Short-Time Well Test Data Interpretation in the Presence of Skin Effect and Wellbore Storage

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Introduction
Specifications for modern well testing (drawdown or buildup) are usually written in such a way that a well will be tested for a period of time long enough to reach and define a proper "straight line" when test data are plotted in conventional manners. Pressure data obtained before the straight line is reached are not often analyzed, despite the fact that a number of publications have advanced methods for doing so.¹⁻³ One reason for this situation is that many factors are known to affect the short-time data. "Short-time data" signify data obtained before a conventional straight line is reached. Some of the factors are the effects of wellbore storage, perforations, partial penetration, and well stimulation such as fracturing or acidizing.

Although the effects of such factors are generally known, the duration and importance have not been clearly defined in all cases — particularly when these effects are combined in a well test. However, recent studies have revealed a great deal of potentially useful information concerning the analysis of short-time well test data.⁴⁻⁶ Our purpose here is to illustrate the interpretation of short-time well test data through presentation of field examples. Factors to be considered will include wellbore storage, well damage, and fractured wells.

Wellbore Storage and Skin Effect
The effect of wellbore storage or unloading was originally considered by van Everdingen and Hurst.⁷⁻⁸ These studies called attention to the fact that the storage or unloading of fluid contained within the wellbore could cause a significant difference between the surface production rate and the sand-face flow rate in a well immediately following sudden changes in production rate. Gladfelter et al.¹ presented a method for correcting pressure buildup data for the changing sand-face flow rate. In another publication² a method was presented for estimating the duration of the storage effect, and the Gladfelter et al. correction was generalized to apply to drawdown data.

Fundamentally, wellbore storage can occur in several ways. Fluid can be stored by compression of the fluid in a completely filled wellbore, or by movement of a gas-liquid interface. Russell³ presented a method for analysis of the latter case, pointing out that the virtue of the method was that it was not necessary to know the sand-face flow rate as was the case in the Gladfelter et al. method. One problem with Russell’s method was that only a portion of the short-time data was used, yet no criterion for selection of the data was presented.

Recently, Agarwal et al.⁴ re-examined this problem and presented dimensionless pressure-dimensionless time plots for the case of a well in an infinitely large reservoir, producing at constant surface production rate, and having wellbore storage and a skin effect. Fig. 1 presents a portion of their results. The usual definitions of dimensionless groups were employed.

The log-log type-curve described here shows clearly the presence and duration of wellbore storage as well as the presence of linear flow due to fracturing. It can be used to obtain quantitatively the information normally obtained from pressure buildup analyses and to identify the proper straight line in pseudo-radial flow for a fractured well.
Dimensionless time:
\[ t_D = \frac{0.000264kt}{\phi \mu c r_w^2} \]  

Dimensionless pressure:
\[ p_D = \frac{kh (p_i - p_w)}{141.4q \mu B} \]

Dimensionless storage constant:
\[ \bar{C} = \frac{C}{2\pi h \phi c r_w^2} \]

The skin effect, \( s \), is the van Everdingen-Hurst dimensionless skin effect:
\[ s = \frac{kh(\Delta P_{skin})}{141.4q \mu B} \]

It was pointed out that the effect of wellbore storage was to cause a line of unit slope on Fig. 1 at times immediately following start of production, or shut-in, of a well. Actual buildup or drawdown data can also be graphed on a log-log plot as the logarithm of the difference between the initial pressure at the start of the change and the pressure at any time after the change vs the logarithm of the time after the change. The result should be a plot identical with that shown in Fig. 1. This sort of plotting has been called “type-curve” analysis by groundwater hydrologists. The log-log type-curve for field data can be plotted on the same size coordinate on tracing paper and matched with Fig. 1 by comparison of shapes. Once a match is obtained, values for permeability-thickness and hydraulic diffusivity can be obtained from Eqs. 1 and 2 and the known coordinates of any matching point on the field data plot and the analytical type-curve. The value of the skin effect can be read from the analytical curve.

The basic reason for type-curve matching can be seen if we take the logarithm of Eqs. 1 and 2:
\[ \log p_D = \log \left( \frac{kh}{141.4q \mu B} \right) + \log(p_i - p_w) \]  

Thus the only difference between a log-log plot of dimensionless pressures and times and real \( \Delta p \) and real time is a translation of both coordinates by appropriate constants. Type-curve matching permits determination of the constants, i.e., the first group in brackets on the right-hand side in both Eqs. 5 and 6.

