Summary

Traditional decline methods such as Arps’ rate/time relations and their variations do not work for wells producing from supertight or shale reservoirs in which fracture flow is dominant. Most of the production data from these wells exhibit fracture-dominated flow regimes and rarely reach late-time flow regimes, even over several years of production. Without the presence of pseudoradial and boundary-dominated flows (BDFs), neither matrix permeability nor drainage area can be established. This indicates that matrix contribution is negligible compared with fracture contribution, and the expected ultimate recovery (EUR) cannot be based on a traditional concept of drainage area.

An alternative approach is proposed to estimate EUR from wells in which fracture flow is dominant and matrix contribution is negligible. To support these fracture flows, the connected fracture density of the fractured area must increase over time. This increase is possible because of local stress changes under fracture depletion. Pressure depletion within fracture networks would reactivate the existing faults or fractures, which may breach the hydraulic integrity of the shale that seals these features. If these faults or fractures are reactivated, their permeabilities will increase, facilitating enhanced fluid migration. For fracture flows at a constant flowing bottomhole pressure, a log-log plot of rate over cumulative production vs. time will yield a straight line with a unity slope regardless of fracture types. In practice, a slope of greater than unity is normally observed because of actual field operations, data approximation, and flow-regime changes. A rate/time or cumulative production/time relationship can be established on the basis of the intercept and slope values of this log-log plot and initial gas rate.

Field examples from several supertight and shale gas plays for both dry and high-liquid gas production, and for oil production were used to test the new model. All display the predicted straight-line trend, with its slope and intercept related to reservoir types. In other words, a certain fractured flow regime or a combination of flow types that dominate a given area or play because of its reservoir-rock characteristics and/or fracture-stimulation practices all produce a narrow range of intercepts and slopes. An individual-well performance or EUR can be derived that is based on this range if the best 3-month average or the initial production rate of the well is already known or estimated. The results show that this alternative approach is easier to use, gives a reliable EUR, and can be used to replace the traditional decline methods for unconventional reservoirs. The new approach is also able to provide statistical methods to analyze production forecasts of resource plays and to establish a range of results of these forecasts, including probability distributions of reserves in terms of P90 (lower side) to P10 (higher side).

Introduction

Unconventional reservoirs, especially wet shale gas and oil, are currently being pursued aggressively for new development in both the US and Canada. Forecasting production and estimating reserves accurately for these resource plays have become more urgent and important than ever before.

Mattar et al. (2008) have discussed various techniques for production analysis and forecasting of shale-gas reservoirs, while Lee and Sidle (2010) have analyzed some of the commonly used procedures for forecasting key strengths and limitations of these techniques. The commonly used method for decline-curve analysis is Arps’ hyperbolic rate decline (Arps 1945), if adequate production data are available. Fetkovich type curves (1980) are not appropriate because the matched b value is usually greater than unity. For horizontal wells with multiple transverse fractures, numerical-simulation modeling is a highly preferred option. There are, however, some validity and applicability issues with these techniques.

The hyperbolic decline equation is conveniently used because it can have a “best fit” for the long transient linear-flow regime observed in shale-gas wells with b values greater than unity. Lee and Sidle (2010) showed that values of b equal to or greater than unity can cause the reserves to have physically unreasonable properties. To avoid this drawback, “stretched-exponential” models have been proposed recently (Valkó 2009; Ilk et al. 2008, 2009).

The current simulation models that are available in the market are still using conventional technologies for unconventional reservoirs. Some of the assumptions used in simulation modeling are inconsistent with the field-data observations such as pressure initialization and radial (or elliptical) transient flow. Field data indicate that the pressure transition through a shale-gas zone is at a disequilibrium state, while pressure initialization in simulation modeling is based on an equilibrium state with its fluid gradient. Newsham and Rushing (2002) showed that pressure gradients measured in the Lower Bossier can be as high as 63 kPa/m (3 psi/ft) or more than 25 times its saturated gas gradient or three times the lithostatic gradient. Spencer (1987) presents some pressure gradients in overpressured and gas-bearing shale plays in the Rocky Mountain region. The interpreted pressure profile across the active zone for wells in the Piceance basin shows a pressure gradient of approximately 36 kPa/m.

The concept of stimulated rock volume (SRV) that is supported by microseismic monitoring of hydraulic-fracture treatments is not confirmed by the observed radial flow from a numerical simulator, as pointed out by Mattar et al. (2008). These authors claim that the radial flow of fractured shale wells observed in a numerical simulator is more likely to be a false radial flow and may not be representative of the enhanced permeability of the SRV.

This paper introduces an empirically derived decline model that is based on a long-term linear flow in a large number of wells in tight and shale-gas reservoirs. On the basis of this model, a new method has been developed for production analysis and forecasting of unconventional reservoirs. This method also uses probability distributions of reserves in forecasting resource plays, to represent any uncertainty in reserves estimation.

