A Study of Anomalous Pressure Build-Up Behavior

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A Study of Anomalous Pressure Build-Up Behavior

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ABSTRACT

In one field in South Texas, approximately 75 per cent of the pressure build-up results show a characteristic "hump" (i.e., the pressure builds up and then falls off), which makes interpretation by standard methods impossible. Correlation of size and time of humps with formation permeability, well productivity index, and method of completion led to the tentative conclusion that the humps were caused by segregation of gas and oil in the wellbore after completion. This conclusion was confirmed by performance of simple laboratory bubble-flow experiments, by theoretical bubble-elastic time calculations, and by a detailed evaluation of PVT behavior in the wellbore of a particular well on which accurate surface and bottom-hole pressure measurements were made.

The hump behavior has since been found to occur in many other fields. The cause, however, is not the same in all cases. In some of the humps the traceable to leaks in the tubing that allow influx of gas from the annular space after completion. In other cases the hump is traceable to leaks in the device separating pay horizons in dually completed wells.

It is concluded that the recording of both surface and bottom-hole pressures is desirable in wells where such an anomalous build-up behavior occurs. A number of field examples are discussed where use of both sets of measurements can be of assistance in a case for anomalous behavior to be found, and a reasonable interpretation of bottom-hole pressure to be made.

INTRODUCTION

Theoretically the pressure build-up in an infinite reservoir should be a linear function of in [1 - e^(-t/(2rt))], where t is the production time and r is the skin factor time. Some of the variations from this behavior are well known, such as the curve tailing immediately after shut-in which results from after-production and skin effect, and the flattened portion which results from boundary effects in a limited reservoir. The effect of stratified producing zones and irregular geometrical drainage patterns may also contribute unusual characteristics in build-ups. However, the effect of still another phenomenon—anomalous behavior due to fluid redistribution—is not considered in the current theory. This phenomenon has been observed in many fields where there is a significant difference in permeability and pressure gradients between the reservoir and the wellbore. The pressure build-up is characterized by an initial rapid increase followed by a slower rate of increase, which may continue indefinitely or approach a constant value.

ANOMALOUS BEHAVIOR

The first stage of the build-up is characterized by an instantaneous pressure rise, followed by a more gradual increase. This initial rapid rise is due to the influx of high-pressure oil from the reservoir, while the subsequent gradual increase is due to the slow diffusion of gas from the reservoir into the wellbore. The pressure build-up is limited by the pressure drop across the wellbore and the reservoir, as well as the permeability and thickness of the reservoir.

FIELD OBSERVATIONS ON HUMP BUILD-UPS

In approximately 75 per cent of the wells in a medium-sized field in South Texas the pressure build-up curves due to a maximum and then decline to what appears to be a stabilized reservoir pressure as shown in Fig. 1. Similar behavior has also been observed in wells in other fields in Texas and Louisiana. Because of the possibility of such behavior being a mechanical failure of the production well, two bombs were run in tandem into one well which had been known to exhibit this behavior. When both bombs recorded identical curves with the characteristic hump, it was inferred that the usual build-up is a result of well or formation behavior, rather than a recording instrument failure.

In order to understand more fully the mechanisms involved in these build-ups, a statistical study of 120 of these humping builds in one South Texas field was made. The results indicated that the humps may be of various sizes, have an average size of about 45 psi, and always occur at the end of or shortly after the typical "beads-neck" part of build-up. The time of occurrence generally varies from 100 to 1000 minutes after shut-in, with the greatest number of curves humping at 300 to 400 minutes. In all cases in which the hump is later than 360 minutes, the build-up part of the curve is also late and there is a sizeable rate of build-up within 350 minutes from the time of shut-in.

Apparently, there is no correlation in this field between this unusual build-up and rate of production of the well. Variations in the build-up pressure, such as sufficient to negate use of the data for computing permeability or stabilized build-up pressure. Examples of each of these phenomena are discussed below.

THEORETICAL AND EXPERIMENTAL STUDIES ON PHASE REDISTRIBUTION

STAGE THEORY AND EXPERIMENTS

To illustrate the theory, suppose a frictionless piston separates a gas and a liquid phase below. Assume the piston is isothermal and incompressible, and the pressure at the top is p. The pressure below the piston is fixed at p and the pressure at the bottom is the total pressure (p + p). Thus, by changing the relative position of the phases in the column, the absolute pressure at both top and bottom is changed. The difference between the top and bottom pressure remains constant, however, and is equal to the weight of the fluids between the two points divided by the cross-sectional area.

The experiment described above was made in the laboratory, and the results were found to be in good agreement with the theory.

DYNAMIC EXPERIMENTS

Following this static phase-distribution experiment, a second experiment, which more closely simulated waterflood performance, was carried out using equipment shown in Fig. 2. The reservoir was initially flooded with glycerol, and with Valve B open and Valve C closed, air was introduced through the core at the base of the column. When the air bubbles were sufficiently dispersed throughout the column, the air supply (Valve A), and

any application of this phenomenon is made to a well, a discussion of the theoretical effect of phase redistribution will be made.

the outlet at the top of the column. (Valve B) were extended simultaneously. A total pressure buildup at both top and bottom of about 33 m of water was observed, due simply to bubble rise.

An explanation of the pressure rise in this case may be seen by considering the bubbles at the bottom to be compressed by the fluid head just as the gas is compressed beneath the piston. The subsequent rise of gas through the liquid lessens the pressure on the bubble due to fluid head, but because the gas cannot expand in the closed system, it exerts a pressure on the liquid as a gas-liquid interface. This pressure is transmitted to the foam, and when added to the hydrostatic pressure, gives an additional pressure associated with bubble rise.

Several runs of this type were made in which the amount of gas initially at the top of the column and then the volume of bubbles contained in the liquid were varied. It was found that (1) the pressure buildup due to bubble rise is inversely proportional to the square of the top gas volume, and (2) the pressure buildup due to bubble rise is directly proportional to the total volume of bubbles in the column.

In order to reproduce more closely the effect of a pressure buildup in a well, an inverted flask containing both glycerine and air was attached to the bottom of the column, this flask being analogous to the porous reservoir around the wellbore. Again air was flowed through the core into the bottom of the column. This time the top valve was open, Valve C was slightly cracked, and gas was allowed to bubble up. The air-pressure in the flask was adjusted to value slightly above the flowing bottom-hole pressure.

When a steady-state dispersion of bubbles throughout the column was reached, the air-supply was shut off and the top valve was closed. This time the pressure buildup resulted solely from influx from the reservoir and from the bubble rise. However, the pressure buildup in bubble rise shortly after shut-in was so much greater than that from influx that the bottom-hole pressure rose above the reservoir pressure (see Fig. 4). This occurred because the liquid in the column was unable to flow back into the reservoir fast enough to prevent the anomalous high bottom-hole pressure caused by bubble rise. After rising above reservoir pressure the liquid in the column broke free from the reservoir until the bottom-hole pressure declined to the reservoir pressure.

In this model is an analogy of a well having a high skin factor, simulated by the partially closed valve, and a considerable quantity of bubbles entrapped in the borehole liquid of shut-in.

