimate the wellbore storage coefficient, $C = 0.005 \text{ RB/psi}$. Then we can estimate the location of the horizontal asymptote from Eq. 5:

$$\Delta t = \frac{h}{C_v} = \frac{24C_v}{24(0.0942)} = 29 \text{ psii/hr}.$$  

Proceeding through Steps 4 and 5, we get the match indicated in Fig. 3. We use Eq. 7 to calculate

$$\frac{kh}{\mu} = \frac{(0.942)(4200)}{\frac{1}{1}} = 400 \text{ md ft/cp}.$$  

This agrees well with results from other tests in the same area. Skin is estimated from Eq. 8:

$$s = \frac{1}{2} \ln \left[ \frac{0.20(10^2)(12.5)(10^4)}{0.89359(0.096)} \right] = -0.1.$$  

Thus, we conclude there is essentially no damage or improvement.

**Example 3**

This example demonstrates the magnitude of errors that might occur in using the technique presented here. We analyze the calculated data presented as Fig. 4 by McKinley.** The log-log plot of $\Delta p/\Delta t$ vs $\Delta t$ is shown in Fig. 4, along with two possible type-curve matches. The solid line is the match for the $C_v = 10^5$ curve of Fig. 1. The dashed line is for $C_v = 10^6$. The shapes of these two curves are very similar; it is not obvious which one, or which curve between, gives the best match. Table 2 compares results calculated from these two matches with McKinley's match and the data used to calculate his curve. For these calculations we used $q = 175 \text{ STB/D}$ and $B = 1.0 \text{ RB/STB}$. Although McKinley does not specifically give a value of $q_{\text{real}}/e$, we assume it to be $1.028 \times 10^{-4} \text{ cp sq ft/(md ps)}$, the value used for his type curves.

This example indicates that errors of a factor of 2 to 3 can occur in the permeability calculation. Nevertheless, it is encouraging that even though $C_v = 10^5$ varied by a factor of 10 in this example, $kh/\mu$ varied by a factor of only 2. When all other methods fail, it is better to know permeability within a factor of 2 or 3 than not at all. Unfortunately, the uncertainty in skin factor can be worse. If the uncertainty in $C_v$ is $10^5$, then the skin factor will be uncertain by about the additive quantity, $1.15 \times$.

**Example 4**

Fig. 5 is a type-curve plot of McKinley's calculated data (McKinley's Fig. 6a and 6b). The triangles represent calculated test response for a well with $s = 8$. The circles represent response calculated for a well with $s = 2$. McKinley's other data are $kh/\mu = 1,000 \text{ md ft/cp}$, $q = 100 \text{ STB/D}$, $B = 1.0 \text{ RB/STB}$. $C = 0.178 \text{ bbl/ps}$, $(\phi Ae_1)/h = 1.028 \times 10^{-4} \text{ cp sq ft/(md ps)}$. Fig. 5 also shows the type-curve matches obtained using Fig. 1. The results given in Table 3 compare quite well with the parameters used by McKinley to calculate the data.

**Example 5**

Russel** provides the field data shown in the type curve of Fig. 6. These data are also used by McKinley in his Example F-2.** Since there are no data points at very early times, these data are difficult to analyze. Equally good data matches can be obtained using the $C_v = 10^5$ or the $C_v = 10^6$ curves, shown as the dashed and solid lines, respectively, in Fig. 6. Using Russell's data of $q = 157 \text{ STB/D}$, $B = 1.6 \text{ RB/STB}$, $s = 0.3 \text{ cp}$; $h = 4.0 \text{ ft}$; $\phi = 0.10$; $c_i = 2 \times 10^{-4}$, and assuming $c_r = 0.25 \text{ ft}$, we obtain the results shown in Table 4. Both Russell's and McKinley's results are shown in the table for comparison.

In spite of the marginal quality of the data, the most useful results are obtained using the matching procedure presented in this paper. If several $C_v$ curves seem to match the data equally well, then we can estimate a range of transmissibility. This gives an idea of the accuracy of the results. In this case, we conclude the transmissibility is between 370 and 640 and that the well is badly damaged.

