Modern Time-Rate Relations
Orientation

Time-Rate Relations:
● New time-rate relations which utilize the following components:
  — Hyperbolic and modified-hyperbolic relations,
  — Power-law/stretched exponential relations, and
  — Exponential relations (e.g., the Fulford model).
● The basis for the proposed relations are data diagnostics/characteristics.
● Model basis:
  — Power-law component for approximating early-time behavior.
  — Hyperbolic and exponential components for representing late time behavior.

D-parameter: \([\frac{dq}{dt}/q]\)
● Basis:
  — Based on the definition of loss-ratio.
  — Power-law behavior for almost all tight gas/liquid-rich shale reservoirs.
  — Modified-Hyperbolic model valid for most gas shales.
● Power-law behavior of the D-parameter yields the stretched exponential function.

Conclusions:
● Modeling time-rate behavior with different functional forms reduces uncertainty.
● Power-law exponential has a very strong correlation for tight gas/shale oil cases.
● Stretched exponential should be considered a valid model for tight gas/shales.
● Modified-hyperbolic model remains primary "currency" in time-rate analysis.
● \(\beta_{q,cp}\)-derivative function is primarily dependent on well completion and geology.
SPE 116731

Exponential vs. Hyperbolic Decline in Tight Gas Sands — Understanding the Origin and Implications for Reserve Estimates Using Arps' Decline Curves

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Rate Function Definitions:

- **Loss Ratio:**
  \[
  \frac{1}{D} \equiv -\frac{q}{dq/dt}
  \]

- **Derivative of Loss Ratio:**
  \[
  b \equiv \frac{d}{dt}\left[\frac{1}{D}\right] \equiv \frac{d}{dt}\left[\frac{q}{dq/dt}\right]
  \]

- **Exponential and Hyperbolic Rate Relations:**

  *(Exponential Decline)*
  \[
  D = \text{con} \rightarrow q = q_i \exp[-D_i t]; \quad \text{or} \quad b = \text{con} \rightarrow q = \frac{q_i}{[1 + bD_i t]^{(1/b)}}
  \]

  *(Hyperbolic Decline)*

Discussion:
- Hyperbolic relation is mis-applied to transient data.
- What is the "characteristic behavior" of the \(D\) and \(b\)-parameters? Evaluate *continuously using data.*
**SPE 116731 — "Power-Law Exponential" Rate Result**

**Observed Behavior of the "Decline" Parameter \([D(t)]\):**

\[
D = -\frac{1}{q} \frac{dq}{dt} \approx D_\infty + n \hat{D}_i t^{-(1-n)} \quad \approx D_\infty + At^{-B}
\]

**Solving for Flowrate \([q(t)]\) Using the \([D(t)]\) Relation:**

\[
q = \hat{q}_i \exp[-D_\infty t - \hat{D}_i t^n]
\]

**Solving for the "Hyperbolic" Parameter \([b(t)]\):**

\[
b = \frac{n \hat{D}_i (1-n)}{[n \hat{D}_i + D_\infty t^{(1-n)}]^2} t^{-n}
\]
Discussion: Small "Waterfrac" Gas Well

- Liquid loading effects are obvious in the latter portion of the flowrate data.
- The onset of the boundary-dominated flow regime is observed.
- We observe a very good match of the flowrate data using $D_{\infty}=0$. 
Discussion: *Large "Waterfrac" Gas Well*

- Erratic rate behavior caused by liquid loading is seen at late times.
- Outstanding matches of the computed $D$- and $b$-parameters with the power-law exponential model are observed.
We convert the "power-law exponential" rate decline model into a dimensionless form.

\[ q = \hat{q}_i \exp[-D_\infty t - \hat{D}_i t^n] \quad \rightarrow \quad q_{Dd} = \exp[-\hat{D}_\infty t_{Dd} - t_{Dd}^n] \]
We develop type curves using the dimensionless form of the "power-law exponential" rate decline model.

\[ q_{Dd} = \exp\left[ -\tilde{D}_\infty t_{Dd} - t_{Dd}^n \right] \]
Discussion:

[Tight Gas Well (Bossier)]