Method Development

Long-Term Linear Flow. Fig. 1 shows the production history of a vertical well producing from the Barnett shale. Production data show a half-slope line, indicating linear flow lasting for more than 5 years of production.

Long-term linear flow in a large number of wells in tight or shale gas has been observed as early as 1976 (Bagnall and Ryan 1976). Since then, many researchers have tried to explain what
causes this phenomenon. Several authors have shown that this linear flow exists in any type of fractures whether they are infinite- (Agarwal et al. 1979) or finite-conductivity (Cinco-Ley et al. 1978; Cinco-Ley and Samaniego 1981a, 1981b), single or multi-stage (Bello and Wattenbarger 2010), hydraulic (Arévalo-Villagrán et al. 2001), or natural fractures (Arévalo-Villagrán et al. 2006). The difference among these models is the length of this transient linear-flow regime.

To prolong the linear-flow regime in any of those models, all one needs to do is enlarge the term $x^2/k_m$ in which $x$ is the effective fracture half-length of the fracture system and $k_m$ is the matrix permeability. For any given shale-gas reservoir, matrix permeability is fixed. Thus, the connected fracture density of the fractured area must be increasing over time to support this fracture flow over the life of a producing well. This increase is achievable because of local stress changes under fracture depletion (Warpinski and Branagan 1989). Pressure depletion within fracture networks would reactivate the existing faults or fractures, which may breach the hydraulic integrity of the shale that seals these features. If these faults or fractures are reactivated, their permeabilities will increase, facilitating enhanced fluid migration. A fractured shale model that imitates this long-term linear flow and nonequivalent pressure distribution will be briefly discussed later and is the subject of another paper.

If a fracture flow regime (either linear or bilinear) is prolonged over the life of a well, the gas-flow rate $q$ will be

$$q = q_1 t^{-m}, \quad \text{for linear flow,}$$

$$q = q_1 t^{1-n}, \quad \text{for bilinear flow,}$$

where $n$ is one-half for linear flow, $n$ is one-quarter for bilinear flow, and $q_1$ is the flow rate at Day 1.

The gas cumulative will be

$$G_p = \int_0^t q \, dt = q_1 \left(1 - t^{-m}\right).\quad (2)$$

This gives

$$\frac{q}{G_p} = \frac{1}{t} \left(1 - t^{-m}\right).\quad (3)$$

Field Applications. Field data from several shale-gas plays were used to test Eq. 3. These field data were operated under the actual field conditions that may violate some of the ideal assumptions used to derive Eq. 3. Some of the results are shown in Fig. 2 in which log-log plots for $q/G_p$ vs. time in days were constructed.

These plots all give a log-log straight line with a negative slope, $-m$, and an intercept of $a$. That is,

$$q = at^{-m}. \quad (4)$$

Note that the slope is negative but $m$ is always positive. As shown later, $m$ is always greater than unity for shale reservoirs. If $m$ is less than unity, it may indicate a conventional tight well.

Equations for $q$ and $G_p$ can be derived as shown in Appendix A:

$$\frac{q}{q_1} = e^{\frac{a}{m}} t^{-m-1} \quad (5)$$

Fig. 1—A log-log plot of field production and pressure data for a shale-gas well. Production data show a half-slope line, indicating linear flow lasting for more than 5 years of production.

Fig. 2—A log-log plot of $q/G_p$ vs. $t$ for various shale-gas wells with more than 5 years of production. The data all form a straight line with a slope $m$ and an intercept $a$. A slope of higher than unity is normally obtained, to account for any deviations from the ideal cases.
Fig. 3—A correlation between a and m for various gas plays.

It will be shown later that a can be eliminated from Eqs. 5 and 6 if they are rewritten in forms of dimensionless rate and time. This indicates that a and m are related variables, as shown in Fig. 3, obtained from various gas plays. Using this correlation to convert a into m, type curves for \( q/t \) vs. time in days can be constructed, as shown in Fig. 4.

Fig. 4 shows that there is a maximum flow rate, \( q_{max} \), and corresponding time \( t_{max} \), for each \( m \) value of greater than unity. The parameters \( q_{max} \) and \( t_{max} \) as a function of \( a \) and \( m \) are found in Appendix B. Fig. 4 also indicates that \( t_{max} \) is normally less than 30 days. Thus, the well will flow at the maximum flow rate within the first month of production if there are no well constraints such as rate, WHP, or bottomhole-flowing-pressure limits.

During production, a well may deviate from fracture flow and may experience early water (aqueous) breakthrough, gas dropout, and liquid dropout/holdup in the fracture system and wellbore. The characteristic of a linear relation between the best 3-month average rate and the corresponding time in days for both vertical and horizontal wells was plotted against their cumulative productions of 1, 2, and 5 years, as shown in Fig. 5. A very good match is obtained. See more details in the Analog Type Curves subsection of the Field Examples section.