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**Table 2 — RESULTS FOR EXAMPLE 3**

<table>
<thead>
<tr>
<th>$C_v = 10^5$</th>
<th>$C_v = 10^6$</th>
<th>McKinley</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (bbl/ps)</td>
<td>$1.64 \times 10^{-4}$</td>
<td>$1.76 \times 10^{-4}$</td>
<td>$1.74 \times 10^{-4}$</td>
</tr>
<tr>
<td>$kh/\mu$ (md ft/cp)</td>
<td>0.7</td>
<td>4.6</td>
<td>0</td>
</tr>
</tbody>
</table>

---

**Table 3 — RESULTS FOR EXAMPLE 4**

<table>
<thead>
<tr>
<th>Circles (s = 8)</th>
<th>McKinley Curve Match</th>
<th>Triangles (s = 8)</th>
<th>McKinley Curve Match</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_v$ (bbl/ps)</td>
<td>$0.185$</td>
<td>$1.018$</td>
<td>$0.178$</td>
</tr>
<tr>
<td>$kh/\mu$ (md ft/cp)</td>
<td>$1.400$</td>
<td>$500$</td>
<td>$1.000$</td>
</tr>
<tr>
<td>$s$</td>
<td>$3.4$</td>
<td>$&gt;25$</td>
<td>$&lt;0$</td>
</tr>
</tbody>
</table>

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**Table 4 — RESULTS FOR EXAMPLE 5**

<table>
<thead>
<tr>
<th>Circles (s = 8)</th>
<th>McKinley Curve Match</th>
<th>Russell</th>
<th>McKinley Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (bbl/ps)</td>
<td>$0.0105$</td>
<td>$0.0105$</td>
<td>$0.011$</td>
</tr>
<tr>
<td>$kh/\mu$ (md ft/cp)</td>
<td>$370$</td>
<td>$636$</td>
<td>$310$</td>
</tr>
<tr>
<td>$s$</td>
<td>$18$</td>
<td>$10$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

---

**Footnotes:**


**Slide — 19**

![Image of graph showing pressure transient testing](link-to-graph)
Example 6

Fig. 7 shows data from Example 6-3 of McKinley, a buildup test on a gas-lift well. The test is so dominated by wellbore storage that conventional data analysis is impossible. The only other data McKinley gives are $q = 3.3 \text{ STB} / \text{D}, B = 0.1 \text{ RB} / \text{STB}, \mu = 2 \text{ cp},$ and $k = 6 \text{ ft}.$

The data plotted in Fig. 7 can be matched reasonably well with values of $C_a$ and $C_N$ from Table 6. Using these matches we calculate the results shown in Table 5. McKinley does not give enough data to estimate $C_a.$ However, we know $a$ for $C_a e^{at} = 10^4$ will be about 6.9 above the value for $C_a e^{at} = 10^3.$ The type-curve matching procedure gives a transmissibility value that varies by a factor of 3. Even so, without some kind of type-curve matching procedure, this data could not have been analyzed.

Conclusions

1. The curve-matching technique presented in this paper can be used to estimate formation transmissibility, skin factor, and wellbore storage coefficient from short-time transient test data. The method should not be used if well known semiempirical methods can be applied.

2. To use this technique it is necessary that formation porosity, formation thickness, system compressibility, and wellbore radius be known or estimated. The test must involve a known constant flow rate.

3. The type-curve matching technique presented here applies to pressure drawdown, pressure buildup, injectivity, and pressure fall-off testing.

4. Results tend to be more accurate if the wellbore storage coefficient, $C_a$, can be determined independently of the transient test. However, if that is not possible acceptable results can still be obtained.

5. During this study, we demonstrated numerically that the skin factor can be included with the wellbore storage coefficient in constructing type curves for short-time transient behavior.

Nomenclature

$B =$ formation volume factor, RB/STB
$c =$ compressibility, psi$^{-1}$
$c_t =$ total system compressibility, psi$^{-1}$
$C =$ wellbore storage coefficient, RB/psi

TABLE 5—RESULTS FOR EXAMPLE 6

<table>
<thead>
<tr>
<th>$C_a$</th>
<th>$C_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^{at} = 10^4$</td>
<td>$e^{at} = 10^3$</td>
</tr>
<tr>
<td>McKinley</td>
<td>McKinley</td>
</tr>
<tr>
<td>$C_a =$ dimensionless wellbore storage coefficient, Eq. 1</td>
<td></td>
</tr>
<tr>
<td>$a =$ acceleration due to gravity, ft/sec$^2$</td>
<td></td>
</tr>
<tr>
<td>$q =$ units conversion factor, $32.17 \text{ lb} / \text{ft}$</td>
<td></td>
</tr>
<tr>
<td>$h =$ formation thickness, ft</td>
<td></td>
</tr>
<tr>
<td>$p =$ pressure, psi</td>
<td></td>
</tr>
<tr>
<td>$\delta =$ pressure change during the test, psi</td>
<td></td>
</tr>
<tr>
<td>$P =$ flow rate, STB/D</td>
<td></td>
</tr>
<tr>
<td>$w =$ wellbore radius, ft</td>
<td></td>
</tr>
<tr>
<td>$t =$ time, hours</td>
<td></td>
</tr>
<tr>
<td>$b =$ wellbore volume per unit length, lb/ft $^3$</td>
<td></td>
</tr>
<tr>
<td>$\mu =$ viscosity, cp</td>
<td></td>
</tr>
<tr>
<td>$\rho =$ density, lb/ft $^3$</td>
<td></td>
</tr>
<tr>
<td>$\phi =$ porosity, fraction</td>
<td></td>
</tr>
</tbody>
</table>