**ANOMALIES RESULTING FROM PHASE SEGREGATION**

**Assumptions Used in Calculating Well Behavior**

- **Temperature:** Calculations take into account the effect of temperature gradient in a wall on the volatility properties of gases, but neglect the effect of changing temperature on liquid properties.
- **Subsurface Properties:** When gas-oil saturation is specified, the gas and oil are assumed to be in equilibrium at all times.
- **Frictional Loss:** Loss in pressure drop in the tubing due to friction was negligible in all cases considered in this report.

**Basic Calculations**

An approximation of the limiting upward velocity of bubbles is needed to determine the length of time during which phase redistribution may be expected to occur. These limiting values were taken from the correlation of Pimble and Gruber.

**TIME OF BUBBLE RISE IN SOUTH TEXAS WELL**

Assuming that no very small bubbles (i.e., radius < 0.0015 ft) occur, the minimum rate of rise obtained from this correlation is 0.55 ft/sec. Thus, the maximum time of rise in a 2,900 ft South Texas well is 377 minutes. The maximum rate of bubble rise is 0.55 ft/sec, which leads to a maximum rise time of 240 minutes.

Earlier it was stated that the humps in the South Texas field never were more than 350 minutes removed from a predictable rate of after-production. To cause a sizable rise of buildup in a considerable quantity of gas and water, the following must be true: (a) resolving the hump in less than 150 minutes of humping. The time required for such gas to rise to the top is 240 to 377 minutes, (b) the right order of magnitude to account for the hump.

**ANALYSES OF BEHAVIOR OF SOUTH TEXAS WELL 1**

**Pressure Behavior**

Bottom-hole pressure (BHP) buildup data on Well 1 in South Texas field obtained by the Shell Development Co.'s BHP telemetering device. This device showed presence of gas influx to less than 0.05 psi on an accurate time scale.

Fig. 1 shows the change in BHP (after shut-in) minus flowing BHP) and the change in tubing pressure (tubing pressure after shut-in minus flowing tubing pressure). The difference between these two curves is also plotted in Fig. 1. This difference curve is seen to reach, rise, and then decline to a constant value. Since the friction losses at any time is negligible, the value of the difference curve is proportional to the amount of fluid which has entered after shut-in between the BHP bomb and the tubing, or the rate of change of the phase distribution. Any increase in the difference curve indicates an increase in fluid in the tubing, and, conversely, a decrease indicates an outward flow of material flow. Because the wall has a pocket, the fluid leaving the tubing flow only into the formation.

In view of these observations it is possible to analyze the pressure behavior shown in Fig. 1. The difference curve rises after shut-in, indicating a normal after-production into the wellbore; however, at about 300 minutes it reaches a maximum. This time coincides with the time at which the BHP curve first reaches the eventual stabilized pressure. As one might expect, the direction of flow after the BHP exceeds the stabilized BHP from the tubing into the formation. This is seen in the fact that the difference curve decreases after the 300-minute point. The BHP continues to rise even after the pressure at this point exceeds the stabilized formation pressure because the pressure rise caused by the rise of gas occurs at a greater rate than the pressure decrease due to oilflow. However, since no more gas is entering the tubing, the rate of pressure increase to bubble rise finally decreases, and at 390 minutes equals the rate of pressure decrease due to oilflow. This is the point at which the BHP reaches a maximum and begins its decline. After this time the tubing pressure will continue to rise, this rise being attributable to the rise of gas which has just entered the tubing. When most of the gas has risen to the surface (500 minutes) the tubing pressure will finally begin to decline because of continued negative after-production. The final state of buildup is a stabilization of all three curves to constant values.

**TOTAL PRESSURE RISE CAUSED BY PHASE SEGREGATION**

The effect of phase redistribution on the South Texas Well 1 buildup discussed previously will now be computed by: (a) finding the amount of gas and oil in the tubing at the time of closing-in, (b) finding the rate and amount of after-production, (c) calculating the pressure increase due to bubble rise, (d) attributing the difference between the observed and calculated pressure increase to phase segregation, and (e) determining the effect of this phase segregation on the total pressure rise.

The amount of gas and oil in the tubing at the time of closing-in is calculated by the method discussed in Appendix 1. The results indicate that at time of closing-in there were 2,910 ft of gas (at tubing conditions) and 3,990 lb of liquid in the tubing.

The rate and amount of after-production are calculated in Appendix 2. Rate results are shown in Fig. 4: the initial rate is about 20 bbl/min, but this drops rapidly to about 1 ft/min at the end of 200 minutes. Note that the results do not follow the often-used assumption that the rate of influx is proportional to the rate of change of BHP. However, the rate of influx is in proportion to the rate of change of the difference between tubing and bottom-hole pressure. To a good approximation the total volumetric rate of influx is also proportional to the rate of change of difference between tubing and bottom-hole pressure. This suggests that tubing, rates, and BHP's should be measured during build-up or when it is desired to make calculations of after-production for pressure buildup interpretation. Then by using the difference between top-hole and bottom-hole pressures one can calculate the rate of after-production accurately.
The fact that wells without packers have smaller bumps than wells with packers seems reasonable in view of our experiments with bubble columns. There was found that the amount of the pressure build-up caused by bubble rise diminished as the initial gas volume in the bubble increased. Since the wells without packers had much more gas initially in the wellbore than wells with packers, they should have had smaller bumps. It was also observed that wells without packers had the hump later than wells with packers. This is reasonable because more influx will be required to bring the pressure in the wellbores up to that required to balance the reservoir pressure.

Having concluded that bubble rise is probably the cause for most of the bump anomalies at this South Texas field, an examination of an anomaly slightly different from those in South Texas Well W 1 was made.

**Analysis of Behavior of South Texas Well 2**

An example of a well in which phase redistribution influences the build-up, but is not large enough to cause the BHP to exceed the reservoir pressure, is shown in Fig. 7 for South Texas Well 2. The difference curve for this well shows a maximum at about the same time as that for South Texas Well 1, indicating that bubble rise conditions were similar at time of closing-in. However, the continued slow rise in pressure in Well 2, which tends to mask the hump, indicates that the formation permeability around this well is somewhat lower than that around Well 1.

On comparing the behavior of Well 2 with Well 1, it appears that there is probably an optimum permeability for humping; formation of low permeability will still be building up at the time of separation of water and gas, but the hump may be partially masked, as in Well 2. In formations of high permeability and no skin effect, equilibrium can always exist between wellbores and formation with the result that no hump will occur. The optimum conditions appear to be in formations of moderate permeability where there is a considerable skin effect.

**ANOMALIES RESULTING FROM TUBING OR PACKER LEAKS**

**Analysis of Behavior of North Texas Well A**

This well exhibits a behavior slightly different from the two previous examples, as shown in Fig. 8. The phase redistribution build-up occurs early, the tubing pressure reaching a maximum at about 120 minutes. This time the BHP exceeds the adjacent reservoir pressure so much that it reaches a maximum and hump even though the well has not built up to average reservoir pressure. Following the temporary decrease, the build-up continues toward the stabilized reservoir pressure, with the complication that a slow leak in the tubing head causes this pressure to decrease. To balance this decrease in tubing pressure, the difference curve increases, i.e., the rate of gas and liquid formation increases. Because the rate of this leading (after-production) will influence the BHP, it appears that a straight-line portion is not reached during this build-up.