In summary, if the value any one of the three parameters \( 3q_{max} \), \( q_{max} \), or \( q_{max} \) is known, reserves evaluations can be established for any individual well, group of wells, development area, or play with a given set of \( a \) and \( m \). The \( q_{max} \) is preferred because it uses a longer production epoch and, hence, reduces data uncertainties.

Fig. 4 gives a set of type curves for a typical field-data plot because they are based on the real time (days) and a real relation between \( a \) and \( m \). To construct type curves in a dimensionless form, the curves in Fig. 4 were normalized using \( t_{max} \) and \( q_{max} \). The dimensionless time \( t_{n} \) and dimensionless rate \( q/q_{max} \) will be

\[
q/q_{max} = e^{-a m t_{n}}
\]

and

\[
t_{n} = t/t_{max}
\]

The general type curve is shown in Fig. 6 for an \( m \) range of 1 to 1.96. Note that \( a \) has been eliminated from the type curves (see Eq. 8).
For comparison, the Arps rate/time relations (Arps 1945) were reconstructed in the same plot using dimensionless time $t_D = D t$ and dimensionless rate $q_D$, for their normal range of $b$ from 0 to 1. Note that $q_D$ is defined as the dimensionless rate at $t_D=1$, not the same as normally based on $q$. The results show that Arps’ curve of $b=0.5$ is equivalent to the new curve of $m=1.96$ at late time ($t_D>10$), as shown in Fig. 6. For $b>0.5$, Arps curves give more-optimistic forecasts compared to the new approach.

### Field Examples

In this section, several field examples from different unconventional gas plays were used to test the new approach. Examples were selected to illustrate the decline behaviors in various completion and production scenarios such as the cases of retrograde gas, newly developed or developing plays, and vertical or horizontal wells. For changes in flow regime, $a$ and $m$ values will be highlighted for each example in the following discussion. A comparison of results is also made with other methods.

#### Retrograde-Gas Case

Wet-shale-gas plays have become more desirable lately because of higher liquid contents and attractive liquid price. Unfortunately, very little is known about their flow characteristics and how the liquid dropout in the formation and fracture network behaves.

The following example contains production data from four horizontal wells from a new emerging play. The production rates from both gas and liquid were recombined and converted into gas-rate equivalent and finally were normalized on the basis of the highest rate from these wells, as shown in Fig. 7. Rates on Wells 2 and 3 were reinitialized because of some significant downtimes.

Fig. 7a shows that the flow regime varies from bilinear to linear flow. Generally, wells with bilinear flow regime will have lower deliverability rates and/or bilinear flow may be caused by liquid holdup in fractures or by fracture choking.

Values of $a$ and $m$ obtained from a retrograde shale gas have covered a large range of values; these vary from 1.02 to 1.67 and from 1.04 to 1.21, respectively, as shown in Fig. 7b for this example.

#### Bakken Oil

An abandoned vertical well that produced from the Bakken formation in the Viewfield pool, Saskatchewan, Canada, for more than 26 years is shown here as an example for oil production from a fractured shale or tight formation. Fig. 8a shows the well was under a transient linear flow (half slope) for all of its production life except for a few months at the start of production. The late-time flow that indicates a big rate drop in oil rate can be mistaken as BDF, but in reality it is caused by an unexpected increase in water production.

The normalized decline-trend plots for the well are shown in Fig. 8b. If $q/N$, is based on the total liquid produced, the plot gives a good fitted straight line ($R^2$ of 0.9824) with $a$ and $m$ values of 1.7894 and 1.1477, respectively. The values are slightly higher if only the oil stream is used in the calculation. Thus, an increase in water production from a fractured shale or tight formation.

![Fig. 6](image-url)

**Fig. 6**—A comparison between Arps’ and the new method’s type curves for dimensionless rate and time.

### Diagrams

- **Fig. 7**—A condensate/shale-gas example. Flow regimes that change from linear to bilinear in this retrograde play may be caused by liquid dropout in the fracture. There is a wider range for $a$ and $m$ observed in this play compared to other types.
in water production will cause both $a$ and $m$ to increase in value (Fig. 8b). Because of the unexpected increase in water production at the late time, an overprediction of only 8% in the well’s EUR is projected.

**Individual-Well Analysis.** In this example, a step-by-step procedure is shown in Fig. 9 of how to perform decline analysis on an individual-well basis. A vertical well taken from the Barnett shale has been selected for this illustration. The well has been in production for the last 5 years and operated with minimum water production.

1. **Step 1: Data Check and Correction.** The production-data histories such as flowing WHPs, gas rate, and water rate are plotted as shown in Fig. 9a. The well was initially choked back at the surface on the basis of the observed flat rate and high WHPs. A correction was made for the rate on the basis of average operating pressures. The wet-gas rate should also be corrected to the dry-gas rate equivalent if there is a high condensate/gas ratio.

