Natural Gas Engineering

A Quadratic Cumulative Production Model for the Material Balance of Abnormally-Pressured Gas Reservoirs


T.A. Blasingame, Texas A&M U.
Department of Petroleum Engineering
Texas A&M University
College Station, TX 77843-3116
+1.979.845.2292 — t-blasingame@tamu.edu
Executive Summary: — "\( p/z-G_p^2 \)" Relation

The rigorous relation for the material balance of a dry gas reservoir system is given by Fetkovich, et al. as:

\[
\frac{p}{z} [1 - \overline{c}_e(p)(p_i - p)] = \frac{p_i}{z_i} - \frac{p_i}{z_i} \frac{1}{G} \left[ G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{inj} B_w - W_e) \right]
\]

Eliminating the water influx, water production/injection, and gas injection terms; defining \( \omega G_p = c_e(p)(p_i - p) \) and assuming that \( \omega G_p < 1 \), then rearranging gives the following result:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( \frac{1}{G} - \omega \right) G_p - \frac{\omega}{G} G_p^2 \right]
\]
Simulated Dry Gas Reservoir Case: Abnormal Pressure:
- Volumetric, dry gas reservoir — with $c_f(p)$ (from Fetkovich).
- Note extrapolation to the "apparent" gas-in-place (previous approaches).
- Note comparison of data and the new "Quadratic Cumulative Production" ($p/z-G_p^2$) model.
Executive Summary: — "$p/z-G_p^2$" Relation

Case 3 — Anderson L (SPE 02938)
Validation of New Material Balance Concept for Abnormally Pressured Gas Reservoirs (Variable Compressibility Only)
"Gan Plot 3" $p/z$ Versus $G_p$

Legend:
- $p/z$ Data
- $p/z \omega$-constant (Quadratic) Model
- $p/z$ "Apparent Gas-in-Place" Line
- $p/z$ "Abnormal Pressure" Line
- $p/z$ "Normal Pressure" Line
- $p/z$ "Inflection Point"

Gas-in-Place, $G = 73.5$ BSCF

Anderson L Reservoir Case: Abnormal Pressure:
- South Texas (USA) gas reservoir with abnormal pressure.
- Benchmark literature case.
- Note performance of the new "Quadratic Cumulative Production" model.
Presentation Outline:

● Executive Summary
● Objectives and Rationale
  ■ Rigorous technique for abnormal pressure analysis.
● Development of the $p/z-G_p^2$ model
  ■ Derivation from the rigorous material balance.
● Validation — Field Examples
  ■ Case 1 — Dry gas simulation ($c_f(p)$ from Fetkovich).
  ■ Case 3 — Anderson L (South Texas, USA).
● Demonstration (MS Excel — Anderson L case)
● Summary
● Recommendations for Future Work
Objectives and Rationale:

- **Objectives:**
  - Develop a rigorous functional form (i.e., a model) for the $p/z$ vs. $G_p$ behavior demonstrated by a typical abnormally pressured gas reservoir.
  - Develop a sequence of plotting functions for the analysis of $p/z$—$G_p$ data (multiple plots).
  - Provide an exhaustive validation of this new model using field data.

- **Rationale:** The analysis of $p/z$—$G_p$ data for abnormally pressured gas reservoirs has evolved from empirical models and idealized assumptions (e.g., $c_f(p) =$ constant). We would like to establish a rigorous approach — one where any approximation is based on the observation of some characteristic behavior, not simply a mathematical/graphical convenience.
Development: The $p/z-G_p^2$ Model

**Concept:**
- Use the rigorous material balance relation given by Fetkovich, *et al.* for the case of a reservoir where $c_f(p)$ is NOT presumed constant.
- Use some observed limiting behavior to construct a semi-analytical relation for $p/z—G_p$ behavior.

**Implementation:**
- Develop and apply a series of data plotting functions which clearly exhibit unique behavior relative to the $p/z—G_p$ data.
- Use a "multi-plot" approach which is based on the dynamic updating of the model solution on each data plot.
- Develop a "dimensionless" type curve approach that can be used to validate the model and estimate $G$. 
The rigorous relation for the material balance of a dry gas reservoir system is given by Fetkovich, et al. as:

\[
\frac{p}{z} [1 - \bar{c}_e(p)(p_i - p)] = \frac{p_i}{z_i} \left[ \frac{1}{G} \left( G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g} (W_p B_w - W_{inj} B_w - W_e) \right) \right]
\]

