Productivity of Vertically Fractured Wells Prior to Stabilized Flow

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Introduction

Several authors have presented papers on the effects of hydraulic fracturing on well productivity. Most of this work has been concerned with the effects of fracturing on "stabilized" or "semisteady state" productivity. Prats et al. investigated analytically the effects of vertical fractures of infinite flow capacity on well productivity as a function of time. Their results showed a significant variation in productivity over the range of dimensionless times ($t_0$) from 0.01 to 0.5. Productivity for one set of reservoir conditions showed a variation of about eightfold over this range of dimensionless time. Morse and Holditch indicated fractured well productivities varying by factors in excess of 10 over a real-time range of a few weeks or months under some reservoir conditions.

The objective of this work was to make a systematic study of both constant rate and constant pressure well productivities for a range of reservoir rock and fluid properties and for vertical fractures of various lengths.

Constant Rate Results

We noted that the work of Russell and Truitt was presented in sufficient detail to allow calculation of productivity index—defined here as $q_f/(\bar{p} - p_w)$—as a function of dimensionless time. Details of this calculation are shown in the Appendix. Fig. 1 shows the results of such calculations from the data of Russell and Truitt. This figure shows the calculated well productivity index ratio $J_f/J_s$ vs $t_0$, for various ratios of fracture radius to drainage radius.

To show the relationship between relative productivity ($J_f/J_s$), real time, and rock permeability, Fig. 2 was calculated from Fig. 1 for a given set of reservoir rock and fluid properties using the dimensionless time equation

$$t_0 = \frac{0.006328 k}{\rho \mu \phi c r_f^2}.$$  

A fracture length of 330 ft ($x_f/x_r = 0.5$) and a 40-acre square drainage area were assumed. The constant value of $\phi c r_f^2$ used to construct Fig. 2 could represent a rock porosity of 0.20 and either of the following:

(1) gas with viscosity of 0.01 cp and compressibility of $10^{-3}$,

(2) oil with viscosity of 1.0 cp and compressibility of $10^{-3}$.

In other words, Fig. 2 represents the relative-productivity vs real-time relationship for a typical gas or undersaturated oil reservoir.

These results show that for these broadly typical conditions the time necessary for stabilized flow conditions may vary from 7 days to 7,000 days for reservoir permeabilities from 10 md to 0.01 md. Also, it is readily apparent that in this permeability range a short-time well productivity measured in the first 10 days of production can differ from the stabilized productivity by a factor of 10 or more. For the 0.01-md permeability the time for stabilized flow is in excess.

Using numerical simulation, a systematic study has been made of the productivity of vertically fractured wells from the start of production until the attainment of stabilized flow. Productivity data are presented for wide ranges of fracture lengths, reservoir conditions, and fluid properties. Results show that for low-permeability reservoirs, well productivity varies greatly with time.
of 19 years. If the drainage area were 640 acres, all the times shown in Fig. 2 would be increased by a factor of 16.

The results for the unfractured 0.01-md well show that the total range of indicated well productivities differs less than a factor of 3 from the stabilized productivity. As shown by Fig. 1, the greater the ratio of fracture radius to drainage radius \(x_f/x_d\) the greater the variation in relative productivity from early times to the time of stabilized flow.

From these results it is apparent that for fractured wells in low-permeability reservoirs, forecasts of production based on the "stabilized flow" concept and known reservoir conditions can lead to grossly conservative values for the first months or years of production. Similarly, long-term forecasts based on single-point productivity tests early in the producing life of a well can be very greatly in error on the high side.

It appears possible that complete payout of a project may be attained during a period of high rate of production preceding stabilized flow. For this reason a study was made of the production rates attainable from fractured wells in which the wellbore pressure was held at a constant level. The work of Russell and Truitt was for constant rate of production and, hence, not applicable to this situation.

**Method of Solution**

Numerical simulation was used to solve for the relationship between production rate and time; for constant wellbore pressure, a square drainage area with closed boundaries, and fractures of various lengths. A general simulator was used for the solution. The only unique feature found necessary was the use of flow cells in which the dimensions varied over a wide range. To achieve results of acceptable precision it was necessary to use small dimensions in the vicinity of any discontinuity such as the edge or end of the fracture. Larger dimensions were acceptable at greater distances from these discontinuities. Fig. 3 shows the model used to simulate the performance of a quadrant containing a well with \(x_f/x_d = 0.5\). This model arrangement allowed simulation results that checked within less than 1 percent of those calculated from the tabulated results of Russell and Truitt over the complete range of dimensionless time.

