Estimating Oil Reserves From Production-Decline Rates

By PARK J. JONES
Petroleum Engineer, The Texas Company

1. Rate of decline.—The rate of production decline of most fields, leases, and wells decreases from a high initial rate down to a relatively low rate toward abandonment. Quantitatively,

\[
\text{Per cent rate of decline} = 100 \times \frac{Q_0 - Q_1}{Q_0} \times \frac{100}{D}
\]

Where:

\[
D = \text{fractional rate of decline, bbl/year} \\
Q_0 = \text{last year's production, bbl/year} \\
Q_1 = \text{this year's production, bbl/year}
\]

Example 1.—The following rates of decline on a certain lease which produced in accordance with the listed values is obtained by Equation 1.

\[
\begin{array}{cccccc}
\text{Year} & 1933 & 1934 & 1935 & 1936 & 1937 \\
\text{Bbl./year} & 100,000 & 70,000 & 50,000 & 40,000 & 40,000 \\
\text{Decline rate, } D & 62.0 & 70.7 & 55.7 & 62.0 & 10.0
\end{array}
\]

The rate of decline as defined by Equation 1 is the average rate during a given year, and it is understood that the rate of decline means the average rather than the instantaneous rate.

If the rate of decline is constant,

\[
D = Q_0 - Q_1 = Q_1 - Q_2 = \ldots = Q_n - Q_{n+1}
\]

and the production during any year is equal to the production of the previous years divided by \(1 + D\), that is

\[
Q_n = \frac{Q_0}{1 + D}
\]

Although the rate of decline is not constant during the early life of a field, lease, or well, it is convenient to consider constant rates of decline first.

2. Annual production and accumulated recovery.—If the rate of decline is constant and equal to \(D\) and if last year's production is represented by unity, then

\[
\text{Annual production} = (1 + D)^{-n}
\]

times last year's production and by the end of the nth year the

\[
\text{Accumulated recovery} = \sum_{i=1}^{n} (1 + D)^{-i}
\]

times last year's production.

The solution of Equations 1 and 2 corresponding to rates of decline ranging from zero up to 30 per cent are listed in the second and third columns of the appraisal tables of Appendix 1.

3. Annual factor and discounted accumulation.

If \(i\) is the rate of interest and on the assumption that the income from oil production during a given year is received as a lump sum in the middle of that year, the annual present worth factor or, more briefly, the

\[
\text{Annual factor} = P_i = (1 + i)^{-t}\times\text{Discounted accumulation} = \sum_{i=1}^{n} P_i (1 + D)^{-i}
\]

This is the concluding installment of this series, which began in the issue of May 28, 1942. Titles of previous articles in the series, and the dates they appeared are as follows:

May 2—"Effective Porosity, Specific Permeability and the Geometry of Spacing."
June 9—"Flow of Homogenous Fluids Through Linear Systems."
June 10—"Flow of Homogenous Fluids Through Radial Systems."
June 18—"Water in Virgin Oil and Gas Plays."
June 25—"Flow of Oil-Water Mixtures Through Media of Uniform Permeability."
July 2—"Flow of Gas-Oil Mixtures Through Media of Uniform Permeability."
July 9—"Behavior of Natural Gases."
July 16 and 23—"Recovery From Gas Reservoirs."
July 30 and August 6—"Recovery From Oil Reservoirs."
August 13—"Estimating Oil Reserves From Production-Decline Rates."

Present worth of annual production = \(P_i (1 + D)^{-n}\) times last year's production. Whence,

\[
\text{Discounted accumulation} = \sum_{i=1}^{n} P_i (1 + D)^{-i}
\]

The annual factor for 3 per cent and 6 per cent interest rates is listed in Appraisal Table 1. The discounted accumulation is given for the various rates of production decline at the two said rates of interest.

4. Present worth factor.—The present worth of unity deferred for \(n\) years is equal to \((1 + i)^{-n}\) where \(i\) is interest rate. The present worth factor for 3 per cent and 6 per cent interest is listed in Table 1 for convenience.

5. Composite factor.—In certain problems of evaluating oil properties it is necessary to know the composite present value of the accumulated recovery. This factor is obtained from the tables by dividing "Discounted Accumulation" by the corresponding "Accumulated Recovery."

6. Reserve factor.—Let \(V\) denote the reserve in barrels; \(q\), the production in barrels/day; \(s\), the economic limit, barrels/day; then the reserve factor \(F\) in barrels/barrel decline in production is given by

\[
F = \frac{V}{q - s}
\]

The reserve factors for various constant rates of decline are listed in Appraisal Table 31. A proof of the equations may be had by comparing Table 31

7. Illustrative applications.—The daily production of a certain property during the past year averaged 200 bbl per day; the economic limit is 50 bbl per day; the reserve, 450,000 bbl. What is the present worth of the reserve? Solution: The reserve factor equals 450,000/(200 — 50) or 3,000 bbl/bbl, corresponding to which rate of decline (Table 31) is 12 per cent annually. As the production at abandonment is 5/20 or as the accumulated recovery is 450,000/200 = 2,250, the life of the property (Table 12) is almost 12 years corresponding to which the composite factor is 0.774, and the present worth of the property, 0.774 x 450,000 = 348,000 bbl.

