Pressure-Derivative Approach to Transient Test Analysis:
A High-Permeability North Sea Reservoir Example

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Summary
A recently developed method of transient test analysis, which uses the derivative of pressure data vs. time rather than the pressure/time data alone, is discussed in general terms with regard to its application to very-high-permeability reservoirs. The extension of this method to the various well/reservoir system models considered by several authors is summarized.

The procedure involves the combined use of existing type curves in both the conventional dimensionless pressure form (pD) and the new dimensionless pressure derivative grouping (p' D × tD / C D).

The procedure proved theoretically and through examination of data from the Oseberg field to be a powerful tool in transient test analysis. The identification and interpretation of reservoir heterogeneities, both linear and dispersed, as well as earth tidal effects, which are often not easily discernible through existing methods, are greatly facilitated by the new method.

Introduction
The interpretation of transient test data is the main source of information on the dynamic behavior of a reservoir, especially if permeability is high, because a test can investigate an appreciable volume of the formation.

In recent years, the science of transient well test interpretation has progressed rapidly, most notably through the increased use of type curves, the introduction of new reservoir models, and the advent of computer interpretation packages. These advances greatly augmented the conventional analysis approaches that are based on the homogeneous reservoir system and manual, graphical methods. However, many engineers still have reservations about the usefulness of type curves and the reliability of test information from anything but the simplest responses.

Improvements in well-testing techniques—especially the introduction of the latest generation of electronic bottomhole pressure (BHP) transducers and recorders—have dramatically improved data quality and, consequently, the amount of information that can be obtained from test analysis.

An analysis method that deals directly with rate of pressure change, rather than absolute pressures, greatly simplifies the main problems of interpretation, model diagnosis, and evaluation of parameters. Consequently, much more confidence can be placed in the results obtained with the derivative approach.

This paper covers extensions of the derivative approach to various models proposed by different authors. Its application to high-permeability (> 1 darcy) reservoirs is demonstrated. All previous work on the subject has been confined to reservoirs with permeabilities less than 100 md. Field examples are used to illustrate the application’s validity.

Interpretation of Transient Tests
Traditional and Derivative Theory. Practical transient test interpretation methods have become polarized in recent years between two analysis techniques—conventional and global. The former basically consists of fitting straight lines to data regions (for example, in the Horner analysis) or their multirate extensions or superpositions (see Appendix A). The latter involves the use of various type curves to include the entire data set in the process of reservoir/well system diagnosis, flow-regime identification, and evaluation of system parameters. It is commonly accepted that great confidence in interpreted results is obtained by an iterative combination of the two techniques that starts with the global approach.

The recently documented pressure-derivative approach has combined the most powerful aspects of the two previously distinct methods into a single-stage interpretative plot.

The most useful traditional type curves depict, on a log-log scale, the evolution of the dimensionless pressure, pD, with the dimensionless time group tD / C D. For the basic, most common model of a well with wellbore storage and skin in a homogeneous reservoir, the individual curves are dependent on the wellbore condition group CDe2S.

These type curves have two main drawbacks, especially when associated with high-permeability reservoirs. First, there is the uniqueness of the diagnosis. The various curves have similar shapes, particularly when the effect of wellbore storage is very short-lived, as in the highly permeable Oseberg field. The main regime of in-
terest for the evaluation of the reservoir parameters, infinite-acting radial flow, has no characteristic shape on a log-log plot. Second, late- and intermediate-time deviations from the diagnosed trend are compressed to the extent that recognition is unlikely. Both necessitate the supplementary use of specialized, conventional analysis or semilog scale plots to obtain more accurate results and to help recognize and improve evaluation of nonhomogeneous behaviors.

In dimensionless terms, infinite-acting radial flow is conventionally written as

\[ p_D = 0.5 [\ln(t_D/C_D) + 0.80907 + \ln(C_D e^{25})]. \quad (1) \]

This is the "semilog approximation" and is valid only after the wellbore storage effect has become negligible. When fluid movement is confined entirely to expansion or compression in the wellbore, pure wellbore storage is given by

\[ p_D = t_D/C_D. \quad (2) \]

This rarely has been observed as a pure, dominant flow regime on Oseberg, but the diminishing transient effect it has on the sandface flow rate is noticeable at the beginning of most of the flow periods. The combined response appears as the thick curves in Fig. 1.

When the semilog slope of the dimensionless pressure response is plotted in place of the dimensionless pressure, the thin lines in Fig. 1 are obtained. The \( y \) axis is the derivative of pressure with respect to the natural log of time:

\[ \frac{dp_D}{d \ln(t_D/C_D)} = \frac{t_D}{C_D} p_D'. \quad (3) \]

When applied to the infinite-acting radial-flow equation, for which the slope is constant on a semilog scale, Eq. 1 becomes

\[ t_D/C_D p_D' = 0.5, \quad (4) \]

a more easily diagnosed horizontal line on a log-log plot. Pure wellbore storage becomes

\[ t_D/C_D p_D' = t_D/C_D. \quad (5) \]
This again is a unit-slope line on log-log paper, as was the underived form. Hence, the endpoints of all the curves are fixed by two common asymptotes with a hump-shaped transition whose shape is controlled by the wellbore condition group $C_D e^{-2}$.

Several types of heterogeneities commonly are characterized by either middle-time transitions or late-time deviations from infinite-acting radial flow, represented by a straight line on semilog paper and a horizontal line in the log/time derivative form. In general terms, the pressure-derivative method can be considered as a normalization of a semilog plot. In doing this, other characteristic behaviors that often are not easily discernible by traditional methods are accentuated, facilitating model diagnosis, flow regime identification, and parameter evaluation.