2. **Step 2: $a$ and $m$ Determination.** A log-log plot of $q/G_p$ (or $q/N_p$ for oil) is constructed to determine these values, as shown in Fig. 9b. Data are examined to determine which section should be used to obtain the representative $a$ and $m$ parameters for the well. The $R^2$ value is used to determine the best fit of the data. An $R^2$ value of more than 0.95 is recommended.

3. **Step 3: Rate Forecast.** $q_1$ Determination. To obtain $q_1$, gas-flow rate should be plotted against $t(a, m)$. From Eq. 5, we have

$$q = q_1 t(a, m)$$

where $t(a, m) = t^{-e^{g(q_1/a)}}$. The $q$-vs.-$t(a, m)$ plot should give a straight line through the origin with slope of $q_1$. However, this may not be the best selection in some cases because of the current wellbore operating conditions. In such cases, Eq. 9 becomes

$$q = q_1 t(a, m) + q_\infty$$

where $q_\infty$ is the rate at infinite time, so it can be zero, positive, or negative. In either case, the abandonment rate should be set on the basis of economics.

---

**Fig. 9**—Four steps for using the proposed approach.
Once $q_1$ (and $q_2$) have been determined, gas-rate forecasts can be made by using Eq. 9 or 10.

4. **Step 4: Reserves Estimate.** Eq. 4 \( G_r = \frac{q}{a} t^{-m} \) is used for cumulative calculation with \( q > q_{eco} \), whereas EUR is estimated by Eq. 11:

\[
\text{EUR} = \frac{q_{eco}}{a} t^{-m} \]  

where \( q_{eco} \) is the minimum economic rate, and \( t_{eco} \) is at \( q = q_{eco} \).

**Analysis Comparisons.** To compare the results, other methods such as Arps’ hyperbolic with \( b \) greater than unity and the recently developed power-law-exponential method (Ilk et al. 2009) were selected. Texas A&M University has developed a rate/time spreadsheet on using the power-law-exponential approach (Ilk et al. 2008, 2009). Data from Example 3 from this spreadsheet was employed here for the purpose of comparison. The analysis results are shown in **Fig. 10**. The results from the power-law-exponential approach and Arps’ hyperbolic with \( b \) of 1.9 are included in **Fig. 10** for comparison.

The results that were calculated on the basis of time limit of 30 years and economical rate of 2.83×10^3 m^3/d (100 Mscf/D) are summarized in **Table 1**.

**Table 1** shows that the new approach provides conservative reserves estimates compared with the other two methods.

**Analog Type Curves.** A field example was used to illustrate how a developing play can be used as an analog for other similar plays.

Once $q_1$ (and $q_2$) have been determined, gas-rate forecasts can be made by using Eq. 9 or 10.

4. **Step 4: Reserves Estimate.** Eq. 4 \( G_r = \frac{q}{a} t^{-m} \) is used for cumulative calculation with \( q > q_{eco} \), whereas EUR is estimated by Eq. 11:

\[
\text{EUR} = \frac{q_{eco}}{a} t^{-m} \]  

where \( q_{eco} \) is the minimum economic rate, and \( t_{eco} \) is at \( q = q_{eco} \).

**Analysis Comparisons.** To compare the results, other methods such as Arps’ hyperbolic with \( b \) greater than unity and the recently developed power-law-exponential method (Ilk et al. 2009) were selected. Texas A&M University has developed a rate/time spreadsheet on using the power-law-exponential approach (Ilk et al. 2008, 2009). Data from Example 3 from this spreadsheet was employed here for the purpose of comparison. The analysis results are shown in **Fig. 10**. The results from the power-law-exponential approach and Arps’ hyperbolic with \( b \) of 1.9 are included in **Fig. 10** for comparison.

The results that were calculated on the basis of time limit of 30 years and economical rate of 2.83×10^3 m^3/d (100 Mscf/D) are summarized in **Table 1**.

**Table 1** shows that the new approach provides conservative reserves estimates compared with the other two methods.

**Analog Type Curves.** A field example was used to illustrate how a developing play can be used as an analog for other similar plays.

**Conventional Tight Gas Wells.** All examples illustrated thus far result in an \( m \) value greater than 1. However, \( m \) can be less than 1, as shown in **Fig. 3**. The cases in which \( m \) is less than unity in **Fig. 3** are from a shallow and tight gas formation. A typical rate-decline analysis is shown in **Fig. 14**.

Notice the slightly concave-up shape of the rate plot, as shown in **Fig. 14d**. This concave-up shape, which is caused by a late time pseudoradial flow is characteristic for \( m<1 \). For \( m=1 \), a log-log plot of rate vs. time will display a concave-down feature (Fig. 4).
This feature distinguishes flow characteristics of a conventional (late-time pseudoradial flow with \( m < 1 \)) vs. unconventional (lack of late-time pseudoradial flow or \( m > 1 \)) reservoir.