**Analysis of Behavior of Williston Basin Well 1**

Three build-up curves on two wells in a Williston Basin field showed an anomaly caused by leakage between zones of different pressure. These wells were completed in two zones which were separated by a

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**References**


**Appendix I**

**Gas and Oil in Tubing at Time of Closing-In**

It is possible to approximate the volume of gas and oil in the tubing at flowing conditions assuming PVT

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equilibrium at all points, neglecting friction drop, and
assuming that the G/L ratio (the ratio of standard cubic
feet of free and dissolved gas to barrels tank oil) is
the same in any interval in the column.

With the above assumptions one may solve, by trial
and error, for a curve of pressure vs depth which fits
the known surface and bottom-hole flowing pressure.
For South Texas Well 1 it was found that the cal-
culated curve of pressure vs depth fits the observed
end points when a gas-liquid ratio equal to 475 scf/STB
is used (the flowing GOR was 700). In the final
calculation the effect of a temperature gradient from
246 to 130°F at tubing head was included. These
results were only slightly different from those assum-
ing constant temperature of 246°F in the flowing
tubing system.

From this same set of calculations the gas saturation
at any level, and, therefore, the total amount of gas,
can also be found. For South Texas Well 1 it was
found that there are 2,950 ft of free gas and 4,590 ft
of liquid in the tubing under flowing conditions.

It is realized that the assumption of a constant
gas-liquid ratio throughout the tubing is not necessarily
true. However, the method seems to give reliable results
in South Texas wells, as will be discussed more fully
in Appendix 2. At this point it should be mentioned,
however, that the curve of pressure vs depth obtained
here is a shape similar to the curves obtained by
Gilbert* from field measurements.

**APPENDIX 2**

**RATE AND AMOUNT OF AFTER-PRODUCTION**

**RATE OF AFTER-PRODUCTION**

The difference between the change in bottom-hole
pressure and the change in tubing pressure may be
related directly to the amount of material flowing
into the well. If the flowing gasoil ratio is assumed to
remain after shut-in,** the incremental difference
relationship is:

$$\frac{\Delta p}{\Delta t} = \frac{\Delta (\Delta BHP - \Delta TP)}{\Delta t}$$

$$= 0.633 \left( \frac{\Delta G}{\Delta t} + \frac{\rho_o \Delta L}{\Delta t} \right)$$

where

**Note that the gasoil ratio (GOR) is not the same in the
bottom-hole pressure, THP, as in the tubing, TP. The flow of
correlation between the bottom-hole pressure and the tubing
pressure may not be the same as the GOR at standard conditions of the
total fluid leaving the tubing at the well. However, if liquid build-up
and gas-free gasoil ratio are constant, then the G/L ratio at the
time of flowing fluid can be assumed to remain after shut-in.

*Note that the incremental change in pressure and
after-production are approximately equal.

**Compression Caused by After-Production**

Knowing the cumulative volume of oil and gas
flowing into the well at any time, the compression of
the gas phase and the consequent increase in tubing
head pressure can be calculated from

$$p_t - p_i = \Delta p_g = \left( \frac{V_c}{L + G} \right) \rho_o$$

where $V_c$ is the volume of gas in tubing when flowing,
$L/\Delta t$ is not a flowing gas pressure at tubing head;
and $p_i$ is the initial gas pressure at tubing head.

If it is desired to take solution of gas into account,
$G$ is replaced by $G'$ in the above equation, where
$G' = G - (\text{compressibility in volume due to solution} -
\text{increase in oil volume}) = G - (\rho_o \frac{dV_c}{dp})$.

The volume of gas going into solution per unit cross
sectional area is:

$$\Delta V_c = \left( \frac{dV_c}{dp} \right) \Delta p \cdot \frac{dF_R}{dV} \cdot \frac{dG}{dV} \cdot \frac{dR}{dG}$$

The increase in oil volume per unit cross sectional
area is:

$$\Delta V_o = \left( \frac{dV_o}{dp} \right) \Delta p$$

where $R_t$ is the dissolved gas, $M$ is the feet of oil in
tubing, and $\Delta p$ is the average pressure change.

Results of these calculations are shown in plotting
Fig. 6, where $\Delta p_g$, is obtained from Eq. 1 by substitut-
ing G' for G.

Trans., AIME (1958) 213, 44.
A Comparison of Theoretical Pressure Build-Up Curves with Field Curves Obtained from Bottom-Hole Shut-In Tests

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Table 1 - Summary of Well Data.

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Fig. 2 - Bottom-Hole Shut-In and Surface Shut-In Pressure Build-Up Curves of South Texas Well No. 1 (Bottom-Hole Flowing Pressure: 1,528 Psi).

Fig. 4 - Bottom-Hole Shut-In and Surface Shut-In Pressure Build-Up Curves of West Texas Well No. 1 (Bottom-Hole Flowing Pressure: 865 Psi).
A Comparison of Theoretical Pressure Build-Up Curves with Field Curves Obtained from Bottom-Hole Shut-In Tests

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ABSTRACT

Interpretation of pressure build-up data obtained in the conventional manner has often been difficult because of the deviation from theoretical behavior. Major causes of this deviation have been attributed to damage and afterflow, and to fluid redistribution in the wellbore which, in extreme cases, can cause the pressure build-up to be quite different from the theoretically expected pressure build-up in the early period of the build-up curve.

Theoretical investigations show that bottom-hole pressure is definitely influenced by phase redistribution in the tubing column during surface shut-in tests, and that the magnitude of this effect is closely related to the producing gas-oil ratio and stabilized rate of flow in the well.

For field experiments a wire-line tubing packer, which can be run in the tubing against a stabilized flow rate, was developed for bottom-hole shut-in tests. By use of this bottom-hole shut-in method, pressure build-up data obtained in surface shut-in tests were completely eliminated and the effects of afterflow minimized.

Analog and digital computer studies have been made to obtain theoretical curves for comparison with field curves, and remarkable agreement between the results of bottom-hole shut-in tests and theoretical curves has been obtained.

INTRODUCTION

Pressure build-up data from shut-in wells have been used by the petroleum industry to determine the porosity of the formation, to estimate wellbore damage, to evaluate the static reservoir pressure and to evaluate reservoir volumes. Calculation of these factors is based on methods of analysis developed for theoretical systems whose build-up curves have a characteristic shape when the wells are shut-in at the sand face. However, field curves obtained by conventional methods do not always exhibit this characteristic shape. Such factors as stratification, rock heterogeneities and irregular reservoir geometry can cause the character of a build-up curve to deviate from that predicted by theory for a simple system. In addition, all field build-up data are affected by the methods used in obtaining the measurements.

Conventional surface shut-in techniques allow two effects to occur which contribute directly to the manner in which pressure builds up at the sand face. First, afterflow, has been recognized for some time and methods are available for estimating the magnitude of its effect and for correcting its influence. The second, phase segregation in the tubing column after shut-in, has been reported only recently and appears to exert a considerable influence on the character of field build-up curves. In extreme cases, phase segregation can produce "pressure humps" in the early portion of the build-up curve. Such curves have been considered anomalous and have defied analysis.

It is the purpose of this paper to discuss the effect of phase segregation in the tubing string during build-up and to present a bottom-hole shut-in technique for obtaining field build-up data which minimizes the influence of afterflow and eliminates phase segregation effects.

It will be shown that reliable reservoir information can be calculated from build-up measurements obtained using this method, even though conventional surface shut-in tests on the same wells yield anomalous data.