**Well/Reservoir System Analysis.** Some of the most commonly observed well/reservoir system responses are summarized in Fig. 2. For each model, the two typical, traditional plots, log-log and semilog, along with the corresponding form in the pressure-derivative format, are given. The abscissa for all the figures is the log of the elapsed time, which increases to the right. All curves are for the case of a drawdown from initial conditions. As will be discussed later, the shapes obtained during buildup can be more complex and can even alter in trend.

**Homogeneous Reservoirs.** The first three columns in Fig. 2 consider a homogeneous reservoir in which reservoir properties can be represented by a single system model. Departures from this infinite-acting radial-flow model are caused by inner and outer boundary conditions. The most common case, that of wellbore storage and skin (Col. 1 in Fig. 2), was described in terms of pressure derivative by Bourdet et al. and in this paper. It has been included as the inner boundary condition on all the models with the exception of fractured wells, where it is omitted for clarity.

1. Hydraulically fractured wells. For hydraulically fractured wells, both infinite-conductivity and uniform-flux models are shown (Col. 2 in Fig. 2). The use of the derivative for fractured well analysis was discussed by Bourdet et al. and Puthigai and Tiab. Unless obscured by wellbore storage, the half-slope line characteristic of fracture linear flow initially is observed on a traditional log-log plot. In dimensionless terms, this is given for both models by

$$p_D = \sqrt{\pi t_{Df}}. \quad \ldots \quad (6)$$

When differentiated with respect to the log of the time function, this becomes

$$t_{Df}p_D' = 0.5 \sqrt{\pi t_{Df}}. \quad \ldots \quad (7)$$

again giving a half-slope line on log-log. In radial flow, the pressure response is approximated by

$$p_D = 0.5 \ln t_{Df} + 2.2 \quad \ldots \quad (8)$$

for infinite conductivity and

$$p_D = 0.5 (\ln t_{Df} + 2.81) \quad \ldots \quad (9)$$

for uniform flux.

The difference in the constant term manifests itself on log-log paper by the stabilization of the final curves at different levels (Fig. 2). Radial flow again is represented by a horizontal line.

$$t_{Df}p_D' = 0.5. \quad \ldots \quad (10)$$

The two models differ only in the transitional period and give different curves that are analogous to different skin ($C_D e^{-25}$) values on the wellbore storage and skin type curve (Fig. 1).

2. Bounded systems. The two most common outer boundary conditions are barriers to flow, such as faults and pinchouts, and pressure maintenance from a gas cap or aquifer. Neither of these has an easily observable characteristic form on a log-log scale. A semilog plot normally is used for conventional or type-curve analysis (Col. 1, Fig. 2).

A single no-flow boundary results in the establishment of a semiradial flow regime with an increased rate of pressure change. On semilog plots this produces a doubling of the slope $(m)$. Consequently, after a transition period, the boundary appears on the derivative plot as a horizontal line (Col. 1, Fig. 2):

$$(t_{Df}/C_D)p_D' = 1. \quad \ldots \quad (11)$$

A further no-flow boundary can double this to 2.

Pressure-maintenance boundaries, on the other hand, result in a reduction in the pressure change at late times. This manifests itself as a flattening on the semilog plot as the maximum pressure differential is attained; on the derivative plot, the curve slopes downward to zero.

In a closed system (Col. 3, Fig. 2), the late-time pseudosteady-state response can be written as

$$p_D = a(t_{Df}/C_D) + b, \quad \ldots \quad (12)$$

where $a$ and $b$ are constants dependent on reservoir size, shape, and properties. On the log-log plot, the curve tends asymptotically to a unit slope (Col. 3, Fig. 2). When the derivative with respect to the log of time is taken, the second constant is lost.

$$(t_{Df}/C_D)p_D' = a(t_{Df}/C_D), \quad \ldots \quad (13)$$

which gives a unit-slope straight line on a log-log scale.

**Heterogeneous Reservoirs.** There are many reservoirs where the pressure response is the product of the interplay of more than one conductive medium. Most current models are composed of two homogeneous media dispersed throughout the reservoir, with a large permeability contrast between them. The bias toward high-contrast systems is caused by the averaging nature of transient testing to conceal lesser degrees of heterogeneity.

The models currently used most extensively are the two considered for double-porosity systems. Here the observed response is the result of two media, usually...
considered as fissures and blocks in naturally fractured reservoirs. For simplicity, this terminology is often extended, as it is here, to represent the high- and low-permeability layers, respectively, in multilayered reservoirs. Initially, flow is almost entirely from the high-permeability, low-storativity fissure system. Eventually, there is pressure support from the high-storativity block system before the two systems finally stabilize; the subsequent response is that of the total system. This behavior has been best described with component type curves\textsuperscript{11} that use the concept of two homogeneous system responses (fissure and total) with a transition regime during the period of pressure support. Two main flow types between blocks and fissures, pseudosteady-state and transient, with different transition responses have been envisaged. The top two illustrations in Fig. 2 (Cols. 4 and 5) show idealistic versions of the double-porosity responses, demonstrating the differences in the transition between the fissure system (F) and total system (T) response curves. Additionally, the theoretical possibility of the existence of two parallel lines on semilog plots is shown, along with the less likely case of the half-slope (m/2) region that can be observed with transient interporosity flow.