References


APPENDIX

This appendix presents the reasoning behind the type-curve matching technique presented in the paper. We start by defining three dimensionless variables used commonly in reservoir fluid flow.

Dimensionless time:

$$t_d = \frac{t}{C_a}$$

Dimensionless wellbore storage coefficient:

$$C_b = \frac{C_a}{2 \pi \rho \rho_c}$$

Dimensionless pressure:

$$p_d = \frac{p_c}{C_a}$$

Dimensionless time, dimensionless wellbore storage coefficient, and skin factor (always dimensionless) are usually taken as the independent variables in fluid flow work. Dimensionless pressure is commonly expressed as a function of dimensionless time. It is possible, however, to express $p_d$ as a function of $t_d$, $C_b$, and $s$. Adopting this convention and using an approximation given for $p_d$ at short time, we write:

$$p_d = p_0 + p_{ac} e^{-t_d}$$

Rearranging Eq. 4-4 and applying Eqs. A-1, A-2, and A-3:

$$p_{ac} = p_0 e^{-t_d}$$

at very short times. Furthermore, from Eqs. A-1 and A-2, we find:

$$p_d = \frac{p_0}{1 + \frac{t_d}{t_0}}$$

Since $p_0$ is a function of $C_a$ and $C_b$, and we suspect from Eqs. A-5 and A-6 that a plot of $p_0 C_b / t_0$ vs $t_d / C_a$ should give a family of curves parametric in $C_b$ and $t_0$, with all curves asymptotically approaching $p_0 C_b / t_0 = 1$ at small $t_d / C_a$. This indeed happens, but the number of curves is so great that the plotting approach seems to be of little value.

The multidi of curves can be reduced to one family of curves by defining an effective wellbore radius, $r_e^{eff}$. In terms of this effective wellbore radius, the dimensionless storage and time terms become $C_b r_e^{eff}$ and $t_d / C_a r_e^{eff}$, respectively.

$$t_d / C_a r_e^{eff}$$

Note that Eqs. A-7 and A-8 can be reduced to G

Note that Eqs. A-7 and A-8 can be used to estimate $C_b$ and $t_0$ from the data. This can be done for intermediate times.

This can be rewritten:

$$p_0 = \frac{p_{ac} (t_d / C_d)}{1 - t_d / C_d}$$

Note that this is the same as Eq. A-9 with $t_0$ replaced by $t_d$ and the skin term omitted. If we use this modified dimensionless time and the modified dimensionless storage coefficient of Eqs. A-7 and A-8, then the skin factor is correctly included in both the short-time (Eq. A-5) and the long-time (Eq. A-10) expressions for $p_d$.

It is not so simple to show analytically that this can be done for intermediate times. To do this we used numerical simulations of the simulations in our study compared well with the results of McKinley and Kersch for dimensionless storages greater than 100, showing that this can be done with reasonable accuracy.

Petroleum Engineering 648 — Pressure Transient Testing

Lecture 8 — Wellbore Storage: Historical Perspectives

Petroleum Engineering 324
Well Performance

Type Curve Analysis of Well Test Data Including Wellbore Storage and Skin Effects—
the Bourdet-Gringarten Type Curve (Transient Radial Flow Case)

"Man is only truly great when he acts from the passions."
—Benjamin Disraeli (1844)

Topic: Type Curve Analysis of Well Test Data Including Wellbore Storage and Skin Effects—the Bourdet-Gringarten Type Curve (Transient Radial Flow Case)

Objectives: things you should know and/or be able to do

Derivation of the Bourdet-Gringarten Type Curve Plotting Functions: (Transient Radial Flow Case with Wellbore Storage and Skin Effects)