**Discussion**

**Absence of Late-Time Pseudoradial Flow.** Unsurprisingly, the \( m \) value for all shale-gas wells investigated in this study is greater than unity, or a log-log plot of rate vs. time will display a concave-down feature (Fig. 4). This flow characteristic, which indicates an absence of the late-time pseudoradial flow from shale gas reservoirs, can be explained as follows.

Fig. 15 was constructed to distinguish between a conventional (tight) and shale-gas formation with an assumption that Point A, the wellbore location, has the same pressure in both conventional and unconventional formations. If Well A is located in a tight but conventional formation, its pressure will be in equilibrium with its fluid gradients. As in the case of a gas-saturated formation, the pressure gradient should be approximately 2 kPa/m. A hydraulically fractured well located in such a formation will be expected to pass through several distinct flow regimes such as fracture linear, bilinear, elliptical, and pseudoradial flows, and BDF, as discussed by Clarkson and Beierle (2010). An unconventional reservoir may not be working according to such a model because there is no matrix transient flow here to give rise to radial or elliptical flow.

If the reservoir is shale, the pressure gradient will be overpressured. This can be up to 63 kPa/m (3 psi/ft), as recorded in the Lower Bossier play (Newsham and Rushing 2002) and shown in Fig. 16. Spencer (1987) shows some pressure gradients in overpressured and gas-bearing shale plays in the Rocky Mountain region. The interpreted pressure profile for the MWX site wells of the Piceance basin shows that the pressure gradient based on measured pore pressures in the active zone is approximately 36 kPa/m (1.7 psi/ft). The author of this study has also experienced pressure

---

**TABLE 2—PARAMETERS FOR TYPE-CURVE GENERATION FOR BARNETT SHALE**

<table>
<thead>
<tr>
<th>( q_{3,\text{max}} \times 10^6 ) m³/month</th>
<th>Vertical well</th>
<th>Horizontal well</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q, \times 10^3 ) m³/d</td>
<td>( a )</td>
<td>( m )</td>
</tr>
<tr>
<td>Vertical well</td>
<td>P90</td>
<td>P50</td>
</tr>
<tr>
<td>12.19</td>
<td>1.88</td>
<td>1.42</td>
</tr>
<tr>
<td>19.09</td>
<td>1.21</td>
<td>1.14</td>
</tr>
<tr>
<td>32.58</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>
distributions across producing gas shales exceeding the lithostatic gradient, as in the Montney and other studied plays.

With this high pressure gradient in shale gas, the formation is thought to be in a dynamical equilibrium state in which hydrocarbons have been generated and leaked off the formation over geologic time. Under a normal production time scale, shale-gas formations can be treated as impermeable barriers that contain millions and millions of tiny isolated reservoirs (such as sandy laminates, microcracks, or isolated sand bodies); to produce gas from this shale formation, these tiny reservoirs need to be connected by a fracture network, without which they will remain isolated and unproductive.

Because of such isolated characteristics of microreservoirs in shale-gas formations, capillary pressure or relative permeability curves should be applicable in each individual tiny reservoir, but they may not be applicable applied across the entire shale formation. This can explain why the late-time radial- or elliptical-flow regimes are absent from all shale-gas wells studied.

Numerical-Model Comparison. To use the current conventional modeling for fractured shale, some assumptions or modifications must be made. This section proposes and briefly discusses a fracture model consisting of a macrofracture located inside an SRV. To overcome the nonequilibrium issue, permeability within the SRV is assumed to have been anisotropically enhanced by stimulation so that the pressure within it is in a state of equilibrium and can be matched to the post-stimulation transient data. To prolong the linear-flow regime, the drainage area or SRV should be large enough to imitate the gradual increase in the SRV as fractures increase the connectivity.

A commercial reservoir simulator is used here for numerical-modeling work so that various fracture models in both conventional and unconventional reservoirs can be compared. Adsorbed gas

Fig. 13—Type curves for Barnett shale based on a set of 25 wells, for both vertical and horizontal wells.

Fig. 14—A rate-decline example from a shallow and tight gas formation.

Fig. 15—Pressure-gradient distribution in tight and shale gas.
is not considered in the study but can be included in the model. Fig. 17 is a summary of the results in the form of normalized decline-trend plots. The $q/G_p$ ratio of the proposed fracture model is shown as the blue line. If the stimulated region is much smaller, the red or BDF curve proposed by Bello and Wattenbarger (2008) is obtained.

To compare with the proposed analytical method, the thin white line with the matched $a$ and $m$ factors of 0.7785 and 1.0481, respectively, is produced. This line lies exactly on top of the blue line from the numerical simulation result.