Both double-porosity models are based on two infinite, homogeneous systems (Fig. 2, Col. 1) with a reduction in the rate of pressure change during transition. Consequently, when the derivative is taken with respect to the log of time, the main trend is the same as that in Col. 1 of Fig. 2, with a drop in the derivative when the rate of pressure change decreases during transition.\textsuperscript{5,6} The response is similar to that produced by pressure-support boundaries. Radial flow is characterized by a horizontal line at (t_D/C_D)×p'/p = 0.5. Hence, it is easy to understand the controls on two parallel straight lines when one considers the derivative plot. In Col. 4 of Fig. 2, the solid line has reached radial flow in the fissure system before the onset of transition. Alternatively, the dotted line shows the transition starting before the end of the wellbore storage effect. The position of the transition dip is controlled by the interporosity flow coefficient, \( \lambda \), and the depth and length of the transition period are dictated by the storativity ratio, \( \omega \).

With transient interporosity flow, the transition period is usually longer, with the theoretical possibility for the development of a straight line with a slope (m/2) equaling half of that which corresponds to radial flow in the total system (m). When viewed in the derivative format, the half-slope would be represented by a horizontal line at (t_D/C_D)p'/p = 0.25. In practice this is attained only for a slab matrix-block geometry.

The derivative plot gives much more distinctive shapes for the different double-porosity models although the pseudosteady-state model with large \( \omega \) values still can be confused with the transient model. In most cases, if the transition drops below 0.25, then a pseudosteady-state model can be inferred.\textsuperscript{6}

The early and late transition slopes for the double-porosity, pseudosteady-state, interporosity flow model both tend to 45° [0.34 rad] if the transition is long enough for them to establish a straight-line portion. It has been observed in certain tests, previously taken as good examples of double porosity, that the early and late transition curves develop slopes of less than 45° [0.34 rad] when pressure data are derived. This difference has not been observed on traditional interpretation plots. It is possible that this transition slope variation could be a function of the permeability contrast between the two system media in a fully perforated, multilayer reservoir. The contrast would still be high, but the lower permeability could no longer be considered as making a negligible contribution to the total flow capacity.

Comparison of the derivative plots in Cols. 1 and 5 of Fig. 2 reveals that there is still the problem of the similarity of a homogeneous system with a single no-flow boundary and a double-porosity system with transient interporosity flow. When the theoretical curves are compared in detail, there are minor differences, but the quality of derived pressure rarely is good enough to make any distinction.

**Buildup Analysis.** Up to this point, the models and type curves discussed have been for an initial drawdown from static conditions at a constant rate. The production rate generally is not sufficiently stable for analysis of the drawdown. Consequently, most transient test analysis is focused on the buildup when the rate, at the surface at least, is well-defined and is constant—that is, zero.

For conventional analysis methods to be useful for buildups, they must be modified to account for preceding flow periods. The most commonly used plot, semilog, is thus replaced by either a Horner plot or superposition, the latter being used almost exclusively for all tests at Oseberg field. The superposition plot replaces the semilog straight line resulting from infinite-acting radial flow by compressing the later section of the time axis.

For the global approach of type-curve matching, the problem of buildup analysis is more difficult to overcome. In a buildup, the pressure gradually reattains the static reservoir pressure; therefore, the response on a log-log plot is restricted in magnitude to the final flowing-pressure difference of the previous flow period. In fact, the buildup response asymptotically approaches this level, resulting in a flattening of the trend at late time. This flattening means that matching buildup data on drawdown type curves is a very risky process and is normally used only to perform an initial, gross model diagnosis and flow regime identification. Although there have been many methods proposed for extrapolating/converting buildup data to fit on drawdown type curves, by far the best method (used at Oseberg) is the generation of the appropriate multirate type curve for each buildup, thus accounting for the preceding changes in flow rate. This is time-consuming and must be performed with the aid of a computer.

If the derivative of the pressure is taken with respect to the multirate log/time (superposition) function, then the normalizing effect, which restitutes the radial-flow straight line, will also reproduce the characteristic horizontal line of the derivative plot. More precisely, if the preceding drawdown reaches radial flow in the total system, then the buildup data can be matched successfully on drawdown type curves.

The same is true for both homogeneous and heterogeneous reservoirs. This conclusion can be extended to include boundary effects only if the drawdown is sufficiently long for the complete combination of conditions and boundaries of the well and reservoir to have
been encountered. For complex systems, it is still advisable to generate the appropriate multirate derivative type curve, especially if more information than just the values of permeability and skin is to be extracted from the data. These are plotted and matched with the data using the same superposition function as the base for the derivative of both. Radial flow is still represented by a horizontal straight line, but marked changes in transition shapes can be observed. In extreme cases of very short preceding drawdowns, it appears that even the wellbore storage to radial-flow transitional hump can be distorted.

The very high permeabilities encountered in Oseberg mean that virtually all the tests contained buildups with stable preceding flow periods that reached radial flow. In many cases, this required only a few minutes. Unfortunately, the high permeability also meant that outer boundary effects, along with several possible heterogeneous responses, were observed in the vast majority of tests. For this reason, all the final matching was performed with multirate type curves generated for the relevant flow-test history.

**Derivation of Pressure/Time Data.** One of the main problems inherent with the pressure-derivative approach to transient test analysis is that the rate of change of pressure, the quantity under consideration, currently cannot be measured directly and must be extracted from discrete measurements of the absolute pressure evolution. In this study, as already stated, the derivative of the pressure data was obtained with respect to the applicable log/time function.\(^4\) For virtually all the tests on the Oseberg field, several production rates were used before the buildups were studied, the drawdowns never being sufficiently stable for analysis. Thus, to maximize the diagnostic power of the derivative method, the pressure was derived with respect to the superposition time function described in Appendix A, as discussed by Bourdet et al.\(^5\)\(^,\)\(^7\) This had been shown by the abovementioned authors to maintain the reservoir signal trend and to improve its differentiation when associated with extraneous noise from the measuring or recording device.