DISCUSSION OF PHASE REDISTRIBUTION EFFECTS

Some of the effects of phase redistribution on pressure build-up curves have been described by Matthews and Stogemeier, who have presented evidence that phase segregation is responsible for the pressure hump on many build-up curves. Undoubtedly phase redistribution occurs in most surface shut-in tests, but the conditions under which it may occur are not completely understood. In order to better understand these effects, the pressure rise resulting from fluid redistribution after shut-in has been analyzed theoretically using a medium-sized digital computer. The study considered the energy content of gas and oil in a given length of tubing was computed for various flow rates and various tubing pressures. By assuming the tubing string to be shut-in at the surface and at formation depth, the pressure change associated with the phase redistribution of the gas and oil contained in the tubing was calculated. It was readily seen that in many cases the pressure rise at the bottom of the tubing string, resulting from fluid segregation, was of sufficient magnitude to materially affect the build-up curve.

The degree of pressure rise from phase segregation was found to be sensitive to the rate of production and to the producing gas-oil ratio of the well. At low ratios the effects of fluid segregation are apparently most significant. As the producing gas-oil ratio increases, the flow of fluid in the tubing approaches that of a gaseous system, and segregation effects are minimized. At very low rates and highGOR, however, the effects of liquid hang-up or slipage, become very important. Indications are that fluid slipage in the tubing greatly increases the effects of phase redistribution.

Fig. 1 is a field example of the effects that changing wellbore and flowing conditions can have on the character of a build-up curve. This Gulf Coast well, which has been a prolific producer, is 8,300 ft deep, contains a production packer, and has 20 ft of oil sand perforated for casing with 80 ft of formation. Permeability of the formation is in excess of 200 md. In Sept., 1955, this well was shut-in for about 1 minute and later recommissioned. Total production from this well was 425 B.D. and ratio of the well at the time was 570 cu ft/BBL. Both tests showed a marked influence by phase segregation, with the base rate showing the least influence.

In May, 1951, an additional test was made after stabilizing at a rate of 300 B/D, and the influence of phase segregation was found to be diminished. At that time the GOE had risen to 1,350 cu ft/BBL. In April, 1958, additional pressure build-up tests at 250 B/D were performed which were not so obviously affected by phase segregation. The GOE of the well at this time was 2,300 cu ft/BBL. These results are qualitatively consistent with those found in the phase redistribution study.

It appears that pressure build-up data obtained from wells with damage and production packers are more subject to the influence of phase segregation. In the case of a damaged wellbore, the pressure rise at the bottom of the tubing created by fluid segregation cannot be transferred readily to the producing formation. As a result the influence of phase segregation becomes more significant on the final build-up measurements.

Without a packer in the well, the volume and total compressibility of the wellbore is greatly increased and the pressure rise associated with fluid segregation in the tubing is more readily absorbed. At the same time, however, an afterflow of gas into the formation into the wellbore is maximized. Either one or a combination of these effects may produce a misleading build-up curve.

It seems reasonable to conjecture that phase redistribution effects are cumulative as a wellbore is extended in larger volumes and longer times are involved in the segregation of the fluids. With these complications, it is obvious that a new and more accurate method of analysis can be developed to correct for the combined wellbore effects.

BOTTOM-HOLE SHUT-IN TOOL

To eliminate the segregation effects and to minimize afterflow, a tool has been developed to shut a well in at formation depth. This tool, known as the collar lock pressure gauge plug, was designed and constructed to be run in the tubing against the stabilized flow of the producing well by means of a wire line. In this manner production of the well is not interrupted until the desired time of shut-in at the bottom of the tubing string. Field experiments have verified the applicability of the tool with flow rates as great as 225 B/D.

Tools are lubricated into the tubing string and retracted in the conventional manner using a 24-ft lubricator and the usual wire-line tools. Except for running small size gauges, the tool operations are similar to those used with other chokes and plugs. Pressure build-up tests may be attached serially to the plug by means of a shock absorber, and field tests have shown that this manner is not subjected to excessive shock.

EXPERIMENTAL PROGRAM

The collar lock pressure gauge plug just described has been used in several wells to obtain bottom-hole shut-in build-up curves for comparison with data obtained from conventional surface shut-in tests and from theoretical investigations. The wells chosen for this experimental work were equipped with production packers which were checked for packer leaks prior to each test. In addition observations of casing pressure during the test indicated that the packers were not leaking, and eliminated the possibility that similar fluids above the packer exerted any influence upon the pressure measurements. The presence of packers in these wells also enabled the bottom-hole shut-in to be more closely approximated shut-in at the sand face.

Wells have been selected to make a comparison between a surface shut-in test and a bottom-hole shut-in test in the same well. Individual wells were stabilized for equal time intervals and at equal production rates prior to both tests. The same pressure elements and bombs were used for both tests in each well. In the surface shut-in test, two recording bombs were run where possible—one containing a 60-hour clock to measure production over some portion of the build-up and the other, a 72-hour clock, for recording pressures at later times. Bottom-hole shut-in gauge measurements were taken using two bombs; the bomb containing the 72-hour clock was run on a tubing hanger just below the tubing top, while the bomb containing the 60-hour clock was run on a tubing hanger just above the tubing top.

Fig. 1—Pressure Build-Up Curves of Gulf Coast Well No. 1 (SPE 11330).
in where the producing formation has been isolated from the tubing columns.

The surface-in-tube curve shows that afterflow was predominant to 60 minutes and that humping occurred after approximately 1.5 hours. It is apparent that this curve is not amenable to analysis using standard methods. On the other hand, the curve obtained from the bottom-hole-shut-in shows no hump and a minimum of afterflow. In order to compare this bottom-hole-shut-in curve with a theoretical curve, the methods previously outlined were applied to obtain the pertinent reservoir parameters. Since the producing-rate ratio of this well was considerably above solution ratio, it was apparent that two-phase flow was occurring within the reservoir, consequently, the data were being dealt with multiphase flow and gave the following values; $m = 9.3$ psig/cycle; $k = 100$ reperm, $B_o = 6$ and $J_o = 430$ md/cycle.

These calculated values along with porosity, sand thickness and permeability were sufficient to characterize an ideal system, and enabled a theoretical build-up curve to be obtained from both the digital and analog approaches. A reasonable value for the drainage radius was considered to be 660 ft. Best results were obtained by computing the reservoir fluid compressibility under the assumption that no gas went into solution during the time of build-up and by weighting the oil and gas compressibilities according to the distance to the fractional portion of the hydrocarbon pore volumes occupied by each phase.

**Methods of Analysis**

The methods of analysis used in this paper are well known and are those presented by Miller, Dyes and Hardcastle (by reference). To establish, however, that these standard methods of analysis, based on shut-in data at the land side, can be advantageously used even though damage and afterflow are present, electrical analog and digital computer approaches were used to create theoretical curves for detailed comparison with the field curves obtained from the bottom-hole-shut-in tests. The authors of this paper are indebted to P. E. Chesney, in the operating departments of Sun Oil Co. for their splendid co-operation in this project. In addition we wish to thank W. A. Pinner for his assistance in the computer studies and the management of Sun Oil Co. for permission to publish this paper.

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**References**


See API System List in Trans., AIME (1956) 208, 142, for other system definitions.

Pressure Buildup Analysis
With Wellbore Phase Redistribution

Walter B. Fair Jr., SPE, Shell Oil Co.

APRIL 1981
Fig. 8 - Pressure buildup data, Example 1.