The production of a property is declining at 14 per cent annually; the production at the economic limit is 0.4 of last year's production. What is the reserve and its present worth? Solution: 4.29 times last year's production is the reserve and 3.63 times last year's production is the present worth of the reserve (Table 14).

The allowable of a well is 48 bbl per day; the reserve producible at the allowable rate, 180,000 bbl.; the stripper reserve, 70,000 bbl.; the economic limit, 10 bbl per day. What is the present
worth of the 200,000 bbl? Solution: The lift of the well prior to the stripper stage is 180,000/46 × 365 or 10.7 years corresponding to which the present worth of 180,000 bbl. (Table 1) 180,000 × 0.744 or 134,000 bbl. The reserve factor is 70,000/36 or 1,940 bbl./bbl. and the corresponding rate of decline (Table 31) is approximately 19 per cent annually. As at the economic limit the production is 10/46 or 0.217, the stripper life of the well is nearly 9 years (Table 19) and the corresponding composite factor is approximately 0.82. Whence, the present worth of the stripper reserve relative to the log-log to its of the stripper stage is 57,400 bbl. As the present worth factor for 10.7 years is 0.53 (Table 1) the present worth of the stripper reserve is 57,400 × 0.53 or 30,400 bbl. So, the present worth of the 250,000 bbl. is 164,400 bbl.

8. Variable decline rate—If the production from a field, lease, or well is declining at variable rates, such variation may be approximated by a straight line on log-log paper, Fig. 37. The corresponding equation is

\[ \log D_n = \frac{\log n}{\log n_0} (\log D_t - \log D_0) + \log D_n \]

where

- \( D_n \) = rate of decline during nth year, fraction
- \( D_0 \) = rate of decline during first year, fraction
- \( D_t \) = rate of decline during final year, fraction
- \( n \) = final year.

A convenient notation for variable rates of decline is \( D/D_n \). For example; a decline rate from 100 per cent during the first year down to 10 per cent during the tenth year in 10 years is 100/10/10. If the rate of decline carried to abandonment after 20 years by 8 per cent in the foregoing example, the notation would be 100/10/10—8.

9. Annual production and accumulated recovery.—The annual production for variable decline rates is given by,

\[ Q_n = \frac{Q_0 - 1}{1 + D_n} \]

and

\[ \text{Accumulated recovery} = \frac{Q_0}{1 + D_1} + \frac{Q_1}{1 + D_2} + \frac{Q_2}{1 + D_3} \]

(2)

Figs. 38 and 39 are illustrated solutions of Equations 1 and 2 for the variable decline rates shown in Fig. 37 and carried through 20 years at \( D = 10 \) per cent after the tenth year.

10. Payoff of investment.—As producers of oil operate on limited capital funds and as the first law of survival is continued solvency, it follows that payoff regulates the flow of capital. One method for determining payoff is as follows: Based on straight-line amortization plus average interest, which is sufficiently accurate for payoffs up to 10 years, the cost of capital recovery. C (dollars per year), is equal to the amount amortized during a given year plus interest thereon and on the unamortized fraction of the total investment; that is

\[ C = \frac{I}{n + 1} \]

(1)

where

- \( I \) = total investment, dollars
- \( r \) = rate of interest
- \( n \) = number of years required to amortize \( I \).

Whence, if \( A \) is the allowable in bbl./year and if \( P \) is the income after royalty, taxes, and operating costs in dollars/barrel of oil produced, then

\[ AP = C \]

(2)

and Table 32 is a solution of Equations 1 and 2 for 6 per cent and 3 per cent interest rates.

Lastly, it is convenient to designate the expression \( AP/I \) as the break-even factor, for reasons illustrated in the following applications:

Application A.—How many years will elapse before a 14,000 bbl.-per-year allowable can amortize a $35,000 investment if the income after royalty, taxes, and operating costs is $6.00 barrel of oil produced?

**Solution.**—In Table 32 the break-even factor corresponding to a 3-year payout is 0.873. Whence, 7,000 \( \times 0.873 \) or $13,100 is the maximum investment that can be amortized by the end of the third year.

Application B.—Amortization, C.—The leasehold cost plus the development cost of a certain property is $1,400,000. The income for amortization is 0.62 dollar/barrel of oil produced. What must the allowable be in order to pay out the investment by the end of the seventh year?

**Solution.**—Corresponding to a 7-year payout, the break-even factor is 0.177. Whence, 1,400,000 \( \times 0.177 \) or 240,000 bbl. per year is the required allowable.

**References**

1. Any text on Mathematics of Finance.

**Safety Tip**

When using jacks to move equipment into position, never have the bar in position unless the jack is being raised or lowered. The jack may slip out or the trip may slip, throwing the bar and causing injury to a worker. Walking into a bar projecting from the jack handle is also a common accident. The delay to the job is negligible.