Several algorithms have been proposed to obtain the derivative of pressure/time data.\(^7\) The main criterion is that the reservoir/well system response must be preserved while the spurious noise is removed. All the algorithms considered are a balancing process, with the ability to suppress the noise on one side and the degree of data distortion on the other. The main uncertainty is that the true derivative of the data is not known.

Two algorithms were found to be effective in producing good derivatives from the pressure/time data. In both cases, the smoothing parameters \(l\) and \(L\) are expressed in terms of the superposition time function.

**Algorithm A.** This is basically a least-squares method with Parameter \(l\) defining the region around the point to be considered when the line representing the gradient at that point is fit (Fig. 3a). This algorithm effectively derived the pressures obtained from gauges that incorporate data-reduction algorithms. Unfortunately, it has a tendency to give unpredictable end effects, which limits its use when pressures deviate from the horizontal radial flowline, as occurs with boundary effects and heterogeneous systems.

\[ m_B = \frac{\Delta P_1 \cdot \Delta t_2 + \Delta P_2 \cdot \Delta t_1}{\Delta t_1 + \Delta t_2} \]

\( o = DATA POINTS \)
\( \bullet = POINT TO BE DERIVED \)
\( \ast = POINTS USED FOR THE CALCULATION \)
\( m = VALUE OF DERIVATIVE \)

**Fig. 3—Data derivative algorithms.**

**Algorithm B.** Bourdet et al.\(^7\) describe this algorithm. Basically, the weighted mean of the slopes to a preceding (left) and a following (right) point are placed at the point of interest (Fig. 3b). The parameter \(L\) is used to define the minimum abscissal distance to these points and to smooth out noise with points farther away.

The effect is not generally as efficient as Algorithm A in smoothing very noisy data, but there is much less distortion of the late-time data resulting from end effects. Because the latter was an important consideration for Oseberg test interpretation, this was the algorithm used in most cases. Algorithm A was used only when it highlighted particular peculiarities. In some cases, different values of \(L\) were used for early-time data to smooth them without overly distorting an already smooth late-time curve. This principle is also discussed by Bourdet et al.\(^7\)

**Data Quality.** The quality of the pressure data has a major influence on the application of the derivative method. In our experience, the data from electronic gauges are normally sufficiently dense and of high enough resolution to be derived easily. In reservoirs with high mobility-thickness products, the noise from many gauge types can be of the same magnitude as the pressure gradient, which makes useful derivation difficult. The best data in these cases were obtained with crystal gauges, as shown in Example Test A. Here the low point density, which resulted from the use of a data compression algorithm in the recorder, still produced a good derivative on a very shallow buildup slope.
Oseberg, A Field Example

The Reservoir. The Oseberg field is located 140 km [87 miles] west of Bergen in the Norwegian sector of the North Sea. Norsk Hydro is the operator for the license areas in Blocks 30/6 and 30/9. Statoil and Saga Petroleum are partners in both licenses and are joined by Elf Aquitaine, Mobil, and Total in Block 30/6.

In 1979, Well 30/6-1 was drilled and proved gas. Ten more wells subsequently drilled in the field (Fig. 4) have shown the reservoir to contain a 656-ft [200-m] oil column with an average sand thickness of 131 ft [40 m] in the Brent formation. Structurally, the Oseberg field can be characterized as a relatively narrow, eastward-dipping (7°), fault block complex with three major compartments, designated as Alpha, Alpha North, and Gamma.

The Brent is made up of the Etive and Ness subdivisions. Most of the reserves are in the Etive formation, which is thought to be a well-developed, massive, delta-front sandstone unit. The overlying Ness formation consists of interbedded sandstones, shales, and coals and is thought to have been deposited in a delta plain environment with possibly some marine incursions.

The field generally has very good reservoir properties with permeabilities commonly more than 2 darcies in the oil zone. The nature of the Ness formation makes reservoir evaluation by core and log analysis difficult. As a result, it was tested intensively in the later wells; a total of 16 flow tests were performed on it in eight wells. No test on the Ness formation produced a response completely interpretable as a homogeneous, infinite-acting system. The high permeability often resulted in boundary effects influencing the behavior very early. In several cases, the interpretation is ambiguous, but generally the Ness could be subdivided behaviorally into two units: the extensive Cycle I and Upper Brent sand units, where the boundary effects are associated mainly with fluid contacts and faults, and the Intra Ness sands, whose apparently limited extent results in extremely complex responses.

To obtain adequate bottomhole data in such high-permeability formations, electronic pressure gauges are almost obligatory. In the early tests, Sperry Sun MRPG recorders were used. These were replaced in the later tests by Flopetrol SDP gauges that incorporated a data-compression algorithm.

Because of operational restrictions, the maximum effluent production rate used for testing was approximately 6,000 B/D [954 m³/d], with most tests being performed with a main flow of about 2,500 B/D [397 m³/D]. The low rates exacerbated the problem of shallow buildup gradients produced by high permeabilities.

Both tests given as examples are from the Ness formation.

Interpretation Methodology. The same methodology was applied to the interpretation of tests from all except the last two wells in Oseberg. A first interpretation, the iterative global/conventional approach, used type curves for model and flow regime diagnosis, followed by specialized, superposition analysis. Rigorous type-curve matching with computer-generated, multirate curves for various models was used to check the specialized analysis and to evaluate late-time effects. Finally, the entire test was simulated to verify the overall applicability of the analysis.

With the development of the pressure-derivative approach, most of the tests were re-evaluated. In this case, the data were analyzed directly with the combined global/conventional type curves.

For the last two wells (Wells 30/6-I3 and 30/9-2), the analysis was performed primarily with the multirate, derivative type-curve method. The traditional analysis was given for reference only.