- Excellent match of the data with the type curve for \( n=0.2 \) — this yields an upper bound for the reserves (≈ 5.34 BSCF).
- The lower bound for the reserves \( (G_{p,max}) \) is estimated by the second type curve match. \( \bar{D}_\infty = 10^{-3.75} \)
SPE 123298

A Simple Methodology for Direct Estimation of Gas-in-place and Reserves Using Rate-Time Data

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Quadratic rate-cumulative production relation can be rearranged to yield a plotting function as:

\[
\frac{q_{gi} - q_g}{G_p} = D_i - \frac{1}{2} \frac{D_i}{G} G_p
\]

The plotting function \( (q_{gi} - q_g)/G_p \) versus \( G_p \) yields an intercept in the \( x \)-axis of \( 2G \) — i.e., use to estimate \( G \)
Boundary-dominated flow regime can be identified using the $\alpha$-parameter through the modification of the rate-cumulative production relation:

$$\alpha = \left[ \frac{G_p}{G} - \frac{1}{2} \left( \frac{G_p}{G} \right)^2 \right] \left( 1 - \frac{q_g}{q_{gi}} \right)$$

The plotting function, $\alpha$ versus $G_p/G$ has a diagnostic value in establishing the boundary-dominated flow regime (i.e., $\alpha = 2$ as $q_g \to 0$ and $G_p \to G$).
SPE 123298 — $q_g$-$G_p$ Relation

The plotting functions $q_g/q_{gi}$ versus $G_p/G$ and $q_g$ versus $G_p$ are used in conjunction with the previous plotting functions to yield the best estimate for $G$.

Discussion:
- $q_{gi}$, $D_i$, $G$ parameters are calibrated using the plotting functions.
- We iterate on all plots until the best match is obtained.
SPE 123298 — Tight Gas Well

a. Plotting Function 1: (Tight Gas Well) \( \frac{q_{gi} - q_g}{G_p} \) vs \( G_p \) Plot (Cartesian scale).

b. Plotting Function 2: (Tight Gas Well) "\( \alpha \)" Diagnostic Plot — reverse solution for the \( \alpha \)-parameter (Cartesian scale).

c. Plotting Function 3: (Tight Gas Well) Model Validation Plot — \( \frac{q_g}{q_{gi}} \) versus \( \frac{G_p}{G} \) (Cartesian scale).

d. Plotting Function 4: (Tight Gas Well) Model Validation Plot — \( q_g \) (data and model) versus \( G_p \) (log-log format).
SPE 125031

Decline Curve Analysis for HP/HT Gas Wells: Theory and Applications

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Discussion: Rate-Time Gas Flow Relation (Knowles et al.)
- Basis is the linearization of the nonlinear $\mu_g c_g$ term (Ansah, et al.).
- $D$-function and $b$-function are formulated using the definitions for loss-ratio and the derivative of the loss-ratio.
**Discussion: Rate-Cumulative Gas Flow Relation**

- Definition of the loss-ratio can be re-cast in terms of rate and cumulative production.
- A quadratic relationship exists between rate and cumulative production.
Discussion: Methodology

- The main goal is to match the data with the model using the definitions for the \( q-D-b \) functions during the boundary-dominated flow regime.
- \( b \)-function \( \rightarrow 0.5 \) for high drawdown cases (almost constant behavior).
Field Example: Application of the Methodology

- 3.5 years of daily data are available for a hydraulically fractured well completed in a HP/HT gas reservoir.
- Well clean-up effects, liquid-loading, and operational changes are observed in the data trends.
- The flowrate data are reviewed prior to analysis; and any erroneous/redundant data points are removed.
- The half-slope trend is evident in the rate-integral derivative function.