The yellow curve shows one of the results of a normal reservoir model where the virgin area surrounding the SRV is also in pressure communication. This yellow curve, which includes a late-time linear flow from the virgin region into the SRV, can also be obtained using the approach from Anderson et al. (2010). Another result is shown as a green line where both stimulated and virgin regions are the same or under a homogeneous state. Pseudoradial flow will be achievable to make $m$ a value of less than unity. Fig. 14 illustrates a field example of a gas well producing from a tight reservoir.

Method Limitations. Downtime Effect. Because of some operational conditions, a well may have to be shut in for some periods of time during its production life. If the downtime of each shut-in is significant, rate initialization is required as in traditional curve-analysis methods. To use $q/G_p$, both rate and cumulative production should be initialized with pressure correction. Fig. 7 shows the example plots of rate initializations for Wells 2 and 3.

EUR Constraints. In most cases, without the presence of BDF in shale reservoirs, drainage area cannot be established for EUR prediction. In this method, the drainage area will be defined only when the fracture-expansion process is brought to an end. Thus, EUR is not based on the traditional concept of drainage area (BDF) but on the constraints of the latest trends of $(q_i, q_\infty)$ with both time and economic rate limits. Because of this, results are valid only where analog wells are similar in both interwell and fracture spacings.

In case of spacing differences, a numerical reservoir modeling (see the Numerical Model Comparison subsection) should be used for any EUR adjustment.

Latest Trends. During a well production, many factors can contribute to the deviation from a fracture flow to accelerate the decline rate such as water production or liquid dropout/holdup in the fracture system and wellbore. The values of $q_i$ and $q_\infty$ (Eq. 10 and Fig. 9) are designed to account for this acceleration in rate decline in late time. An increase in water production will also cause both $a$ and $m$ to increase in value (Fig. 8).

Time Limit vs. Rate Limit. Fig. 12 shows a typical EUR prediction for an infill in the Barnett shale for both vertical and horizontal wells. The prediction based on a time limit alone would give an

---

**Fig. 16**—Two examples of high pressure-gradient distribution across the producing shale gas: (a) the Lower Bossier play (Nesham and Rushing 2002) and (b) the Piceance basin (Spencer 1987).

**Fig. 17**—Normalized decline trend plots for various simulated fracture models in shale or tight reservoirs: (a) for all production data and (b) at late time.
optimistic value. Note that the EUR shifts to the left when the rate limit was imposed on the time-limit plot, as shown in Fig. 12b. The lower the initial rate is, the higher and more optimistic the value of the predicted EUR will be. This observation is especially applicable to vertical wells because of their lower initial rates. The recommendation is to use both time and economic rate limits for predicting EUR.

Conclusions
1. A new approach that is easy and simple to use for predicting the future rate and EUR has been developed for fracture-dominated wells in unconventional reservoirs.
2. The new approach is also able to provide a statistical method to analyze production forecasts of resource plays and establish a range of results for these forecasts, including probability distributions of reserves in the form of P90 to P10.
3. A log-log plot of rate over cumulative production vs. time is observed to fit a straight line in all unconventional-reservoir cases studied so far. The slope and intercept are related to reservoir-rock characteristics (tight or shale), fracture-stimulation practice, operational conditions, and (possibly) liquids content.
4. EUR for the new approach is not based on the traditional concept of drainage area (BDF) but on the constraints of the latest trends in unconventional wells in unconventional reservoirs.
5. Pressure initialization used in numerical modeling based on fluid gradients may be incorrect. Results from such numerical modeling may not be representative of the shale-gas flow characteristics. Some modifications of flow characteristics in both stimulated and undisturbed regions are needed.

Nomenclature

\[ a = \text{intercept constant defined by Eq. 4, } d^{-1} \]
\[ b = \text{Arps’ exponent, dimensionless} \]
\[ D_f = \text{Arps’ initial decline rate, } d^{-1} \]
\[ G = \text{original gas in place, m}^3 \]
\[ G_p = \text{cumulative gas production, m}^3 \]
\[ k_f = \text{fracture permeability, } 10^{-3} \text{µm}^2 \]
\[ k_m = \text{matrix permeability, } 10^{-3} \text{µm}^2 \]
\[ m = \text{slope defined by Eq. 4} \]
\[ n = \text{fracture time exponent, dimensionless} \]
\[ N_p = \text{cumulative oil production, m}^3 \]
\[ q = \text{rate, m}^3/d \]
\[ q_1 = \text{rate at Day 1, m}^3/d \]
\[ q_{1D} = \text{rate at } t_D = 1, \text{m}^3/d \]
\[ q_{m3} = \text{best 3-month average rate, m}^3/\text{month} \]
\[ q_{eco} = \text{economic cutoff rate, m}^3/d \]
\[ q_{max} = \text{maximum rate, m}^3/d \]
\[ q_{infty} = \text{rate at infinite time, m}^3/d \]
\[ t = \text{time, days} \]
\[ t_D = \text{dimensionless time defined by } t_D = D_f t \]
\[ t_{Dis} = \text{dimensionless time based on fracture half-length} \]
\[ t_n = \text{dimensionless time based on Eq. 7} \]
\[ t_{max} = \text{time at maximum flow rate, days} \]
\[ t_p = \text{prediction time, days} \]
\[ t(a,m) = \text{time function based on Eq. 9} \]
\[ x_f = \text{fracture half-length, m} \]
\[ \varepsilon = \text{decline rate defined by } \varepsilon(t) = \frac{q}{Qt}, d^{-1} \]