One important result of the Agarwal et al. study was the realization that wellbore storage (if evident at all) was the controlling effect at times immediately after start of a test, and that during the storage period as indicated by a unit slope on a log-log type-curve, absolutely nothing could be learned concerning formation flow capacity, diffusivity, or well skin effect. Furthermore, the type-curve shows clearly whether or not storage is controlling well behavior, and thus it sets a lower time limit on utility of well-test data for normal purposes.

Another important result was the realization that the physical nature of the skin effect could influence interpretation of the short-time data for times after complete storage control. That is, the depth of formation damage could change the shape of the transition from a line of unit slope to the beginning of the usual straight line. These effects were studied by Wattenbarger and Ramey, but will not be reviewed here.

**Field Example of Damaged Well with Storage**

Fig. 2 presents a pressure buildup curve for a well with a prominent storage effect. This set of data was presented in detail by Russell (his Well A) to illustrate a method for analyzing the data in the curving portion of the plot indicated by the arrow on Fig. 2. Fig. 3 presents a type-curve plot of the same data, i.e., the logarithm of \( (p_i - p_w) \) vs the logarithm of the buildup time, \( \Delta t \). This plot is valid for pressure buildup if the producing time, \( t \), is long compared with the buildup time. For pressure drawdown, \( (p_i - p_w) \) would be plotted vs log \( t \). Either pressure buildup or pressure drawdown can be plotted on a...
type-curve and compared with dimensionless type-curves such as Fig. 1.

The first buildup points on Fig. 2 do fall on a line of unit slope, indicating that buildup behavior was completely controlled by storage until a buildup time of about 0.67 hours. There is no possibility of obtaining information about formation flow capacity or skin effect from buildup data obtained before 0.67 hours of buildup. But this buildup information is important on the type-curve since it graphically displays that storage is the controlling factor to 0.67 hours. This casts a light of new importance upon measuring pressures immediately after shut-in.

Any single point of corresponding buildup time and pressure difference from the line of unit slope can be used to compute the unit storage factor $C$ (in reservoir bbl/psi). From Fig. 2, a point is 560 psi at 0.67 hours. Thus,

$$C, \text{ res. bbl/psi} = \frac{qB\Delta t}{(p_{res} - p_{oil})} = \frac{(157 \text{ STB/D})(1.6 \text{ res. bbl/STB})(0.67 \text{ hr})}{(560 \text{ psi})} (24 \text{ hr/D})$$

$$C = 0.125 \text{ res. bbl/psi}.$$

Several comments are pertinent here. First, the value of the storage constant can be determined accurately regardless of the mode of fluid storage, i.e., whether due to compression of a single-phase fluid, or due to rise of a liquid level in the wellbore. Second, the effective volume of the wellbore can be found from the storage constant by considerations reviewed in an earlier work. Normally, one would expect a reasonably close check with the known volume obtained from well-completion data. But there is the interesting possibility of estimating the volume of reservoir voids that communicate with the wellbore when the measured volume is larger than the known wellbore volume. An example of this situation will be shown later. Third, it should be clear that the mode of storage of fluid in the wellbore could change during a test, and thus the storage constant could change abruptly as well. An example would be a damaged well producing at a pressure below the bubble point, whose mean reservoir pressure is considerably above the bubble point. Upon shut-in, the liquid level might rise until the wellbore is filled with liquid and all gas has gone into solution, then storage would continue due to compression of the undersaturated liquid. The same effect would be common in pressure fall-off in water injection wells. Peculiar effects that could be attributed to this factor have been noticed in short-time well test data.