- Be able to derive the “Earlougher-Kersh” relation that validates $C_D e^{C_D t}$ as the appropriate correlating parameter for the transient radial flow case (including wellbore storage and skin effects). This “long-time” relation is given as:
  $$p_{w}(t) = \frac{1}{2} \ln \left( \frac{4t_{f} C_D}{e^{2} C_D} \right) + \frac{1}{2} \ln \left( C_D e^{C_D t} \right)$$
  (where $e = 0.577216$ ... Euler’s constant)

- Be familiar with the “Gringarten” Laplace domain approach for the validation of $C_D e^{C_D t}$ as the appropriate correlating parameter for the transient radial flow case (including wellbore storage and skin effects). This is a fully analytical result and is valid for all times. This result is given by:
  $$\bar{p}_{w}(w) = \frac{1}{w} \left[ \ln \left( \frac{4t_{f} C_D}{e^{2} C_D} \right) + \frac{1}{2} \ln \left( C_D e^{C_D t} \right) \right]$$

- Application of the “Bourdet-Gringarten” Dimensionless Type Curve:
  - Be able to use the “Bourdet-Gringarten” dimensionless type curve ($p_{w}$ vs $t_{f} C_D$) for the analysis of well test data which exhibit transient radial flow behavior, including wellbore storage and skin effects. You should be able to develop and apply the following “match point” relations for the analysis of well test data using the “Bourdet-Gringarten” type curve:
    - **Formation Permeability:**
      $$k = \frac{141.2 \sqrt{t_{f} p_{w}}} {h}$$
    - **Dimensionless Wellbore Storage Coefficient:**
      $$C_D = 0.0002637 \frac{k}{\sqrt{\frac{p_{w}}{t_{f} C_D}}}$$
    - **Skin Factor:**
      $$s = \frac{1}{2} \ln \left( \frac{C_D e^{2}} {C_D} \right)$$
Lecture Outline:
Derivation of the Bourdet-Gringarten Type Curve Plotting Functions:
(Transient Radial Flow Case with Wellbore Storage and Skin Effects)
- Consider the transient radial flow relation
  \[ p_{D}(t) = \frac{1}{2} \ln \left( \frac{4}{e} t \right) + s \]
- Expand the skin factor term in an exponential-logarithmic form
  \[ s = \frac{1}{2} \ln \left( e^{2s} \right) \]
- Substitute the skin factor relation, then multiply and divide the \( t_D \) term by \( C_D \)
  \[ p_{D}(t_D) = \frac{1}{2} \ln \left( \frac{4}{e} t_D C_D \right) + \frac{1}{2} \ln \left( e^{2s} \right) \]
- Expand the \( C_D \) multiplier term as a separate logarithmic term
  \[ p_{D}(t_D) = \frac{1}{2} \ln \left( \frac{4}{e} t_D C_D \right) + \frac{1}{2} \ln \left( C_D \right) + \frac{1}{2} \ln \left( e^{2s} \right) \]
- Collect the \( C_D \) and \( s \) logarithms terms to yield the "Earlougher-Kersch" result
  \[ p_{D}(t_D) = \frac{1}{2} \ln \left( \frac{4}{e} t_D C_D \right) + \frac{1}{2} \ln \left( C_D e^{2s} \right), \text{ therefore, } p_{D}(t_D) = f(t_D/C_D, e^{2s}) \]
- Presentation of the "Gringarten" Laplace domain result for \( C_D e^{2s} \)
  - Begin with the definition of \( p_{D}(t_D) \) in the Laplace domain, \( \tilde{p}_{D}(s) \)
    \[ \tilde{p}_{D}(s) = \frac{1}{s} \ln \left( \frac{4}{e} C_D s \right) + \frac{1}{2} \ln \left( C_D \right) + \frac{1}{2} \ln \left( e^{2s} \right) \]
  - Recall the \( p_{D} \) solution for radial flow in the Laplace domain, \( \tilde{p}_{D}(s) \)
    \[ \tilde{p}_{D}(s) = \frac{1}{2s} \ln \left( \frac{4}{e} s \right) + \frac{1}{2} \ln \left( s \right) \]
  - Multiplying \( \tilde{p}_{D}(s) \) by \( s \) gives us
    \[ \tilde{p}_{D}(s) = \frac{1}{2} \ln \left( \frac{4}{e} \right) + \frac{1}{2} \ln \left( e^{2s} \right) \]
  - Dividing \( \tilde{p}_{D}(s) \) by \( u \) returns us to a more compact form of \( p_{D}(t_D) \)
    \[ \tilde{p}_{D}(u) = \frac{1}{2u} \ln \left( \frac{4}{e} u \right) \]