**Constant Pressure Results**

The results of the constant well pressure simulations are presented in Fig. 4 in the same form as for the constant rate presentation of Fig. 1. Here, again, \(J_0\) refers to the stabilized productivity index for constant producing rate. The results for the two sets of data are generally similar but show significant quantitative differences. Note on Fig. 4 that for each of the different fracture lengths semisteady state is reached at a \(t_0\) of approximately 0.2.

For the same reservoir and fluid properties used in Fig. 2, the ratios of productivity indices \((J_f/J_0)\) are shown in Fig. 5 vs real time for the constant pressure case. As the permeability decreases, the real time to
achieve semisteady state increases. As Fig. 5 shows, the semisteady state $J_s/J_s$ is constant at 6.6 for each of the different permeability reservoirs since they all have the same fracture length of 330 ft.

The relationship between dimensionless time and the production ratio $q_{10}/q_{10}$ is illustrated in Fig. 6. At the later values of dimensionless time the effect of depletion of the reservoir is felt at the well. The effect is felt first in the well with the longest fracture since it has the highest initial production rate and the largest cumulative production.

For $x_0/x_r = 0.5$ and the same rock and fluid properties as used in Figs. 2 and 5, Fig. 7 shows the relationship between $q_{tr}/q_{tr}$ and real time.

Fig. 8 presents the ratio of cumulative production $(N_{pt}/N_{pt})$ vs dimensionless time. The cumulative production ratio $N_{pt}/N_{pt}$ for each of the fracture lengths is approaching 1.0 as a terminal value. That is, the unfractured well would ultimately recover the same amount of oil as the fractured well if it were allowed to produce long enough. But for low-permeability formations this would not be economically feasible.

Fig. 9 shows the cumulative production ratio vs real time for the same reservoir situations as illustrated in Fig. 7. Fig. 9 indicates that during the first few months the cumulative production for a fractured well may be 100 or more times that of a stabilized unfractured well. For example, in a reservoir with a permeability of 0.01 md and the other reservoir properties as listed on Fig. 9, it can be shown that for a certain wellbore pressure, the cumulative production is increased from 100 to 2,540 bbl of oil for the first
100 days by inducing a fracture extending 330 ft from the wellbore.

From Fig. 5 it can be seen that if the reservoir permeability is more than 10 md the productivity ratio soon approaches a constant multiple of that of a stabilized unfractured well. This multiple indicates the magnitude of increase that would be predicted from the published semisteady-state charts of McGuire and Sikora and Craft et al. However, for a well in a reservoir having a permeability of 0.01 md the cumulative fluid produced during the first 100 days is about 4 times that predicted from the semisteady results (see Fig. 9). Thus, it is quite possible that the actual production could be very attractive economically, whereas the semisteady-state rates would be marginal or economically unattractive.

It should be emphasized that the performances shown in Figs. 5, 7, and 9 are for a specific set of reservoir fluid and fracture properties. Performance may be calculated for other fluid and rock properties from the generalized data of Figs. 4, 6, and 8. It should be noted that all figures apply only to fracture flow capacities that are very large relative to the formation flow capacity. If the flow capacity of the fracture is less than about 10,000 times that of the formation, additional simulation data must be derived for the lower fracture flow capacity.

Also, it should be noted that all figures are calculated for 40-acre spacing and \( r_e / r_w \) of 1.320. The real time for any other spacing will be the time shown multiplied by \( A/40 \), where \( A \) is the spacing in acres per well. The time shown just to achieve stabilized flow for 0.01 md, 40 acres, and \( x_{fr} / x_e = 0.5 \) is more than 19 years. It seems highly unlikely that reservoirs of such low permeability will be exploited on very wide spacings even with mammoth vertical fractures. For example, for 0.01 md and 400-acre spacing, it would take 190 years just to reach stabilized flow.

Since the performance results presented were derived for a constant-compressibility fluid, they will apply quantitatively only to undersaturated oil reservoirs. Compressibility factors for gas or gas-oil mixtures increase greatly with reduced pressure. It is to be expected that for these situations even greater difference will be observed between the actual performance of fractured wells and that predicted from semisteady-state estimates.