Once the storage constant, $C$, has been determined, it is possible to compute the dimensionless storage constant, $\overline{C}$, from Eq. 3. For Well A,

$$\overline{C} = \frac{C}{2\pi h \phi \epsilon r_w^2}$$

$$= \frac{(0.0125)(5.615 \text{ cu ft/bbl})}{(2\pi)(4 \text{ ft})(0.1)(2 \times 10^{-5} \text{ psi}^{-1})} (0.0247 \text{ sq ft})$$

$$= 5.68 \times 10^4.$$

The formation constants used were obtained from Ref. 3. Since the equation of any unit-slope straight line on Fig. 1 is known to be

$$p_v(s, \overline{C}, t_d) = \frac{t_d}{\overline{C}}, \ldots \ldots \ldots \ldots (5)$$

the dimensionless storage constant can be used to locate the unit slope line and thus the approximate position of match on the type-curve as shown on Fig. 1. The field data plot is moved along the $\overline{C}$ unit slope line of $5.68 \times 10^4$ until the field and analytical curves appear to be a reasonable match. Once a match has been obtained, any matching point from the two type-curves can be used to compute the formation flow capacity, $kh$, and the hydraulic diffusivity, $k/(\phi \mu c)$.

Such a match is shown on Fig. 4 for Well A:

$$\Delta t = 10 \text{ hrs}, (p_{res} - p_{oil}) = 10^4 \text{ psi},$$

$$t_d = 3.17 \times 10^6, p_D = 68.3.$$ 

From Eq. 2:

$$p_D = \frac{kh(p_{res} - p_{oil})}{141.4 q \mu B}$$
Substitution analysis, even when sufficient information is available calculated from the conventional skin-effect equation.

curve matching do not represent two independent from Fig. This information is sufficient to find comparison found here can be more accidental than

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if the work is done casually. However, careful type-curve matching should produce skin effect values accurate to one and often two significant figures. Normally, extreme accuracy is not required (nor is it often obtained) for assessment of well condition. But uncertainty in the skin effect can lead to uncertainty in estimation of static formation pressure. Although not illustrated here, static pressure can be obtained at this stage in several ways. For example, the slope of the straight line can be calculated, and the pressure at 1 hour's buildup time on the straight line can be calculated from the conventional skin-effect equation. This information is sufficient to find \( p^* \), and thus the static pressure.

Type-curve plotting of either buildup or drawdown data offers additional information over conventional analysis, even when sufficient information is available to perform a conventional analysis. Identification of storage effect control has already been stressed. More important is the fact that the type-curve matching procedure is actually a more general solution than the conventional semilog plotting methods. The type-curve presents a completely general solution while the semilog plot represents only a long-time solution to the flow problem. This important difference means that it is possible to extract more information from type-curve analysis (when it is possible to find a match) than from conventional analysis. The example presented previously does not point out sufficiently the generality of the type-curve method. For example, the match shown on Fig. 4 could have been found without calculation of the dimensionless storage constant, \( \bar{C} \). Once the match was identified, both \( \bar{C} \) and \( s \) could have been read as parameters from the type-curve. Recall that it was assumed that the porosity and compressibility were known in calculation of \( \bar{C} \) in the example. This is not necessary. Finally, inspection of usual expressions for calculation of the skin effect (see Ref. 14) reveals that conventional analysis provides a summation of terms involving the skin effect and hydraulic diffusivity. It is not possible to separate this summation by conventional analysis unless one or the other is known. This can be done by type-curve matching. But it should be obvious that type-curve matching is a graphical procedure with certain limitations. It does not replace conventional analysis, but is, instead, an additional tool of great utility.

Stimulated Wells

In the case of a stimulated well (skin effect less than zero), the cause of the negative skin effect can have an important effect upon a type-curve. If a negative skin effect is caused by acidizing at injection pressures too low to cause fracturing, it is reasonable to postulate an annular region of high permeability near the well. If the formation is limestone, both this effect and a physically-increased wellbore radius could result. On the other hand, hydraulic fracturing could result in a high-conductivity fracture of any natural orientation extending from the well into the formation.

For the case of improved permeability near the wellbore, we might expect that type-curves similar to Fig. 1 but including negative skin effects would be satisfactory for test interpretation. Such curves have been presented. But these curves would not be valid for hydraulically fractured wells. It is well known that flow from a fracture plane intersecting a well should initially be similar to flow in a linear system. A
recent paper discussed this problem for gas flow including wellbore storage. It was found that under extreme conditions there could exist an initial period of storage control followed by a period of linear flow. Since linear flow results in pressure's being a function of the square root of time, the result on a type-curve is an initial line of unit slope followed by a second straight line of slope equal to 0.5. The slope of 0.5 would result whether the fracture orientation were horizontal or vertical, or any other inclination.