Acknowledgments
The author would like to thank the management of ConocoPhillips for their support and permission to publish this study. He is also grateful to Dev Mukherjee, Chris Clarkson, Sheila Reader, and Kevin Rateman for reviewing the paper; to coworkers in both the Houston and Calgary offices for peer review of the proposed method; and to Matt Maguire and his group for supplying the Barnett production data.

References
Appendix A—General Equations for \( q \) and \( G_p \)

General equations are developed on the basis of \[
\frac{q}{G_p} = \frac{\varepsilon(t)}{\varepsilon(1)} \] 

(A-1)

Taking a derivative with respect to time, we have

\[
\frac{d}{dt}\left[\frac{q}{G_p}\right] = \frac{d}{dt}\frac{\varepsilon(t)}{\varepsilon(1)} \] 

(A-2)

Using \( q' = dq/dt \) and \( \varepsilon'(t) = d\varepsilon(t)/dt \), Eq. A-2 becomes

\[
q'\frac{\varepsilon(t)}{\varepsilon'(t)} - q\varepsilon'(t)/\varepsilon'(t) = q, \] 

or

\[
dq/dt = q\varepsilon'(t)/\varepsilon'(t) + \varepsilon(t)dt \] 

(A-4)

Integrating both sides from \( t=1 \) to \( t \), we have:

\[
\ln\frac{q}{q_i} = \ln\frac{\varepsilon(t)}{\varepsilon(1)} + \int_1^t \frac{\varepsilon(t)dt}{\varepsilon'(t)} \] 

(A-5)

where \( q_i \) is the theoretical rate at \( t=1 \) day,

\[
q = q_i\frac{\varepsilon(t)}{\varepsilon(1)}e^{\int_1^t \frac{\varepsilon(t)dt}{\varepsilon'(t)}} \] 

(A-6)

and

\[
G_p = \frac{q_i}{\varepsilon(1)}e^{\int_1^t \frac{\varepsilon(t)dt}{\varepsilon'(t)}} \] 

(A-7)

If

\[ q = at^n \] 

(A-8a)

or

\[ \varepsilon(t) = at^n \] 

(A-8b)

Eqs. A-6 and A-7 can be written as:

\[ q = t^n\frac{\varepsilon(t)}{\varepsilon'(t)} \] 

(A-9a)

and

\[ G_p = \frac{q_i}{a}e^{\frac{\varepsilon(t)}{\varepsilon'(t)}(t^n - 1)} \] 

(A-9b)

Appendix B—Specific Equations for \( t \) and \( q \)

\( t_{\text{max}} \) and \( q_{\text{max}} \)

Fig. 4 shows there is a maximum rate flow \( q_{\text{max}} \) for each line where \( m \) is greater than unity. Time \( t_{\text{max}} \), which gives \( q_{\text{max}} \), can be estimated by

\[
t_{\text{max}} = \left( \frac{m}{a} \right)^{\frac{1}{1-m}} \] 

(B-1)

and

\[
q_{\text{max}} = q_i\left( \frac{a}{m} \right)^{\frac{m}{1-m}} \] 

(B-2)

Eq. B-1 was obtained by allowing the rate derivative of Eq. 5, \( dq/dt \), to approach zero.

Best 3-Month Average, \( q_{\text{max}}' \)

Since \( t_{\text{max}} \) is less than 45 days (Fig. 4), the best 3-month average \( q_{\text{max}}' \) will be equal to the average of the first 3 months of production and can be calculated by

\[
q_{\text{max}}' = \frac{q_{i}t_{\text{max}}}{35}e^{\frac{\varepsilon(t_{\text{max}})}{\varepsilon'(t_{\text{max}})}(t_{\text{max}} - 1)} \] 

(B-3)

Eq. B-3 is obtained on the basis of Eq. 6 by calculating the cumulative gas of the first 3 months of production. Gas rate and cumulative gas based on this \( q_{\text{max}}' \) would be

\[
q = 3q_{\text{max}}'at^n e^{\frac{\varepsilon(t)}{\varepsilon'(t)}(t^n - 1)} \] 

(B-4)

and

\[
G_p = 3q_{\text{max}}'e^{\frac{\varepsilon(t)}{\varepsilon'(t)}(t^n - 1)} \] 

(B-5)

Eq. B-5 is used to construct the solid lines in Fig. 3.