Test A Interpretation. This test was performed on a Ness formation sand in Well 2 during July 1983. From core and log data, the unit has been interpreted as a regressive marine sheet sand, representing either a beach or mouth-bar deposit. Consequently, the zone is expected to be laterally extensive. The unit is a coarsening-up sequence, the top half being composed of medium to coarse, well-sorted sand. A coal layer forms the upper limit of the interval; the base is a silt mud with an
TABLE 1—RESERVOIR PARAMETERS FOR ANALYSIS OF TEST A AND TEST B

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test A</th>
<th>Test B</th>
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<tbody>
<tr>
<td>$B$</td>
<td>1.49</td>
<td>1.47</td>
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<tr>
<td>$\mu$, cp</td>
<td>0.445</td>
<td>0.465</td>
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<tr>
<td>$s_w$</td>
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<tr>
<td>$c_p$, psft$^{-1}$</td>
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<td>$17.7 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.23</td>
<td>0.26</td>
</tr>
<tr>
<td>$h$, ft</td>
<td>33</td>
<td>16.4</td>
</tr>
<tr>
<td>$r_w$, ft</td>
<td>0.51</td>
<td>0.51</td>
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TABLE 2—FLOW PERIODS OF TEST A

<table>
<thead>
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<th>Duration (hours)</th>
<th>Production Rate (B/D)</th>
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<tr>
<td>0.067</td>
<td>4,320</td>
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<tr>
<td>1.000</td>
<td>0</td>
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<tr>
<td>0.333</td>
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<tr>
<td>11.667</td>
<td>3,000</td>
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<tr>
<td>18.383</td>
<td>0</td>
</tr>
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</table>

Fig. 5—Log and core description, Example Well A.

Fig. 6—Log-log plot, Test A.
underlying coal (Fig. 5). Reservoir and fluid characteristics are listed in Table 1.

After a short initial flow and buildup, the well was produced for 12 hours before shutting in for a buildup of 18 hours. The flow periods used for analysis are given in Table 2. Pressure data were available from a quartz crystal gauge with a downhole recorder. The pressure change vs. elapsed-time data for the final buildup are listed in Table 3, along with the derived data used in the analysis, which will be discussed later.

The first step, previously considered as a purely diagnostic stage, was to plot both the pressure change vs. elapsed time on a log-log scale (Fig. 6). The raw, unsmoothed derivation of the pressure (Algorithm B) is also shown. Although the data are affected at very early times (less than 30 seconds) by changing wellbore storage, it is immediately obvious that unaffected radial flow is established very quickly. An initial match of the pressure data on the type curve for a homogeneous reservoir with wellbore storage and skin\(^3\) indicates that it is possible that the 10\% slope semilog approximation is reached as early as 40 seconds. Because of the changing wellbore storage, this is only a minimum value from the ordinary pressure plot, but it is confirmed by inspection of the derivative data. As can be seen, the data have flattened out to the horizontal line, characteristic of radial flow in a homogeneous system, after 1 minute.

Following the traditional methodology, the next step was to perform specialized analysis on the region diagnosed as radial flow. Thus, a superposition plot was made, considering four preceding flow rates (Fig. 7). A well-defined straight line was evident from 1 minute onward, and conventional analysis (Appendix B) was performed.
TABLE 4—ANALYSIS RESULTS OF TEST A

<table>
<thead>
<tr>
<th></th>
<th>Superposition</th>
<th>Derivative</th>
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<td>$kh$, md-ft</td>
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<td>102,800</td>
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<td>$k$, md</td>
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</tr>
<tr>
<td>$C$, bbl/psi</td>
<td>—</td>
<td>0.0068</td>
</tr>
<tr>
<td>$C_o$</td>
<td>—</td>
<td>184</td>
</tr>
<tr>
<td>$a$</td>
<td>9.0</td>
<td>9.3</td>
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<tr>
<td>$p_o/\Delta p$, psi$^{-1}$</td>
<td>0.364</td>
<td>0.366</td>
</tr>
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</table>

Fig. 8—Comparison of various derivative smoothing parameters.

Fig. 9—Combined type-curve match, Test A.
Fig. 8 shows the derivative of pressure with respect to the superposition function time product using the two algorithms described with various degrees of smoothing. The unsmoothed data are obviously unsuited to curve-matching. Algorithm A produced smooth data but is dominated by a sine wave at late time, the origin of which will be discussed further. The best algorithm for curve-matching purposes was Algorithm B because the wave is suppressed without unduly distorting the general trend.

A model consisting of an infinite, homogeneous system with wellbore storage and skin initially was used with an apparently perfect match obtained with a multirate type curve on the pressure data (Fig. 9). The \( p_D/p \) ratio obtained from specialized analysis was used to fix the vertical position. Because of the lack of early-time data, this match is poorly controlled.

With the derived data (Fig. 9), a minimum match was obtained for the homogeneous system. Analysis of the match is given in Appendix C. Additionally, a late-time deviation, which was found to be modeled most simply by a barrier to flow, was observed acting at 3,000 ft [914 m] from the wellbore.

Because a high degree of smoothing had been used on the data, it was advisable to verify that this had not deformed the true shape of the data too much. This was done by truncating the \( p_D \) multirate type curve to the same duration as the actual buildup. Then the same smoothing parameters used to produce the derived curve for the pressure data were used. When compared with the unsmoothed curve (Fig. 9), the distortion is negligible, except for an end effect, which was also present in the matched data.

The results obtained independently, from superposition analysis and type-curve matching on a combined log-log plot, are given in Table 4. The differences are negligible.

Interpreting the possible presence, albeit tentatively, of a boundary was important in defining the minimum distance to the updip formation truncation.