Eliminating the water influx, water production/injection, and gas injection terms, then rearranging gives the following definition:

\[
p/z \equiv \frac{p_i/z_i}{(1 - \omega G_p)} \left[ 1 - \frac{G_p}{G} \right] [\text{where } \omega G_p \equiv \bar{c}_e(p)(p_i - p)]
\]
Considering the condition where:

\[ \omega_D = \omega G_p \leq 1 \]

Then we can use a geometric series to represent the \( \omega_D \) term in the governing material balance. The appropriate geometric series is given by:

\[
\frac{1}{1-x} \approx 1 + x + x^2 + x^3 + ... \quad (-1 < x < 1)
\]

or, for our problem, we have:

\[
\frac{1}{1 - \omega G_p} \approx 1 + \omega G_p \quad (-1 < \omega G_p < 1)
\]

Substituting this result into the material balance relation, we obtain:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( \frac{1}{G} - \omega \right) G_p - \frac{\omega G^2}{G^2} \right]
\]
A more convenient form of the \( p/z \)-cumulative model is:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2
\]

\[
\alpha \equiv \left(\frac{1}{G} - \omega\right) \frac{p_i}{z_i} \quad \beta \equiv \frac{\omega}{G} \frac{p_i}{z_i}
\]

We note that these parameters presume that \( \omega \) is constant. Presuming that \( \omega \) is linear with \( G_p \), we can derive the following form:

\[
\frac{p}{z} \approx \frac{p_i}{z_i} - \left(\frac{1}{G} - a\right) \frac{p_i}{z_i} G_p - \left(\frac{a}{G} + b\right) \frac{p_i}{z_i} G_p^2 + \frac{b}{G} \frac{p_i}{z_i} G_p^3
\]

where \( \omega \equiv a - bG_p \)

Obviously, one of our objectives will be the study of the behavior of \( \omega \) vs. \( G_p \) (based on a prescribed value of \( G \)).
Simulated Dry Gas Reservoir Case: Abnormal Pressure

- A linear trend of $\omega$ vs. $G_p$ is reasonable and should yield an accurate model.
- $\omega$ is approximated by a constant value within the trend.
- A physical definition of $\omega$ is elusive — $\omega G_p = c_e (p_i - p)$ implies that $\omega$ has units of 1/volume, which suggests $\omega$ is a scaling variable for $G$. 

Application: $\omega$-Gp Performance (Case 1)
Application: $\omega$-$G_p$ Performance (Case 3) (2/2)

a. Case 3: Anderson L Reservoir Case (South Texas, USA) — Plot of $\omega$ versus $G_p$ (requires an estimate of gas-in-place). Some data scatter exists, but a linear trend is evident (recall that $\omega G_p = c_e(p)(\rho_f - \rho_g)$).

b. Case 3: Anderson L Reservoir Case (South Texas, USA) — Plot of $p/z$ versus $G_p$. Both models are in strong agreement.

Anderson L Reservoir Case: Abnormal Pressure

- Field data exhibit some scatter, method is relatively tolerant of data scatter.
- Constant $\omega$ approximation based on the "best fit" of several data functions.
- The linear approximation for $\omega$ is reasonable (should favor later data).
Validation: The $p/z-G_p^2$ Model

Methodology:

- All analyses are "dynamically" linked in a spreadsheet program (MS Excel). Therefore, all analyses are consistent — should note that some functions/plots perform better than others — but the model results are the same for every analysis plot.

Validation: Illustrative Analyses

- $p/z-G_p^2$ plotting functions — based on the proposed material balance model.
- $\omega - G_p$ performance plots — used to calibrate analysis.
- Gan analysis — presumes 2-straight line trends on a $p/z-G_p$ plot for an abnormally pressured reservoir.
- $p_D-G_{pD}$ type curve approach — use $p/z-G_p^2$ material balance model to develop type curve solution — this approach is most useful for data validation.
Plotting Functions: $p/z-G_p^2$ — Case 1

(a) $\Delta(p/z) = \left[ \frac{p_i}{z_i} - \frac{p}{z} \right]$ vs. $G_p$

(b) $\frac{1}{G_p} \Delta(p/z)$ vs. $G_p$

(c) $\frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p$ vs. $G_p$

(d) $\frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z) dG_p$ vs. $G_p$

(e) $\Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p$ vs. $G_p$

(f) $\frac{1}{G_p} \left[ \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \right]$ vs. $G_p$
Plotting Functions: $\omega - G_\rho$ — Case 1

a. Case 1: Simulated Performance Case — Plot of $c_0(p)(p_r-p)$ versus $G_\rho$ (requires estimate of $G$).