Fig. 9 - Pressure buildup data, Example 3.

Pressure Buildup Analysis
With Wellbore Phase Redistribution

Walter B. Fair, Jr., SPE, Sooth Oil Co.

Abstract
This paper presents an analysis of the effects of wellbore phase redistribution on pressure buildup tests. Wellbore phase redistribution is shown to be a wellbore storage effect and is incorporated mathematically into a new solution of the diffusivity equation. Dimensionless pressure solutions based on an infinite radial reservoir are presented for type-curve matching to analyze pressure buildup tests influenced by wellbore phase redistribution, and example analyses of actual field data are included. The parameters that affect phase redistribution and gas (ramping were documented also. A new information permits analysis of many anomalous pressure buildup tests which previously could not be analyzed quantitatively.

Introduction
Pressure buildup tests and other types of transient pressure tests have been used for many years to evaluate reservoir fluid flow characteristics and well completion efficiency. The basic theory and equations for the analysis of these tests are well documented. Many factors influence the pressure response in transient flow conditions have been investigated—i.e., the effects of reservoir boundaries, heterogeneities, and fractures, wellbore storage of fluids, and various types of well impairments, skin effects, and completion practices. However, little information concerning the effects of the redistribution of gas and liquid phases in the wellbore has been presented.

The phenomenon of wellbore phase redistribution occurs in a well that is shut in with gas in liquid flowing simultaneously in the tubing. As shown by Siegmeier and Matthes, when such a well is shut in at the surface, gravity effects cause the liquid to fall and the gas to rise to the surface. Because of the relative immiscibility of the liquid and gas in the wellbore, the pressure in the wellbore rises until equilibrium is reached. When this occurs, the pressure in the wellbore is relieved through the formation, resulting in a decrease in the pressure drop. The pressure drop is then equal to the pressure drop across the wellbore, and equilibrium is reached.

The redistribution of phases causes a net increase in the wellbore pressure. If the pressure drop is large enough, the pressure buildup in the wellbore is relieved through the formation, resulting in a decrease in the pressure drop. The pressure drop is then equal to the pressure drop across the wellbore, and equilibrium is reached.

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In this form, it is apparent that wellbore phase redistribution is a form of wellbore storage, since when

and when

the pseudosteady-state coefficient becomes negative, indicating a reversal in the direction of flow. When this occurs, a pressure buildup test becomes more like a pseudosteady-state test, and the gas flow rate becomes negative.

By considering the physical process of phase redistribution, certain properties of the pseudosteady-state coefficient can be inferred. In the following section, the pseudosteady-state coefficient will be derived from an analysis of pressure buildup tests.

Mathematical Analysis of Phase Redistribution
If we consider a well where wellbore phase redistribution occurs, it is apparent that wellbore storage also must occur. If the wellbore could store fluids of finite compressibility, the phase redistribution process could either (1) physically not occur or (2) be associated with a zero pressure increase. It is also interesting to note that the techniques presented by Siegmeier and Matthes and Pitzer et al. for minimizing wellbore phase redistribution also minimize wellbore storage effects.

For a well where wellbore storage occurs, the effects of the storage can be described by Eq. 1. The effect of the changing sand-face flow rate on the wellbore pressure also can be obtained from Eq. 2.

\[ \frac{dP}{d\theta} = -C_J \frac{dP}{d\theta} \]  

\[ \frac{dP}{d\theta} = \frac{1}{C_J} \left( 1 - \frac{dP}{d\theta} \right) \]  

To describe the effect of wellbore phase redistribution, note that not all of the pressure change in the wellbore can be attributed to wellbore storage flow rate effects, since some of the pressure change is caused by phase redistribution.

Therefore, it is expected that the pressure drop in the wellbore will not rise quickly and then slowly approach its maximum value C_p. This observation leads to the exponential function in Eq. 7, which satisfies the constraints of Eq. 6. Also, the one available set of unpublished laboratory data on the phase-redistribution pressure drop seems to confirm the following functional representation.*

\[ P_{\text{drop}} = C_p(1 - e^{-t/\tau}) \]  

In Eq. 7, the parameter C_p represents the maximum phase redistribution pressure drop, and \( \tau \) represents the time at which about 63% of the total change has occurred. A small value of \( \tau \) can be obtained by noting that the gas in the wellbore will rise to the surface with the total gas volume remaining constant; this is caused by the relative immiscibility of the liquid in the wellbore. C_p can be derived from Eq. 8, which represents a pseudosteady-state test, and the gas flow rate becomes negative.

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While $\alpha$ is not determined as easily, it is known that it will depend mainly on those factors which control the gas bubble or slug rise time in a well.

Finally, to keep the dimensionless quantities consistent, the dimensionless phase-redistribution pressure function is defined in Eq. 9.

\[
P_{RD} = C_D \left( 1 - e^{-\alpha s} \right), \tag{9}\]

where

\[
P_{RD} = \frac{k h}{C_D} - \frac{1}{\varphi \pi r_w^2}, \quad C_D = \frac{14.2}{q_w f_w^2}, \quad \varphi = 0.00264 \pi, \quad r_w = 0.00264, \quad \text{and} \quad \alpha = \frac{k}{C_D}.
\]

In SPE preferred SI units, replace 141.2 with 7.2 x 10^{-5} and 0.00264 with 3.6 x 10^{-8}.

**Determination of Dimensionless Wellbore Pressures**

To obtain dimensionless pressure solutions for use in the analysis of pressure buildup tests, it is necessary to incorporate the effects of wellbore phase redistribution into the diffusivity equation. For radial flow in an infinite, homogeneous, isotropic reservoir of a fluid of small compressibility, this problem is stated in dimensionless variables as follows. The diffusivity equation is

\[
\frac{\partial P_{RD}}{\partial t_{RD}} + \frac{1}{r_w} \frac{\partial}{\partial r} \left( r_w \frac{\partial P_{RD}}{\partial r} \right) = \frac{1}{C_D} \frac{\partial P_{RD}}{\partial t_{RD}}.
\]

The boundary conditions are

\[
p_{RD}(r_w, 0) = 0, \quad \lim_{r \to 0} \frac{\partial P_{RD}}{\partial r}(r_w, t) = 0, \quad \alpha = \frac{k}{C_D}.
\]

Several authors have shown that this problem also can be written as a convolution integral to account for wellbore storage. This approach leads to Eq. 15.

\[
P_{RD}(t_{RD}) = \int_0^1 \left[ \frac{\partial P_{RD}(r)}{\partial r} - \frac{\partial P_{RD}(r)}{\partial t_{RD}} \right] dr + S \left[ 1 - C_D \right].
\]

Eq. 15 can be solved for $\mathcal{L}(P_{RD})$, the Laplace transform of the desired pressure function. This results in Eq. 16. ($s$ denotes the Laplace transform variable.)

\[
\mathcal{L}(P_{RD}) = \frac{[\mathcal{L}(P_{RD}) + S] + C_D \mathcal{L}(P_{RD})}{s [1 + C_D \mathcal{L}(P_{RD}) + S]}.
\]

Note that the solution is entirely general, since no constraints have been placed on either $P_{RD}$ or $P_{RD}$, except that these functions exist and are Laplace transformable. Thus, if $P_{RD}$ represents any type of reservoir condition, the required pressure solutions for those conditions can be determined in principle. This statement also applies to the phase redistribution pressure.