**Test B Interpretation.** The second example is a drillstem test (DST) performed in Well 30/6-13. The 16.4-ft [5-m] -thick zone tested was part of the Oseberg Intra Ness Unit 1, with basal coal and shale layers and an overlying silty shale (Fig. 10). Interpretation of the core and log data produced a geological model of a distributary channel deposit. Average values of the reservoir and fluid parameters used for the analysis are given in Table 1.

A test sequence consisting of two flow and buildup periods was carried out. The production rates and durations are given in Table 5. The first rate was unreliable because of separation problems. BHP's were recorded with strain-gauge sensors and downhole processors. Table 6 is a listing of the elapsed time, pressure change, and derivative of pressure for the final buildup.

A conventional log-log plot of the data (Fig. 11) again illustrated a very short period of wellbore-storage-affected data. A visual match on the type curve\(^3\) gave the earliest start time for radial flow as 30 seconds. This match assumed a homogeneous reservoir with wellbore storage and skin; late-time effects were visible as a deviation above the curve. The shape could also be diagnosed as resulting from a heterogeneous reservoir, giving a much later radial flow period.

Plots of the derivative of the pressure (with respect to the superposition function) time product vs. time with Algorithm B are shown (Fig. 12) to be much more powerful as a diagnostic tool. The end of wellbore storage, followed by a flattening and then a continuous rise, is clearly seen. The last two are most characteristic of a homogeneous system: the flattening representing radial flow and the increase in the derivative resulting from the influence of no-flow boundaries. At this stage, it still is not possible to discount a heterogeneous system, but the very long transition and lack of a final stabilized period suggests that none of the currently available models would be adequate without the further complexity of outer-boundary effects.

The problems with using adjacent points for deriving data in high-permeability reservoirs is well illustrated by the scattering of the unsmoothed data (Fig. 12). In this case, a high degree of smoothing was required, especially in the early-time region, to give the derivatives listed in Table 6.
Fig. 11—Log-log plot, Test B.

Fig. 12—Log-log plot of various pressure derivations, Test B.
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JOURNAL OF PETROLEUM TECHNOLOGY
The two-stage process, consisting of model diagnosis and flow regime identification on log-log, was followed by specialized analysis; in this case, the superposition plot (Fig. 13) was used. The results given in Col. 1 of Table 7 were obtained for a homogeneous reservoir. Because of the distorted early-time pressures and boundary effects, the exact duration of the semilog approximation is not obvious. Line 1 is taken to be the best slope on the superposition plot.

With computer-generated multirate type curves, the derivative data were matched (Fig. 14) for a homogeneous reservoir with wellbore storage and skin with two perpendicular no-flow boundaries acting at 144 ft [44 m] and 335 ft [102 m] from the well (Fig. 15). Analysis of the match produced the results given in Col. 2 of Table 7. The difference between an infinite system, single and multiple boundaries is illustrated clearly. The doubling effect that flow barriers have on the rate of pressure change is characterized on derivative curves by final stabilization at double the dimensionless pressure,\(^6,7\) that is, \(p'_{D} \times t_{D}/C_{D} = 1\) for single boundaries and 2 for double boundaries as was the case in this test (Fig. 14). This made it easier to decide at an early stage whether a bounded system was a viable solution. The same method was used to check the effect of the smoothing parameters as for Test A. The only variation is a slight flattening at early time, which was considered unimportant. As a final control, the match with pressure data was also plotted (Fig. 14).

Comparison of Cols. 1 and 2 in Table 7 reveals an unacceptable difference (> 10%) between the results of the two analysis methods. This can be explained easily by examination of the derivative curve match (Fig. 14).

For the semilog approximation to be valid, the data must be on the infinite system response (bottom curve) after the wellbore storage effect becomes negligible. It can be seen that this in fact does not occur because of the effect of the boundaries.

The average of the points used to draw the initial straight line (Line 1) on the superposition plot is more than 10% above the level of the semilog approximation. If only the points satisfying this criterion are used, then Line 2 on Fig. 13 is obtained. This yields the values given in Col. 3 of Table 7, which are now in excellent agreement with those from derivative type-curve analysis.

Double-porosity type curves that incorporated either transient or pseudosteady-state interporosity flow\(^5,6\) failed to give a similar match quality in the derivative format, even when additional boundary effects were added.

The homogeneous, bounded model is most consistent with other information, producing a similar average permeability to the core measurements. The skin obtained is consistent with the other zones tested in the well, whereas heterogeneous models produced incongruously negative skins associated with low permeabilities. The bounded nature of the sand unit, determined by type-curve matching, is additional confirmation of the geological interpretation.

An ideal half-width for this type of channel is about 500 ft [152 m], which implies that what is seen in the test is probably a curve on one side of the channel being approximated by the perpendicular boundaries used in the model, as shown in Fig. 15.
Discussion of Examples. Both examples illustrate the power of the derivative approach to test interpretation. Initially, the most obvious improvement is in the identification of the various flow regimes that constitute a pressure response. Conventionally, this is performed by “rule-of-thumb” methods or approximations on type curves. The traditional limits of the radial flow regime are shown on Fig. 16 for the two examples using the position of the 0.5 line found from derivative type-curve matching. As the derivative plot is a plot of the semi-log slope, the source of the original discrepancy in the results from Example B is easily seen. In fact, for Example B, radial flow was never fully established because of the effect of the boundaries.

The characteristic radial flowline makes the type-curve match as accurate as specialized analysis for the evaluation of permeability and skin factors.

The high permeabilities meant that very few data on wellbore storage were obtained. Thus, the second stage of the matching process—i.e., fixing the time match from the unit slope line—could not be performed. This was found not to be a drawback because using a minimum time match defined by the end of the transition hump gave sufficient control for the main interpretation results to be reliable.