Fig. 5 presents type-curve plots prepared from data published by Russell and Truitt for vertically fractured wells. Their study did not consider wellbore storage; but the linear flow effect resulting in straight lines of slope equal to 0.5 is clearly evident. The family of curves on Fig. 5 contains the parameter $L/r_w$, or $L/r_w$, the ratio of the distance from the well center to an end of the fracture to the wellbore radius. Fig. 5 shows that another fairly linear period follows the initial straight line of slope 0.5. Because the second linear portion exhibits differing slopes, depending upon the $L/r_w$ ratio, there is a possibility of determining the length from type-curve matching. But field data often indicate smaller $L/r_w$ ratios than that shown on Fig. 5, and type-curve data are not readily available. Even without appropriate type-curves, a type-curve plot of field data can yield important information. For small values of $L/r_w$ (less than 100), the dimensionless pressure at the start of pseudoradial flow is approximately twice that at the end of the linear flow period. This observation can be used to estimate the start of a proper straight line for a normal buildup plot when the linear flow period is evident on a log-log type-curve. If the $L/r_w$ ratio is greater than 100, the transition period will be longer, and a straight line may be obscured by boundary effects.

In regard to wellbore storage effects for fractured wells, it would be rare that wellbore storage would ever be apparent. This is because for a fractured well the wellbore pressure changes slowly. Thus the wellbore storage changes slowly. However, extreme cases have been observed in field data and will be presented later.

It should be emphasized that our main purpose here is to illustrate the application of short-time well test data analysis. Refs. 2, 4, 5, and 6 present detailed information concerning the origin of the type-curves, as well as background information.

**Field Example of Stimulated Well with Storage**

Normally, one would not expect to see a significant storage effect with stimulated well test data, i.e., a well exhibiting a large, negative skin effect. However, it is possible, as the next set of field data will demonstrate. Fig. 6 presents a conventional pressure buildup plot for a gas well — one that produces natural steam from the Geysers Steam field in California. The well (here called Well B) was drilled with air and completed open-hole. During initial production clean-up, rock and debris were blown from the well. A conventional buildup plot such as Fig. 6 usually shows the characteristics of a stimulated well for other wells in this field; however, Fig. 6 has the character of a slightly damaged well with possible wellbore storage. There appears little doubt that a proper straight line was established during the later portion of the buildup since the extrapolation yields a $p*$ value reasonably close to the static pressure prior to the test. The start of the straight line occurs at a buildup time, $\Delta t$, of about 35 hours.

Fig. 7 presents a type-curve plot for the pressure buildup data shown in Fig. 6. Data taken during the first 0.1 hour of buildup clearly form a line of unit slope and indicate that storage is controlling. At about 0.25 hour, the buildup data form a line of slope 0.5, indicating a long period of linear flow, and the likelihood that this well communicates through a natural fracture. Linear flow persists for this test until a buildup time of about 4 hours. It is possible to estimate the time of beginning of pseudoradial flow by a criterion from Ref. 6: the pressure rise at the end of the linear flow period is approximately half that at the beginning of pseudoradial flow for wells communicating through short vertical fractures. The arrows on Fig. 7 show that the pressure-squared rise at 4 hours was 75 M (psia$^2$), and that a rise of 150 M (psia$^2$) occurs at a buildup time of 35 hours. Thus the estimated start of the straight line on a conventional plot would be at a $\Delta t$ of 35 hours. This value is in reasonable agreement with the start of the straight line previously shown on Fig. 6.
In this case, the type-curve plot was not necessary to establish the proper straight line. However, it has been useful for this purpose in other buildup test analyses. In a number of cases, the type-curve plot has indicated that the proper straight line was not the apparent one when several straight lines appeared on a conventional plot. In one particularly complicated buildup test, the type-curve indicated that the first of three successive straight lines was actually the proper line and that the well was in the corner of intersecting, sealing flow barriers.