In this work, Eq. 9 is used for the phase redistribution effect. Its Laplace transform is:

\[
\mathcal{L}(P_{RD}) = \frac{C_D}{s + 1/\alpha D}.
\]

The required expression for $\mathcal{L}(P_{RD})$ has been presented by Van Everdingen and Hurst\textsuperscript{a} in Eq. 19, where $K_0$ and $K_1$ are modified Bessel functions.

\[
\mathcal{L}(P_{RD}) = \frac{K_0(\nu s)}{s^{3/2} K_1(\nu s)}.
\]

It also has been shown that at long times this simplifies to the line source solution in Eq. 19, since $\nu K_0(\nu s) \rightarrow 1$ when $s \to 0$ or $t_{RD} \to \infty$.

\[
P_{RD}(s) = \frac{1}{s} K_0(\nu s).
\]

A further time-long approximation for $\mathcal{L}(P_{RD})$ can be obtained by noting that as $t_{RD} \to 0$, $s \to 0$.

\[
K_0(\nu s) \approx \frac{1}{\sqrt{s}} e^{-\nu s},
\]

where $\gamma = 0.577 215 664 901 52$... denotes Euler's constant. This gives Eq. 20.

\[
P_{RD}(s) = \frac{1}{s} \left( \frac{1}{\sqrt{s}} e^{\gamma / 2} \right).
\]

Combining the definition of $\mathcal{L}(P_{RD})$ in Eq. 19 with the various forms of $\mathcal{L}(P_{RD})$ from Eqs. 18, 19, and 20 yields required expressions for $\mathcal{L}(P_{RD})$ as follows.

**Cylindrical Source Well.**

\[
\mathcal{L}(P_{RD}) = \left[ \frac{K_0(\nu s)}{s^{3/2} K_1(\nu s)} + 1 \right] \frac{1 + C_D C_D \mathcal{L}(P_{RD})}{s + 1/\alpha D}.
\]

Note that Eq. 19 indicates that a representation very similar to wellbore storage will exist at short times. This is consistent with Earle\textsuperscript{1} and earlier comments.

To obtain dimensionless pressures for use in analyzing pressure buildup tests with wellbore phase redistribution, Eqs. 21, 22, or 23 must be inverted. Since these expressions are too complicated for analytical inversion, the inverse Laplace transforms

\[
P_{RD} = \frac{1}{C_D} + \frac{C_D}{\alpha}.
\]

**Line Source Well.**

\[
\mathcal{L}(P_{RD}) = \left[ (K_0(\nu s) + 1) + C_D C_D \mathcal{L}(P_{RD}) \right] \frac{1}{s + 1/\alpha D}.
\]

\[
\mathcal{L}(P_{RD}) = \left[ (1 + C_D C_D \mathcal{L}(P_{RD})) \right] \frac{1}{s + 1/\alpha D}.
\]

\[
\mathcal{L}(P_{RD}) = \left[ (1 + C_D C_D \mathcal{L}(P_{RD})) \right] \frac{1}{s + 1/\alpha D}.
\]

The long-time approximating form of $P_{RD}$ can be derived from Eqs. 22, 23, or 24 by noting that

\[
\nu^2 \left( \frac{1}{s + 1/\alpha D} \right) \to 0 \text{ as } s \to 0 (t_{RD} \to \infty).
\]

Thus, these equations reduce to the wellbore storage equation given by Agarwal et al.,\textsuperscript{3} which further approaches Eq. 24.

\[
P_{RD} = \frac{1}{C_D}.
\]

The short-time approximation also can be obtained from Eq. 1 by noting that the wellbore storage factor obtained by letting $C_D = 0$ reduces to

\[
\mathcal{L}(P_{RD}) = \frac{1}{s^{3/2} K_1(\nu s)}.
\]

Also, since $\nu^2 \left[ (s + 1/\alpha D) \right]^{-1/2}$ at large $s$, $\mathcal{L}(P_{RD})$ must approach

\[
\mathcal{L}(P_{RD}) = \frac{1}{C_D^2} \frac{1}{s^{3/2} K_1(\nu s)}.
\]

and $P_{RD}$ approaches

\[
P_{RD} = \frac{1}{C_D}.
\]

\[
\frac{1}{C_D} = \frac{1}{C_D} + \frac{C_D}{\alpha}.
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To obtain dimensionless pressures for use in analyzing pressure buildup tests with wellbore phase redistribution, Eqs. 21, 22, or 23 must be inverted. Since these expressions are too complicated for analytical inversion, the inverse Laplace transforms...
were calculated numerically using an inversion technique presented by Seidell and adapted for use on the TI-99 programmable calculator. The type curves shown in Figs. 1 through 6 indicate that the pressure functions may show a tendency toward a damped oscillation. According to Seidell, such an oscillation may render the numerical technique useless unless certain conditions on wavelength of oscillation are met. However, it can be shown that the functions obtained in this work do not oscillate, since the Laplace transform can be written as the sum of three terms. Two of the terms represent monotonic functions, while the inverse transform of the remaining term has a single maximum. Thus, Seidell's criteria of functional 'smoothness' is met on each term and, by the linearity property of the Laplace transform and of the numerical technique, it is valid to use the numerical method with these functions.

Eq. 21 was programmed and inverted for several values of the wellbore-storage coefficient $C_p$ and skin factor $S$. Results and a comparison with data previously reported by Agarwal et al. are shown in Table 1. The excellent agreement indicates that the numerical technique is well suited to the calculator precision. Eq. 22 also was programmed and inverted for several values of $C_p$ and $S$ and again with close agreement with the Agarwal et al. results for the line

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**Table 2** — Comparison of Calculated $P_{s0}$ with Ref. 5 (Line Source Well, $C_p = 0$)

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$C_p = 10000$, $S = 0$

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<tr>
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</table>

$C_p = 100000$, $S = 0$

---

**Table 3** — Comparison of $P_{s0}$ Calculations ($C_p = 0$)

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$C_p = 1000$, $S = 0$

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<th>$P_{s0}$</th>
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<tr>
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<td>8.4063</td>
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$C_p = 10000$, $S = 0$

<table>
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<th>$P_{s0}$</th>
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<tr>
<td>1.00</td>
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<td>8.4063</td>
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</tbody>
</table>

$C_p = 100000$, $S = 0$
source well solution, as shown in Table 2. Finally, Eq. 23 was inverted and, again, the results shown in Table 5 and 6 are in close agreement with previous results. Eq. 23 therefore was used in the remainder of this study.

The use of the dimensionless pressures in buildup analysis, \( C_{DP} \) from Eq. 27 was used as a variable rather than \( \alpha_D \), which is more difficult to determine. The results of this are shown in Tables 5 and 6, and log \( p_{D0} \) vs log \( D_T \) type curves are presented in Figs. 1 through 5. Accuracy is ±0.1%.

A comparison of the phase redistribution type curves with wellbore-storage type curves is shown in Fig. 7. Note that at long times the new curves coincide with the storage curves, while at early times the apparent wellbore storage effect is obvious. At intermediate times, the phase redistribution effect causes the curves to trend away from the apparent storage behavior to the true storage behavior. At large values of \( C_{DP} \), the "gas hump" is apparent, while at small values of \( C_{DP} \) the phase redistribution effect is much diminished. A potential problem in pressure data interpretation also is shown at intermediate values of \( C_{DP} \), since the curve for Case 4 with \( C_{DP} = 100 \) resembles the storage case with \( C_{DP} = 10 \) while the true \( C_{DP} = 1.00 \). An attempt to type-curve match phase redistribution data to a storage curve could give a reasonable type-curve match, but any estimates of reservoir parameters might be greatly in error. Fortunately, this problem can be resolved by comparing the estimates of the true and apparent storage constants as in the following examples.