Taking the derivative of the data normalizes the late-time trend. In Example A, this made visible a sinusoidal component to the pressure behavior (Fig. 17), which, with a 12-hour period, is most likely to be a tidal effect. Generally the two examples and other test interpretations from the field demonstrate that the derivative technique is still effective with very high permeabilities. Additionally, if good data are obtained, the density of the data is not as important as previously suggested. Therefore, pressure recorders incorporating data-reduction algorithms can be used without adversely affecting the interpretability of the data.

Earth Tides vs. Well/Reservoir System Behavior

Exploration wells in high-permeability reservoirs show a rapid pressure buildup to the original reservoir pressure during well tests. In certain cases, a regular pressure oscillation may be observed, which in the exploration phase of a reservoir cannot be explained by well interference effects.

If the oscillation frequency has a period in the range of 2 cycles/D and an oscillation amplitude of $0.02$ to $0.15$ psi [$0.14$ to $1.03$ kPa], it is possible to relate this behavior to earth tides. If the interpretation can be

### TABLE 7—ANALYSIS RESULTS OF TEST B

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<th>Superposition (Line 1)</th>
<th>Derivative Type-Curve Match</th>
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<td>—</td>
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Fig. 16—Comparison of derivative data and the semilog approximation.

Fig. 17—Test A plot: derivative of pressure vs. elapsed time.
justified, additional information on reservoir characteristics may be gained. The ability to observe these phenomena makes research into methods for evaluating them a practical proposition. In the absence of communication between aquifer and sea bottom—the situation in almost all hydrocarbon reservoirs—any tidal effects result from earth tides in a "closed system." In this case, the lunar/solar attraction generates a state of expansion, which is followed by the falling of a mass of earth and the compressing of the fluid contained in the reservoir pores. Thus, the compression/expansion cycles are translated into pressure oscillations whose amplitude depends on the reservoir rock and fluid characteristics. Oscillations can be observed most clearly in Test A when the underlying radial flow trend is normalized by taking the derivative (Fig. 17).

Conclusions
The pressure-derivative approach has proved to be well-adapted to high-permeability reservoirs because of (1) a clearer distinction of flow regimes from more characteristic response forms, (2) high accuracy in reservoir parameter evaluation, (3) magnification of late-time trends (such as boundaries and tidal effects), and (4) the erroneous results semilog analysis gives even in long tests because radial flow may not develop.

Enhancement of model diagnosis and differentiations increases the need for further work on reservoir model development. Much greater system characterization and description from transient test data are indicated for the future.

Nomenclature

\[ B = \text{formation volume factor, bbl/bbl [m}^3/m^3] \]
\[ c_r = \text{total compressibility, psi}^{-1} [\text{kPa}^{-1}] \]
\[ C = \text{wellbore storage constant, bbl/psi [m}^3/kPa] \]
\[ h = \text{formation thickness, ft [m]} \]
\[ k = \text{permeability, md} \]
\[ t = \text{data period length, Algorithm A} \]
\[ L = \text{minimum abscissal distance, Algorithm B} \]
\[ m = \text{semilog slope, psi/cycle [kPa/cycle]} \]
\[ m' = \text{superposition slope, psi/cycle/B/D [kPa/cycle/m}^3/d] \]
\[ p = \text{pressure, psi [kPa]} \]
\[ \Delta p = \text{pressure change, psi [kPa]} \]
\[ p' = \text{derivative of pressure with respect to time} \]
\[ q = \text{flow rate, B/D [m}^3/d] \]
\[ r_w = \text{wellbore radius, ft [m]} \]
\[ S = \text{van Everdingen-Hurst skin factor} \]
\[ \Delta t = \text{elapsed time, hr} \]
\[ V = \text{volume} \]
\[ \lambda = \text{interporosity pseudosteady-state flow parameter} \]
\[ \mu = \text{viscosity, cp [Pa} \cdot \text{s]} \]
\[ \phi = \text{porosity} \]
\[ \omega = \text{storativity ratio} \]

Subscripts

\[ D = \text{dimensionless} \]
\[ f = \text{fracture} \]

\[ M = \text{match} \]
\[ w = \text{wellbore} \]

Acknowledgments
We are grateful to the management of the Oseberg project for permission to publish this paper. We are particularly thankful for the encouragement received from T. Enger and T. Torvund. Appreciation is extended to H. Eide of Norsk Hydro for his work on log interpretation and to J. Ayoub, D. Bourdet, and T. Whittle of Flopetrol-Johnston for their assistance and comments.

References
1. Earlougher, R. C., Jr.: Advances in Well Test Analysis, Monograph Series, SPE, Richardson, TX (1977) 5, 5-264.

APPENDIX A
The superposition time function was used in virtually all analyses at Oseberg field. It facilitates comparison of different flow periods during tests and improves the accuracy of conventional analysis.

The time function also restitutes the most characteristic shape for radial flow—that is, the horizontal line on the derivative plot during buildups. Additionally, the quality of derived data is enhanced as the flattening of the radial flow gradient when use of \( \Delta t \) or \( \log \Delta t \) during buildups is eliminated.

For all analysis purposes considered in this paper, the superposition function is written as

\[
\frac{1}{q_n - q_{n-1}} \times \left[ \sum_{i=1}^{n-1} (q_i - q_{i-1}) \ln \left( \sum_{j=1}^{n-1} \Delta t_j + \Delta t \right) \right] + \ln \Delta t. \quad (A-1)
\]
APPENDIX B

Test A: Superposition Analysis
Equations are used as they are in an example in Ref. 3 (\(m' = mLq\)).