b. Case 1: Simulated Performance Case — Plot of $1/c_0(p)(p_r-p)$ versus $G_\rho$ (requires estimate of $G$).

c. Case 1: Simulated Performance Case — Plot of $\omega$ versus $G_\rho$ (requires estimate of $G$).

d. Case 1: Simulated Performance Case — Plot of $\omega$ versus $G_\rho/G$ (requires estimate of $G$).
Simulated Dry Gas Reservoir Case: Abnormal Pressure:

- Summary $p/z - G_p$ plot for $\omega =$constant and $\omega =$linear cases.
- Good comparison of trends, $\omega =$linear trend appears slightly conservative as it emerges from data trend — but both solutions appear to yield same $G$ estimate.
Gan-Blasingame Analysis (2001): Case 1

a. Case 1: Simulated Performance Case — Gan Plot 1
\( c_0(p)(p_i-p) \) versus \((p/z)/(p_i/z_i)\) (requires est. of \( G \)).

b. Case 1: Simulated Performance Case — Gan Plot 2
\((p/z)/(p_i/z_i)\) versus \((G_p/G)\) (requires est. of \( G \)).

c. Case 1: Simulated Performance Case — Gan Plot 3
\((p/z)\) versus \(G_p\) (results plot).

Gan-Blasingame Analysis:

- Approach considers the "match" of the \( c_e(p)(p_i-p) \) — \((p/z)/(p_i/z_i)\) data and "type curves."
- Assumes that both abnormal and normal pressure \( p/z \) trends exist.
- Straight-line extrapolation of the "normal" \( p/z \) trend used for \( G \).
Type Curve Approach: \( p_D - G_{pD} \) — Case 1

a. \( p_D - G_{pD} \) Type curve solution based on the \( p/z - G_p^2 \) model. \( p_D = \frac{[(p/z_1)-(p/z)]/(p/z)}{p_D} \) and \( G_{pD} = G_p/G \) — both \( p_D \) and \( p_DI \) functions are plotted.

b. Case 1: Simulated Performance Case — Type curve analysis of \( (p/z) - G_p \) data, this case is "force matched" to the same results as all of the other plotting functions.
Plotting Functions: $p/z - G_p^2$ — Case 3 (Anderson L)

\[ \Delta(p/z) = \left[ \frac{p_i - p}{z_i - z} \right] \text{vs. } G_p \]

\[ \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) \, dG_p \text{ vs. } G_p \]

\[ \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) \, dG_p \text{ vs. } G_p \]

\[ \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z) \, dG_p \text{ vs. } G_p \]
Plotting Functions:  $\omega - G_p$ — Case 3 (Anderson L) (2/5)

a. Case 3: Anderson L (South Texas) — Plot of $c_0(p)(p_r-p)$ versus $G_p$ (requires estimate of $G$).

b. Case 3: Anderson L (South Texas) — Plot of $1/c_0(p)(p_r-p)$ versus $G_p$ (requires estimate of $G$).

c. Case 3: Anderson L (South Texas) — Plot of $\omega$ versus $G_p$ (requires estimate of $G$).

d. Case 3: Anderson L (South Texas) — Plot of $\omega$ versus $G_p/G$ (requires estimate of $G$).
Case 3: Anderson L Reservoir (South Texas (USA))

- Summary $p/z - G_p$ plot for $\omega =$ constant and $\omega =$ linear cases.
- Good comparison, $\omega =$ constant and $\omega =$ linear cases in good agreement.
- Data trend is very consistent.
Gan-Blasingame Analysis (2001): Case 3 (Anderson L) (4/5)

a. Case 3: Anderson L Reservoir — Gan Plot 1 $c_e(p)(p_i-p)$ versus $(p/z)/(p_i/z_i)$ (requires est. of $G$).

b. Case 3: Anderson L Reservoir — Gan Plot 2 $(p/z)/(p_i/z_i)$ versus $(G_p/G)$ (requires est. of $G$).

- Gan-Blasingame Analysis:
  - We note an excellent "match" of the $c_e(p)(p_i-p) — (p/z)/(p_i/z_i)$ data and the "type curves."
  - Both the abnormal and normal pressure $p/z$ trends appear accurate and consistent.
  - Straight-line extrapolation of the "normal" $p/z$ trend used for $G$. 