### Analysis of Pressure Buildup Tests

Generally, to analyze pressure buildup test data, the superposition principle is applied to the following dimensionless pressure functions yielding

\[
\frac{dp}{dt} = \frac{p_D(t) + p_m(t) - p_w(t)}{D_T} \quad \text{(28)}
\]

Thus, a log \( p_m(t) - p_w(t) \) vs log \( D_T \) plot of the buildup data can be type-curve matched according to a technique that uses \( Z_{DP} = \frac{p_m(t) - p_w(t)}{p_T(t)} \). If this assumption is not valid, the more general superposition in Eq. 28 must be used;

### Table 5 — Dimensionless Wellbore Pressures with Phase Redistribution

<table>
<thead>
<tr>
<th>( C_{DP} ) = 20, ( C_{DP} = 100 )</th>
<th>( C_{DP} = 20, ( C_{DP} = 1.00 )</th>
<th>( C_{DP} = 20, ( C_{DP} = 10.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{C_{DP}}{C_{DP}} )</td>
<td>( S = 10 )</td>
<td>( S = 20 )</td>
</tr>
<tr>
<td>0.10</td>
<td>1.00</td>
<td>0.95</td>
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<tr>
<td>0.20</td>
<td>0.95</td>
<td>0.97</td>
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<tr>
<td>0.30</td>
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<tr>
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<tr>
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<tr>
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### Table 6 — Dimensionless Wellbore Pressures with Phase Redistribution

<table>
<thead>
<tr>
<th>( C_{DP} = 100, ( C_{DP} = 1.00 )</th>
<th>( C_{DP} = 100, ( C_{DP} = 10.00 )</th>
<th>( C_{DP} = 100, ( C_{DP} = 10.00 )</th>
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</thead>
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<tr>
<td>( \frac{C_{DP}}{C_{DP}} )</td>
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Table 7: Pressure Buildup Data for Field Example 1

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td>q</td>
<td>212 ft³/day</td>
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<tr>
<td>g</td>
<td>4.01 (10⁻⁶) psi⁻¹</td>
</tr>
<tr>
<td>b</td>
<td>0.13 ft³/m³</td>
</tr>
<tr>
<td>h</td>
<td>10 ft</td>
</tr>
<tr>
<td>θ</td>
<td>0.00307 B/D (10⁻²)</td>
</tr>
<tr>
<td>C</td>
<td>0.28</td>
</tr>
<tr>
<td>r₂</td>
<td>0.38 psi (1.0 ft²/m³)</td>
</tr>
<tr>
<td>S₀</td>
<td>0.36 psi (1.0 ft²/m³)</td>
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</table>

In SPE preferred SI units, 1/41.2 is replaced by 7.2 x 10⁻⁸ in Eq. 28 through 30.

In the following section, we discuss the analysis of bottomhole pressure buildup surveys which are influenced by wellbore phase redistribution. Note that not all surveys are analyzed as easily. The tests documented here are taken in gas-lifted oil wells in southern Louisiana, and one factor which makes these tests amenable to analysis is that little free gas enters the wellbore after shut-in. Most of the gas in the tubing string which contributes to the phase redistribution process originates from the annulus through the gas-lift valves. Thus, the true wellbore storage coefficient C₀ is controlled by the rising liquid level of the inflowing fluid which remains essentially constant.

In other cases, it is not as obvious that the density of the inflowing fluid remains constant, since the fluid density or ratio can change as the well is shut-in; this would cause a changing storage coefficient. In addition, the compression of the gas near the surface may not be accounted for correctly, which will also cause a variable wellbore storage. Although the buildup data in gas-lifted oil wells, this gradient must be measured below the point of gas-lift gas entry. Any differences in the flowing and static conditions which cannot be attributed to frictional effects generally lead to an indication of the flow of free gas from the reservoir. The gradients used in the following examples were measured in conjunction with the pressure surveys.

Example Analysis

Example 1 is an actual field pressure buildup data measured in a gas-lifted oil well in southeast Louisiana. The basic data are shown in Table 7 and a log-log plot of the pressure data is shown in Fig. 8.

From the data plot in Fig. 8, a point on the unit slope straight line is estimated to be Δp = 153 psi (1055 kPa) at Δt = 0.1 hour. The wellbore storage coefficient is calculated as in Eq. 30 and the apparent storage coefficient as in Eq. 31. The gradient used in Eq. 30 is calculated from flowing and static-pressure surveys measured in conjunction with the buildup test.

C = Δp / Δt = 0.00173 bbl/psi (0.000 270 m₃/kPa) (30)

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The data are matched on the type curves for $C_p = 100$, $C_{D} = 20$ and the best match is estimated as shown in Fig. 9 to be $C_p = 10$, $S = 5$, $p_{ud} = 2.80$ at $Q_p = 100$ (1568 kPa), and $C_{D} = 320$ in $\Delta t = 1$ hour. From the standard definitions of $p_{ud}$ and $t_{D}$, the permeability is calculated as follows.

From $p_{ud}$ match: $k = 1.45$ md.
From $t_{D}$ match: $k = 2.14$ md.

Fig. 9 also serves to indicate that the last two data points may be close to the semilog straight line. Using semilog analysis, the permeability and skin are estimated to be 1.46 md and 3.4, respectively. Since the straight line may not have been reached and only two points are used to determine the semilog straight line, these estimates are in agreement with the estimates obtained from type-curve matching.

Summary and Conclusions
It has been recognized for some time that wellbore phase redistribution may cause anomalous pressure build-up behavior in oil and gas wells. The basic concepts are much more general. In particular, it is possible to apply the techniques used in this study to other reservoir systems and thereby to obtain techniques for the analysis of data in fractured systems as well as other practical situations.

Nomenclature
$A_c$ = cross-sectional area of the wellbore, $\text{bbl}/(m^2)$
$B$ = formation volume factors, RBST/STB
$C =$ wellbore storage coefficient, bbl/psi
$C_{D} = $ apparent dimensionless storage coefficient, bbl/psi
$C_{D0} = $ apparent dimensionless storage coefficient, $5.6146C_t$ $2.945hr^2$
$C_{D1} = $ effective dimensionless storage coefficient defined in Eq. (5)
$C_{D2} = $ dimensionless wellbore storage coefficient, $5.461C_t$ $2.945hr^2$
$C_{p} = $ phase redistribution pressure parameter, psi/kPa
$C_{pD} = $ dimensionless phase redistribution pressure parameter,
$C_{pD0} = $ dimensionless phase redistribution time parameter,
$C_{D} = $ dimensionless phase redistribution time parameter,
$C = 0.000264 a_{r} e^{-3.65 \times 10^{-6} \frac{t}{hr}}$
$\alpha_{D} = 0.000264 e^{-3.65 \times 10^{-6} \frac{t}{hr}}$
$\phi_{w} = \text{darcy variable of integration}$
$\mu = \text{fluid viscosity, cp (Pa s)}$
$\rho_f = \text{fluid density, kg/m}^3$
$\rho_p = \text{pressure, psi (kPa)}$
$P_{in} = \text{inlet pressure, psi (kPa)}$
$P_{sw} = \text{maldwell pressure, psi (kPa)}$
$P_{wD} = \text{dimensionless wellbore pressure}$
$P_{pD} = \text{dimensionless pressure}$
$P_{PD} = \text{dimensionless phase redistribution pressure}$
$P_{PD0} = \text{dimensionless phase redistribution time parameter,}$

$\text{Acknowledgment}$

The author thanks the Management of Shell Oil Co. for permission to publish this paper.