Anh N. Duong is a principal reservoir engineer and holds the title Reservoir Modeling Director at ConocoPhillips Canada, where he has worked since 2001. E-mail: anhnduong@yahoo.com or anh.n.duong@conocophillips.com. He heads a subsurface team that provides technical expertise in reservoir engineering and modeling to all ConocoPhillips Canada assets. These include oil sands, shale gas, and tight oil and gas reservoirs. Before joining ConocoPhillips, Duong worked 21 years for Gulf Canada Resources as a reservoir engineer. He holds a BSc degree (1978) in petroleum engineering from Petroleum University of Technology (Abadan Faculty), in late 1980s, Duong pursued but did not complete an ME degree in reservoir engineering at the University of Calgary.
Discussion of Rate-Decline Analysis for Fracture-Dominated Shale Reservoirs

Martin Wolff, SPE, Occidental Petroleum Corporation

Duong (2011) presents a new forecasting approach using log-log rate/cumulative production \((q/G_p)\) vs. time \((t)\) plots for wells in unconventional reservoirs which often exhibit long-term linear flow. He also relates parameters of his new method to Arps curves and discusses both linear and bilinear flow. Kupchenko et al. (2008) also discuss linear and bilinear flow and how they relate to Arps parameters but note that rate and/or rate-derivative plots may require sophisticated filtering to become interpretable because of the noisiness of field data. The following discussion presents another approach linked to Arps parameters which complements the methods proposed by Duong and Kupchenko et al.

A classic decline method, the linear-flow plot of cumulative production vs. the square root of time (Nott and Hara 1991), suggests some advantages of using cumulative production with noisy field data. These include the inherent filtering involved with cumulative rather than rate or derivative plots, as well as the ease of fitting straight lines to Cartesian plots. Although such plots are not as diagnostic as sensitive derivative plots, they may have some advantages in allowing relatively robust and repeatable interpretations once suitable time exponents are selected [possibly from plots of \((q/G_p)\) vs. \(1/t\)] as suggested by Eq. 3 in Duong (2011).

The following derivation shows how to construct generalized Cartesian cumulative production vs. time plots for arbitrary \(b\) factors \((\neq 1)\).

Eqs. 1 and 2 define cumulative production \(G_p\) and instantaneous rate \(q\) at time \(t\) for production declines following the Arps hyperbolic form with initial rate \(q_i\) and initial decline, \(D_i\).

\[
G_p = \frac{q_i^b}{(1-b)D_i} (q_i^{1-b} - q^{1-b}) \quad \text{(1)}
\]

\[
q = q_i (1 + b D_i t)^{-1/b} \quad \text{(2)}
\]

Substituting for \(q\) in Eq. 1 and rearranging, we can rewrite Eq. 2 as

\[
G_p = \frac{q_i}{(1-b)D_i} - q_i (1 + b D_i t)^{(b-1)/b} \quad \text{(3)}
\]

When \((bD_i t) \gg 1\), the equation simplifies to the linearized equation

\[
G_p = A t^{(b-1)/b} + B \quad \text{(4)}
\]

where

\[
A = \frac{q_i}{(1-b)D_i} (b D_i)^{(b-1)/b} \quad \text{(5)}
\]

and

\[
B = \frac{q_i}{(1-b)D_i} \quad \text{(6)}
\]

Therefore, we can construct a Cartesian plot of \(G_p\) against \(t^b\) where \(n = (b-1)/b\). This allows easily fitting a straight line which corresponds to Slope A. For the special case of \(b = 2\), \(n = 0.5\) which is the classic linear-flow plot with \(G_p\) plotted against the square root of time. For bilinear flow with \(b = 4\), \(n = 0.75\).

When a range of hyperbolic decline exponents is used to define a range of forecasts for a particular play or field, a set of Cartesian plots with varying \(n\) can be used to fit the data more easily, rather than fitting both \(D_i\) and \(b\) as required by Arps hyperbolic forms. Figs. 1 and 2 show sample field data plotted with fits for \(b = 2\) and \(b = 1.1\) (which could represent a more-conservative projection, possibly for proven reserves). In this case, the 

![Fig. 1—Field example of \(b=2\) linear-flow plot (time\(^{0.5}\)). EUR=estimated ultimate recovery.](image1)

As both Duong (2011) and Kupchenko et al. (2008) note, the Arps hyperbolic form, with all its flaws but also with its well-understood parameters, remains the most common forecasting form used by the industry. Therefore, these Cartesian square-root-of-time plots may have application as simple, repeatable methods for forecasting with relationships to Arps decline factors.
References