Natural Gas Engineering | 26 May - 30 May 2014 (U. Kavala/GREECE)  Tom BLASINGAME | t-blasingame@tamu.edu | Texas A&M U.
Type Curve Approach: \( p_D - G_{pD} \) — Case 3 (Anderson L)

- **Case 3: Anderson L Reservoir (South Texas (USA))**
  - \( p_D - G_{pD} \) type curve solution matched using field data.
  - Note the "tail" in the \( p_D \) trend for small values of \( G_{pD} \) (common data event).
  - "Force matched" to the same results as the other plotting functions.
Example Analysis Using MS Excel: Case 3

- Case 3 — Anderson L (South Texas (USA))
  - Literature standard case.
  - A 3-well reservoir, delimited by faults.
  - Good quality data.
  - Evidence of overpressure from static pressure tests.

- Analysis: (Implemented using MS Excel)
  - $p/z-G_p^2$ plotting functions.
  - $\omega-G_p$ performance plots.
  - Gan analysis (2-straight line trends on a $p/z-G_p$ plot).
  - $p_D-G_{pD}$ type curve approach.
• Developed a new $p/z-G_p^2$ material balance model for the analysis of abnormally pressured gas reservoirs:

$$\frac{p}{z} \approx \frac{p_i}{z_i} \left[ 1 - \left( \frac{1}{G} - \omega \right) G_p - \frac{\omega}{G} G_p^2 \right]$$

where:

$$\omega \equiv \frac{1}{G_p} \bar{c}_e(p)(p_i - p)$$

The $\omega$-function is presumed (based on graphical comparisons) to be either constant, or approximately linear with $G_p$. For the $\omega=constant$ case, we have:

$$\frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2$$

$$\alpha \equiv \left( \frac{1}{G} - \omega \right) \frac{p_i}{z_i} \quad \beta \equiv \frac{\omega}{G} \frac{p_i}{z_i}$$
Summary:

- **Base relation:** $p/z - G_p^2$ form of the gas material balance
  \[ \frac{p}{z} \approx \frac{p_i}{z_i} - \alpha G_p - \beta G_p^2 \]
  \[ \alpha \equiv \left( \frac{1}{G} - \omega \right) \frac{p_i}{z_i} \quad \beta \equiv \frac{\omega}{G} \frac{p_i}{z_i} \]

  a. **Plotting Function 1:**
  (quadratic)
  \[ \Delta(p/z) = \left[ \frac{p_i}{z_i} - \frac{p}{z} \right] \text{ vs. } G_p \]

  b. **Plotting Function 2:**
  (linear)
  \[ \frac{1}{G_p} \Delta(p/z) \text{ vs. } G_p \]

  c. **Plotting Function 3:**
  (quadratic)
  \[ \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \text{ vs. } G_p \]

  d. **Plotting Function 4:**
  (linear)
  \[ \frac{1}{G_p^2} \int_0^{G_p} \Delta(p/z) dG_p \text{ vs. } G_p \]

  e. **Plotting Function 5:**
  (quadratic)
  \[ \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \text{ vs. } G_p \]

  f. **Plotting Function 6:**
  (linear)
  \[ \frac{1}{G_p} \left[ \Delta(p/z) - \frac{1}{G_p} \int_0^{G_p} \Delta(p/z) dG_p \right] \text{ vs. } G_p \]
Summary:

- The plotting functions developed in this work have been validated as tools for the analysis reservoir performance data from abnormally pressured gas reservoirs. Although the straight-line functions ($PF_2$, $PF_4$, and $PF_6$) could be used independently, but we recommend a combined/simultaneous analysis.

- The $\omega-G_p$ plots are useful for checking data consistency and for guiding the selection of the $\omega$-value. These plots represent a vivid and dynamic visual balance of all of the other analyses.

- The Gan analysis sequence is also useful for orienting the overall analysis — particularly the $c_e(p)(p_i-p)$ versus $(p/z)/(p_i/z_i)$ plot.

- The $p_D-G_{pD}$ type curve is useful for orientation — particularly for estimating the $\omega$ or ($\omega_D$) value.
Recommendations for Future Work:

● Consider the extension of this methodology for cases of external drive energy (e.g., water influx, gas injection, etc.).
● Continue the validation of this approach by applying the methodology to additional field cases.
● Implementation into a stand-alone software.
A Quadratic Cumulative Production Model for the Material Balance of Abnormally-Pressured Gas Reservoirs

(End of Presentation)

T.A. Blasingame, Texas A&M U.
Department of Petroleum Engineering
Texas A&M University
College Station, TX 77843-3116
+1.979.845.2292 — t-blasingame@tamu.edu