References
Well-Test Analysis With Changing Wellbore Storage

Peter S. Hegeman, SPE, Debora L. Hallford, SPE, and Jeffrey A. Joseph, SPE, Schlumberger

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SPE Formation Evaluation, September 1993
Fig. 4 - Homogeneous system with exponential storage decrease.

Fig. 5 - Homogeneous system with error-function storage decrease.
Fig. 7 - Example 1, match with constant-storage model and decreasing-storage model (error function).

Fig. 8 - Example 2, match with constant-storage model and decreasing-storage model (error function).
Fig. 9 - Example 3, match with constant-storage model and decreasing storage model (error function).
Fig. 10 - Example 3, match with decreasing storage (exponential).
Well-Test Analysis With Changing Wellbore Storage

Peter S. Hegeman, SPE, Deborah L. Halford, SPE, and Jeffrey A. Joseph, SPE, Schlumberger

Summary. This paper presents a model for analyzing changing wellbore storage during well testing. Our model is based on a modification and extension of a model for phase redistribution. The result is a general solution in Laplace space that can be used to add changing storage to a variety of well/reservoir models. Increasing and decreasing storage cases can be accounted for analytically.

We present several field examples to show the application to well tests with changing wellbore storage.

Introduction

Changing wellbore storage during well testing has been reported in the technical literature for more than 30 years. This class of problems includes wellbore phase redistribution and increasing or decreasing storage in connection with injection well testing. Decreasing storage, usually caused by decreasing wellbore compressibility, is frequently encountered during pressure-buildup testing. Low-permeability gas wells that build up over a large pressure range often show this effect. Although simultaneous measurement of downhole rate and pressure can reduce the severity of changing storage, it does not eliminate the problem when wellbore volume is appreciable below the production logging tool.

Changing storage makes application of analysis methods based on a constant-storage assumption, such as type-curves matching, difficult. Use of these techniques usually results in a systematic mismatch of the models to the measured data at early times. When a well test is run long enough to develop finite-volume radial flow in the reservoir, the most serious side effects of the early-time mismatch will be a virtual impact on the observed and reduced confidence of the interpretation. Furthermore, erroneous insights arise where well-test data are considered uninterpretable because of the continued effects of varying wellbore storage and insufficient matched data. Tests too short can result in poor equipment performance.

In 1972, Ramey and Ashley presented an analytical solution for a step change in wellbore storage. In 1981, Farvani presented a solution for an exponential increase in wellbore storage, which he used to model wellbore phase redistribution. Fair et al. showed that changing wellbore storage caused an apparent lowering of the wellbore storage coefficient.

The storage coefficient could become negative, indicating a reversal in flow direction. In this paper, we present a model for analyzing increasing or decreasing wellbore storage. The model is based on a modification and extension of Fair’s approach. The result is a general solution in Laplace space that can be used to add changing storage to a variety of reservoir models (p functions). Field examples show the applicability of the model to well tests with changing wellbore storage.

Mathematical Development

As van Everdingen and Hurst showed, the effects of constant wellbore storage can be described by

\[
\frac{d\phi}{dt} = \phi
\]

In his study of wellbore phase redistribution, Fair modified Eq. 1 by adding a term to account for the pressure change caused by phase redistribution:

\[
\frac{d\phi}{dt} = \phi \frac{d\phi}{d\phi} + \frac{d\phi}{d\phi} (1)
\]

Thus, phase redistribution was modeled as a changing-wellbore-storage phenomenon. The changing-phase-pressure function, \(f_{\phi}\), has the following properties:

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SPE Formation Evaluation, September 1993

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The buildup was matched with the decreasing-wellbore-storage model by use of the error-function storage term. Fig. 9b shows this match. As in the previous examples, addition of the decreasing storage allowed the entire buildup to be matched. The decreasing-storage term resulted in a significantly lower value for \( C_{w}^{\text{eff}} \) and a corresponding significantly lower value for skin damage \( s = 2.9 \) vs. \( s = 8.7 \) for the constant-storage match.

Fig. 10 shows the match with the exponential-function storage term. Although this match is an improvement over the constant-storage match of Fig. 9a, the buildup data exhibit a sharper, more abrupt transition than the exponential model. The error-function storage transition (Fig. 9b) provides a superior match of the test data.

### Conclusions

1. A model for analyzing changing wellbore storage during well testing has been presented. Increasing- and decreasing-storage cases can be accounted for analytically.

2. The Laplace space solution allows the addition of changing wellbore storage to a variety of well/reservoir models (homogeneous, two porosity, hydraulic fracture, multilayer, etc.) whose solutions also are in Laplace space.

---

**TABLE 3 — WELL AND TEST DATA FOR EXAMPLE 2**

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<tr>
<td>Temperature, °F</td>
</tr>
<tr>
<td>Flow rate, B/D</td>
</tr>
<tr>
<td>Oil density, API</td>
</tr>
<tr>
<td>Water (msc) %</td>
</tr>
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<td>GOR, scf/bbl</td>
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</table>

<table>
<thead>
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<th>Pressure Data</th>
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</tr>
<tr>
<td>0.0160</td>
</tr>
<tr>
<td>0.0320</td>
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<tr>
<td>0.0800</td>
</tr>
<tr>
<td>0.0960</td>
</tr>
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</table>

3. Two forms of the changing-storage transition, exponential and error function, have been investigated. The error function has a sharper, more abrupt transition period.

4. Field data obtained by changing wellbore storage can be interpreted, higher degree of confidence can be placed on the interpretation compared with constant-storage analysis.

5. Using a constant-storage model to analyze buildup data exhibiting decreasing storage may lead to significant overestimation of the skin damage.

---

**Nomenclature**

- \( B = \) PVF, vs. bbl/STB
- \( C_{l} = \) total compressibility, \( \text{L}^2/\text{m} \)
- \( C_{w} = \) wellbore-storage coefficient, bbl/psi
- \( C_{w} = \) apparent (early time) wellbore-storage coefficient, bbl/psi
- \( C_{p} = \) initial production, bbl/day
- \( C_{S} = \) storage coefficient, psi/bbl
- \( k = \) formation thickness, ft
- \( f = \) formation permeability, \( \text{L}^2 \)
- \( m = \) Laplace transform operator
- \( p = \) pressure, psi
- \( q = \) rate, bbl/day
- \( t = \) time, hours
- \( x = \) reservoir variable
- \( \alpha = \) constant, psi/day
- \( \phi = \) porosity

**Subscripts**

- \( D = \) dimensionless
- \( i = \) initial
- \( f = \) final
- \( \text{w} = \) wellbore

**Superscript**

- \( \text{d} = \) derivative

---

**Acknowledgments**

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**References**


SI Metric Conversion Factors

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*Conversion factors in tabular form.*

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