Line parameters from Fig. 7 include the following.
Superposition slope, \(m' = 1.053 \times 10^{-3}\) psi/cycle/B/D,

\[
p (\Delta t = 1 \text{ hour}) = 4,121.9 \text{ psia},
\]

\[
p (\Delta t = 0) = 4,076.4 \text{ psia},
\]

\[
kh = 162.6 \frac{qB\mu}{m'} = 102,385 \text{ md-ft},
\]

\[k = 3,115 \text{ md},\]

\[s = 1.151 \left[ \frac{P_{\text{thr}} - P(t=0)}{m'(q_n - q_{n-1})} \right] - \log \frac{k}{\phi\mu c_t r_w^2} + 3.23 \right] = 8.97,
\]

and

\[
p_d / \Delta p = \frac{1.151}{m'(q_n - q_{n-1})} = 0.364 \text{ psi}^{-1}.
\]

APPENDIX C

Test A: Derivative Type-Curve Match
Equations are used as they are in an example in Ref. 3.
Match parameters from Fig. 9 include the following.

Pressure axis = \(p_d / \Delta p = 0.366 \text{ psi}^{-1}\),
time axis = \((t_d/C_d) / \Delta t = 10,000 \text{ psi}^{-1}\),
match curve = \(C_D e^{2S} = 2 \times 10^{10}\),
\[kh = 141.2 \frac{qB\mu(p_d/\Delta p)_M}{m'} = 102,797 \text{ md-ft},\]

\[k = 3,115 \text{ md},\]

\[C = 0.000295 \frac{kh}{\mu} \frac{1}{(t_d/C_d)/\Delta t}_M = 0.0068 \text{ bbl/psi},\]

\[C_D = 0.8936 \frac{C}{\phi\mu c_t r_w^2} = 184,\]

and

\[s = 0.5 \ln \left[ \frac{(C_D e^{2S})_M}{C_D} \right] = 9.25.\]

SI Metric Conversion Factors

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*Conversion factor is exact.

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\[
\frac{1}{q_n - q_{n-1}} \times \left[ \sum_{i=1}^{n-1} (q_i - q_{i-1}) \right] \times \ln \left( \sum_{j=1}^{n-1} \Delta j + \Delta t \right) + \ln \Delta t.
\]

The lower index on the second \( \Sigma \) should be \( j=i \). This was probably a typographical error, regrettable but difficult to avoid. However, Fig. 7 on Page 2030 needs further explanation. The x-axis is labeled "Superposition Function," but it is not the function as corrected above. Instead, it appears to be

\[
\sum_{i=1}^{n-1} (q_i - q_{i-1}) \ln \left( \sum_{j=i}^{n-1} \Delta j + \Delta t \right) + (q_n - q_{n-1}) \ln \Delta t
\]

\[
\frac{\ln 10}{(q_n - q_{n-1}) \ln \Delta t}.
\]

Eq. D-1 can be written in a simpler form as

\[
\sum_{i=1}^{n} (q_i - q_{i-1}) \log (\Delta t + t_{n-1} - t_{i-1}), \quad \text{(D-2)}
\]

where

\[
t_{n-1} = \text{elapsed time at shut-in},
\]

\[
t_0 = \text{time at start of flow test, 0, and}
\]

\[
t_i = \text{elapsed time when the } i\text{th flow rate stopped.}
\]

The heading \( p' \Delta t \) Group of Table 3 is misleading because the pressure derivative is not being multiplied by any time \( \Delta t \). To get \( p' \Delta t \), first calculate the superposition function for each \( \Delta t \) with Eq. D-2, then calculate the derivative:

\[
p_D' = \frac{d\Delta p}{d(\text{superposition function})} < 0 \quad \text{(D-3)}
\]

with Algorithm B, I assume, then calculate

\[
p' \Delta t = \frac{p' \Delta t (q_n - q_{n-1})}{\ln 10}, \quad \text{(D-4)}
\]

where

\[
q_n = \text{the shut-in flow rate, 0, and}
\]

\[
q_{n-1} = \text{the last flow rate before shut-in.}
\]

The authors have done a good job in presenting a very useful, practical application of the pressure-derivative method that should increase its general acceptance. I hope my comments will help readers trying to follow this paper.
Author's Reply to Discussion of Pressure-Derivative Approach to Transient Test Analysis: A High-Permeability North Sea Reservoir Example

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We would like to thank Zais for pointing out some errors and for indicating areas he feels require some clarification. We present several observations regarding his comments.

The superposition function given in Appendix A is, in fact, that applied to generate the log of time function used in the Flopetrol Johnston derivative method. For the abscissa on the superposition plots, Figs. 7 and 13, the same form of equation was used with decimal logs replacing the natural logs. The reciprocal flow-rate term is neglected because if we do this the function becomes rate normalized, resulting in a very powerful tool for test analysis. The superposition function used, therefore, is

\[
\left[ \sum_{i=1}^{n-1} (q_i - q_{i-1}) \log \left( \sum_{j=i}^{n-1} t_j + \Delta t \right) \right] + (q_n - q_{n-1}) \log \Delta t.
\]

This plot is still required for full analysis because all log-log methods deal in relative values, whereas the superposition plot allows access to and direct comparison of the average reservoir pressure, an absolute value, after the model has been diagnosed.

There is a typing error in the given function in Appendix A. The second summation should start from \( j = i \), not \( j = 1 \) as stated.

The superposition function can be reduced to a single summation taken to \( i = n \), not \( n - 1 \), but this is not conceptually as clear as the function used. Equally, this can be written as a series, but this is less generalized and less satisfactory. Both methods are used in the reference literature.

The term \( p' \Delta t \) is confusing because \( p' \) is not the derivative with respect to elapsed time as stated in the nomenclature. For the first flow period, \( p' \Delta t \) is in fact identical to the derivative with respect to the \( \ln \) (natural log) of time.