Let us return to the test of Well B indicated in Figs. 6 and 7, where the type-curve did indicate useful additional data. Recall that Fig. 6 had the appearance of a slightly damaged well with possible storage. The type-curve Fig. 7 shows clearly that storage is present, but that the well communicates through a fracture and is probably not damaged. The unit slope straight line can be used to find the storage constant as before. If we select the point of 0.04 hours and 4 M (psia\(^2\)) from Fig. 7,

\[
C\text{ lb/psia}^2 = \frac{(114\text{ M lb/hr})(0.04\text{ hr})}{4\text{ M(psia}^2)} = 1.14.
\]

Since the well produces a single-phase fluid (gas), storage volume can be estimated readily.

\[
V = \frac{C(zRT)}{M e_s} = \frac{(1.14)(0.928)(10.73)(815\text{ O}^\circ\text{R})}{(18)(0.0069\text{ psi}^{-1})} = 74,500\text{ cu ft}.
\]

Gas compressibility was evaluated at a mean storage pressure.

The astonishing fact in this case is that the actual wellbore volume is approximately 1,550 cu ft. The measured volume of 74,500 cu ft is considerably greater than the estimated wellbore volume. Part of the difference could be due to sloughing of rock that was blown from the well during clean-up. But in this case, most of the difference is suspected to be natural formation cavities that communicate directly with the wellbore. Drillers reported numerous instances of bit dropping, indicating natural cavities. The large storage volume, coupled with the long period of linear flow, suggests that this well communicates with one or more natural cavities of large total volume and that each cavity is probably large in area but not very thick.

Although the behavior of Well B is not unusual for wells in the Geysers Steam field, it would not be expected that such field data would be encountered often. Although I have seen long periods of linear flow in tests on hydraulically fractured gas wells, the unit slope storage effect has not been apparent on type-curves. Clearly, however, storage should be a predominant feature of wells completed by nuclear fracturing. The analysis presented in this study should find excellent application in this case.

In regard to the foregoing estimate of storage volume, the calculation is only approximate — the compressibility of the gas is a function of pressure. For more precise work, well-test data identified as being under storage control can be analyzed with a rigorous material balance for a volumetric gas container (storage volume).

As for the linear flow period, the presence of damage on the surface of a fracture or non-Darcy (turbulent) flow near a well could prevent or distort a line of slope 0.5 on a type-curve. (See Ref. 6 for a discussion of this point.) In one case with a notch-fractioned well, there was a long period (25 hours) of linear flow, but the type-curve data approached the line of slope 0.5 from above the line. There was no indication of storage for the well, and it was concluded that the notch was partially plugged at the wellbore. The type-curve was the only clue to this information.

Linear flow will also result in a straight line on cartesian coordinates when the pressure is plotted vs the square root of time.\(^9\)\(^10\) Data plotted in this fashion will often yield a straight line when there is no line of slope 0.5 on a type-curve plot, or when data are not properly selected. It is recommended that both type-curve and cartesian plots be used in analyzing data when there is any doubt concerning the interpretation. This remark also applies to conventional well test analysis. The type-curve does not replace conventional buildup or drawdown plotting. The graphs complement each other. An interpretation that explains detail on both the conventional plot and the type-curve plot can be viewed with some confidence.

A few concluding remarks are in order concerning the interpretation of fractured well tests. First, I have encountered many cases for fractured gas wells in which linear flow controlled behavior to 20 or 30 hours of either flow time or buildup time. Common backpressure test analysis based upon radial flow theory is not appropriate in such instances. Second, the presence of an extensive fracture as indicated by the type-curve plot affects the extrapolation of build-up pressures for obtaining the static pressure. Russell and Truitt\(^11\) recommended that Muskat-type semilog plots be used to correct buildup pressures to the static pressure for fractured wells. Their work was based upon computed performance of a vertically fractured well in the center of a square. There is a need for further investigation of static pressure extrapolation for fractured wells. It is significant that Russell and Truitt found that conventional pressure corrections on a Horner-type buildup plot yield static pressures that are too low.

**Discussion and Conclusions**

The preceding examples are only a few of many field cases in which the log-log type-curve reveals a great deal of information not available from conventional buildup plots. The importance of wellbore storage will usually be shown clearly on the type-curve if enough information has been taken immediately after shut-in. This has been observed for both oil and gas wells.