\((p_i = 14000 \text{ psia and } T_R = 260^\circ F)\)
Field Example: Application of the Methodology

- For the computation of $D$- and $b$-parameter data functions we remove the outlying data points; then we perform the numerical differentiation.
- Our analysis using the proposed semi-analytical relation provides a gas-in-place estimate of approximately 8.0 BSCF.
Field Example: Application of the Methodology

- Reasonable matches of the $D$-function with the data using the semi-analytical model is achieved (post-transient flow only).
- The matches of the $b$-function data with the semi-analytical model are problematic — data indicate no unique characteristic behavior.
- Computation of the $b$-parameter data function is severely affected by factors such as liquid loading.
Field Example: Application of the Methodology
- We observe a good match of the flowrate data with the model (except for the early time data affected by "cleanup").
- The "power-law exponential" model yields $G_{p,\text{max}} \approx 8.0$ BSCF.
- Gas-in-place estimates are consistent comparing the methods we used.
SPE 135616

Hybrid Rate-Decline Models for the Analysis of Production Performance in Unconventional Reservoirs

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SPE 135616 — Stretched Exponential Function [Kohlrausch (1854)]

Observed Behavior of Decline Parameter (D):

\[
D \equiv -\frac{1}{q} \frac{dq}{dt} \approx n\hat{D}_i t^{-(1-n)}
\]

Solving for Flowrate:

\[
q = \hat{q}_i \exp[-\hat{D}_i t^n]
\]

Literature:
- Kohlrausch (1854).
- Kisslinger (1993)
- Decays in randomly disordered, chaotic, heterogeneous systems (e.g. relaxation, aftershock decay rates, etc.).

Valkó (2009)

\[
q(t) = \hat{q}_i \exp[-(t / \tau)^n]
\]

Jones (1942) and Arps (1945)

\[
q(t) = q_o \exp \left[ -D_o t^{m-1} \over 100 (m-1) \right]
\]
Discussion: Stretched Exponential Function

- Single, double and four exponentials are used to approximate the data using linear least squares.
- Stretched exponential function can be described as a linear super-position of exponential decays.

$$q(t) = \hat{q}_i \exp[-\hat{D}_i t^n]$$

$$q(t) = \sum_{i=1}^{n} q_i \exp[-a_i t]$$
**Discussion: Rate-Time Gas Flow Relation (Knowles et al.)**

- **Basis** is the linearization of the nonlinear "$\mu_g c_g"$$\text{ term (Ansah et al.).}
- **D-function** and **b-function** are formulated using the definitions for loss-ratio and the derivative of the loss-ratio.
- See Ansah et al. (2000), Knowles et al. (1999), and Ilk et al. (2009) for more details.

**Rate-Time Relation:**

\[
q_{Dd} = \frac{4 p_{wD}^2 \exp[-p_{wD} t_{Dd}]}{(1 + p_{wD}) - (1 - p_{wD}) \exp[-p_{wD} t_{Dd}])^2}
\]

**Conclusions:**

- Theoretical justification for hyperbolic decline relation for gas flow?
- **b** = 0.5 for high drawdown cases ($p_{wf}/p_i \leq 0.05$).
- ONLY valid for **BOUNDARY-DOMINATED FLOW REGIME**.
- Exponential decline at very late times.
**SPE 135616 — $\beta_{q,cp}$-Derivative**

**$\beta(t)$-Derivative: Well Test Analysis** (Hosseinpour-Zonoozi et al. 2006)

\[
\Delta p \beta_d(t) = \frac{d \ln(\Delta p)}{d \ln(t)} = \frac{1}{\Delta p} \frac{d\Delta p}{dt}
\]

**$\beta(t)$-Derivative: Modification for this work (for constant pressure)**

\[
\beta_{q,cp}(t) = -\frac{d \ln(q)}{d \ln(t)} = -\frac{t}{q} \frac{dq}{dt}
\]

**Discussion:**
- Strong diagnostic character of the $\beta_{q,cp}$-derivative function.
- Holly Branch tight gas field production data exhibit similar characteristic behavior.
- Early time data are affected by "non-reservoir" effects.
SPE 135616 — $\beta_{q,cp}$-Derivative
Discussion:
- Fractured vertical gas well with 43 years of production.
Discussion:

- Boundary-dominated flow regime is apparent at late times.
Discussion:

- Horizontal well with multiple fractures with 340 days of production.
**Discussion:**

● Outstanding data quality provides remarkable character.
Discussion: *Rate-Time Models*

- Rate-time models decrease the uncertainty in reserves estimates.