**Conclusions**

1. Estimates of semisteady-state production rates for fractured reservoirs are adequate where the reservoir permeability is more than 10 md.
2. For reservoirs of very low permeability, estimates of semisteady-state production rates for vertically fractured wells will be grossly pessimistic during the first months or years of production.
3. For reservoirs of very low permeability the productivity of a vertically fractured well during the first few months of production may be more than 10 times the final stabilized productivity.
4. Production times up to several months or years are necessary to achieve stabilized well productivities indicative of long-range deliverability for vertically fractured wells in reservoirs of very low permeability.

**Nomenclature**

- \( A \): well spacing, acres/well
- \( B \): formation volume factor, reservoir volume/stock-tank volume
- \( c \): compressibility, psi\(^{-1}\)
- \( h \): formation thickness, ft
- \( J_e \): productivity index of fractured well, BOPD/psi
- \( J_w \): semisteady-state productivity index of unfractured well, BOPD/psi
- \( k \): permeability, md
- \( N_{fr} \): cumulative production from fractured wells, bbl
- \( N_{ps} \): cumulative semisteady-state production from unfractured wells, bbl
- \( p \): pressure, psi
- \( \bar{p} \): average reservoir pressure, psi
- \( p_i \): initial reservoir pressure, psi
- \( p_0 \): dimensionless pressure
- \( q_s \): oil flow rate, B/D
- \( q_{of} \): oil flow rate from fractured well, B/D
- \( q_{os} \): semisteady-state oil flow rate from unfractured well, B/D
- \( r_e \): external radius of drainage, ft
- \( r_w \): wellbore radius, ft
- \( t \): time, days
- \( t_0 \): dimensionless time: \( \frac{0.006328 \, kt}{\phi \mu C_r^2} \)
- \( V \): pore volume of reservoir, bbl
- \( x_e \): distance to boundary, ft
- \( x_{fr} \): fracture half-length, ft
- \( \mu \): viscosity, cp
- \( \phi \): porosity, fraction

**References**


APPENDIX
Calculation of $J_t/J_s$ from data of Russell and Truitt.

\[
c = \frac{1}{V} \frac{dV}{dp}
\]

\[
\Delta p = p_i - p = \frac{\Delta V}{V_c} = \frac{q_o \cdot t \cdot B}{4 \cdot x_e^2 \cdot h \cdot \phi \cdot c}
\]

From Russell and Truitt:

\[
p_i - p_w = \frac{221.8 \cdot q_o \cdot h \cdot B \cdot p_D}{k \cdot h}
\]

By definition,

\[
J_t = \frac{q_o}{p - p_w} = \frac{q_o}{p_i - p_w - (p_i - \bar{p})}
\]

therefore,

\[
J_t = \frac{221.8 \cdot q_o \cdot h \cdot B \cdot p_D}{k \cdot h} - \frac{q_o \cdot t \cdot B}{4 \cdot x_e^2 \cdot h \cdot \phi \cdot c}
\]

For semisteady state,

\[
q_o = \frac{0.007082 \cdot k \cdot h \cdot (\bar{p} - p_w)}{\mu \cdot B \cdot (\ln \frac{r_e}{r_w} - \frac{3}{4})}
\]

\[
J_s = \frac{q_o}{p - p_w} = \frac{0.007082 \cdot k \cdot h}{\mu \cdot B \cdot (\ln \frac{r_e}{r_w} - \frac{3}{4})}
\]

therefore,

\[
J_t/J_s = 1 \left[ \frac{221.8 \cdot \mu \cdot B \cdot p_D}{k \cdot h} - \frac{5.61 \cdot t \cdot B}{4 \cdot x_e^2 \cdot h \cdot \phi \cdot c} - \frac{0.007082 \cdot k \cdot h}{\mu \cdot B \cdot (\ln \frac{r_e}{r_w} - \frac{3}{4})} \right]
\]

From the dimensionless time equation,

\[
t = \frac{\phi \cdot \mu \cdot c \cdot r_e^2 \cdot t_D}{0.006328 \cdot k}
\]

Therefore, assuming $r_e = x_e$,

\[
J_t/J_s = \frac{0.6366 \cdot (\ln \frac{r_e}{r_w} - \frac{3}{4})}{p_0 - t_D}
\]