Again it should be emphasized that type-curve plotting is not intended to replace conventional buildup plotting. The plots are supplementary; each can reveal different kinds of useful information.

The examples presented illustrate some of the simplest applications of the methods. Significant variations in flow rate on drawdown, or production peri-
One interesting application of type-curve plotting is in analysis of buildup data that display a "hump" due to phase segregation within the wellbore. For example, the hump-type buildup shown by Stegemeier and Matthews\(^\text{13}\) as their Fig. 1, replotted as a type-curve, displays an exact straight line of unit slope for the first 40 minutes of buildup, and then normal curvature for a period of time before the hump develops. These data could be analyzed by curve-matching.

Short-time analysis by curve-matching can also be applied to gas well analysis. The damaged gas well buildup example (3-A in the Appendix) in SPE Monograph No. 1, Pressure Buildup and Flow Tests in Wells, yields an excellent type-curve with many points on the unit-slope storage line. If tubinghead data are to be used for gas wells, the flowing pressure used in the type-curve should be that taken at the instant the master valve is closed. Otherwise, the flowing friction should be estimated and added to the tubinghead flowing pressure, or bottom-hole pressures should be calculated. As in conventional gas well test analysis, pressure-squared differences are plotted on a type-curve if the pressure level is under 3,000 psi. High-pressure gas well tests may be handled as equivalent liquid flow or by means of the pseudopressure.\(^6\)

Occasionally, type-curves exhibit strange shapes not entirely similar to those of Fig. 1. One interesting example is the undersaturated oilwell buildup presented in Appendices B and C of Ref. 14. Fig. 9 is a type-curve for this example. Complete details of conventional buildup analysis are given for this well as a sample calculation in Ref. 14. As can be seen on Fig. 9, a very clear period of storage effect control appears for the first 4 hours of buildup. But the type-curve bends sharply upwards and rapidly reaches a conventional buildup straight line. It is not clear from the information in Ref. 14 just what caused this behavior. One reason might be that the storage constant began to diminish rapidly near the end of the unit slope straight line. This might indicate a change in the character of the storage, and might be explained by the presence of some gas in the wellbore at shut-in. But this is not a certain explanation. Storage constants due to liquid compression and liquid level rise can be of the same order of magnitude.

The most common use groundwater hydrologists make of type-curves is in interpreting interference tests. This is a particularly convenient application because the type-curve avoids the trial-and-error solution required to find the argument of exponential integrals as commonly used by petroleum engineers. Descriptions of type-curve matching methods available in the groundwater literature usually indicate that the type-curve is first plotted on log-log graph paper; next, the field data are plotted on a second piece of the same size log-log paper. The two graphs are then matched on a tracing box. It is my experience that the fine grid on both graphs makes matching a confusing and difficult procedure. There is a more convenient method that avoids the need for a tracing box. The type-curve should be on log-log coordinate with a fine grid, but field data can be plotted on a plain piece of tracing paper or typing second sheet. Position the tracing paper over the type-curve and trace only the outlines of the log cycles in both directions. (These lines are needed to keep coordinate scales parallel during matching.) Label the log cycles with appropriate real variable values and plot the field data on the tracing paper using the visible fine grid from the type curve under the tracing paper. The finished field data plot should look like Fig. 3, without the line through the data points. Using two drafting triangles, simply find whether a line of unit slope or a line of slope of \(\frac{1}{2}\) will fit any of the field data. Then match the type-curve to the field data, keeping the coordinates parallel. Choose any intersection of the coarse coordinates on the field plot and read the coordinates of the corresponding point from the fine-grid type-curve under the tracing paper. This procedure also avoids problems that arise when there are slight differences in printed graph paper and photographic distortion in copies of graphs.

The basic conclusion of this study is that type-curve plotting on log-log coordinates can reveal a great deal of useful information not available from conventional buildup or drawdown plotting. The information can be either qualitative or quantitative. Type-curve analysis is a fast, simple procedure that offers definite economies to the analyst. This procedure should become a popular one.

**Nomenclature**

\[ C = \text{wellbore storage constant, res. bbl/psi for oil; lb/psia}^2 \text{ for gas} \]

\[ V = \text{wellbore storage volume, cu ft} \]

All other symbols are standard SPE nomenclature.
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