- Defining a new variable, \( w = uC_D \), and using the "shifting" theorem of Laplace transforms, we obtain
  \[ \tilde{p}_{D}(w) = \frac{1}{C_D} \tilde{p}_{D}(w/C_D) = \frac{1}{w} \ln \left( \frac{4}{e} w/C_D \right) + \frac{1}{2} \ln \left( w \right) \]
- The final result is
  \[ \tilde{L} \tilde{p}_{D}(w) = p_{D}(w/C_D) \]
- This result proves that \( C_D e^{2s} \) is the exact correlating parameter for the case of transient radial flow in an infinite-acting reservoir with wellbore storage and skin effects.

Application of the "Bourdet-Gringarten" Dimensionless Type Curve:
- "Type curve" plots of \( p_{D}(t) \) versus \( t/uC_D \)
- Base plot: Bourdet-Gringarten "type curve"
  - \( C_D e^{2s} > 0.5 \) transient radial flow (unfractured well) model.
  - \( C_D e^{2s} < 0.5 \) transient fractured well model.
- Approximation functions: \( p_{D}(t) \) and \( p_{D}(t/uC_D) \)
- Development of generalized "Match Point" relations
  - The pressure relation is always solved for formation permeability, \( k \).
  - The time relation is solved as follows: (various cases)
    - "Bourdet-Gringarten" type curve—\( p_{D}(t) \) and \( p_{D}(t/uC_D) \)
      - Reservoir Model: Unfractured well in an infinite-acting homogeneous reservoir (transient radial flow) with wellbore storage and skin effects.
      - Correlating Parameter: \( C_D e^{2s} \)
      - Solution Parameter (time axis): \( C_D \)
    - "Ramey-Barker" type curve—\( p_{D}(t) \) and \( p_{D}(t/uD) \)
      - Reservoir Model: Fractured well with an infinite conductivity vertical fracture in an infinite-acting homogeneous reservoir (transient flow) with wellbore storage effects, but NO skin effects.
      - Correlating Parameter: \( C_D \)
      - Solution Parameter (time axis): \( x_f \)
Bourdet-Gringarten "Pressure Derivative" Type Curve (with ratio function).
Bourdet-Gringarten "Pressure Integral" Type Curve (with ratio function).
Bourdet-Gringarten Type Curve (with ratio function, no derivative function).

Legend: Radial Flow Type Curves
- $p_{D1}$ Type Curve
- $p_{D2} = p_{D1}/p_D$ Type Curve

Wellbore Storage Domination Region
- $p_{D1} = \text{Unit Slope Line}$
- $p_{D2} = 1/2$

Wellbore Storage Distortion Region

Radial Flow Region $p_{D2} = 1$
"Constant" $p_{sD}$ Approximation:

$$p_{WD}(t_D, s, C_D) = p_{sD}(t_D) \left[ 1 - \exp \left( \frac{-t_D}{C_D p_{sD}(t_D)} \right) \right]$$
Bourdet-Gringarten Type Curve Generated with a "Linear $p_{sd}$" approximation model (SPE 21826).

"Linear" $p_{sd}$ Approximation:

$$p_{sd}(t_d, C_D) = \frac{1}{p_{sd}(t_d)} + \frac{1}{t_d C_D} \left[ \frac{-t_d}{p_{sd}(t_d)} \right] \left[ 1 - \exp \left( \frac{p_{sd}(t_d) + \frac{1}{t_d C_D}}{p_{sd}(t_d) - p_{sd}(t_d)} \right) \right]$$

$$p_{sd}(t_d) = \frac{p_{sd}(t_d) - p_{sd}(t_d)}{C_D} \left[ \frac{-t_d}{p_{sd}(t_d)} \right] \left[ 1 - \exp \left( \frac{p_{sd}(t_d) + \frac{1}{t_d C_D}}{p_{sd}(t_d) - p_{sd}(t_d)} \right) \right]$$
Bourdet-Gringarten Type Curve Generated with a "Quadratic $\rho_{sd}$" approximation model (SPE 21826).

Blasingame, T.A.: "Type Curve Analysis of Well Test Data Including Wellbore Storage and Skin Effects — the Bourdet-Gringarten Type Curve (Transient Radial Flow Case)," Course Notes Petroleum Engineering 324, Texas A&M University 1999.

Bourdet-Gringarten Type Curve Generated with a "Quadratic $\rho_{sd}$" approximation model (SPE 21826).