Natural Gas Engineering

*Pressure Transient Analysis*

Orientation — Pressure Transient Analysis

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Objectives of Pressure Transient Testing:
- Evaluate reservoir pressure (initial or average pressure).
- Evaluate reservoir fluid (fluid samples collected for lab study).
- Estimate reservoir properties (e.g., $k$, $S$, $x_f$, $\lambda$, $\omega$, etc.).
- Estimate reservoir volumetrics (e.g., fluid-in-place, drainage area).

Input Data:
- BOTTOMHOLE pressure data (accurate to < 1 part in 10,000 or more).
- SURFACE flowrate data (often poorly measured/recorded).
- Fluid properties (e.g., FVF, viscosity, compressibility, ...).
- Reservoir properties (e.g., $h$, $\phi$, $r_w$, $c_p$, ...)

Results of PTA Interpretation:
- Productive capacity of the WELL (damage/stimulation).
- Productive capacity of the RESERVOIR (transmissibility).
- Current average reservoir pressure.
- Reservoir limits (for production to pseudosteady-state).
- Well interference effects.
- Well/Reservoir specific parameters (e.g., $C_s$, $x_f$, $\lambda$, $\omega$, $L_{\text{fault}}$, $r_{\text{comp}}$, $k_v/k_h$, ...).
Field Example: (SPE 12777)
Data match for a case of radial flow —
wellbore storage signature is identified
using the pressure $\beta$-derivative. 
(dimensionless format — $p_{D_{\beta d}}=1$)

Field Example: (SPE 9975)
Data match for the case of a well with
an infinite conductivity vertical fracture
— formation linear flow behavior is
revealed using the $\beta$-derivative. 
(dimensionless format — $p_{D_{\beta d}}=1/2$)
Orientation: Static Data for PTA

● PVT Properties: (Lab report preferred, correlations acceptable)
  - **Black Oil**: $B_o$, $R_s$, $\mu_o$, $c_o$  
    (correlations require: $T$, $\gamma_{g,sep}$, $p_b$, $\gamma_{STO}$)
  - **Dry Gas**: $z$ (or $B_g$), $\mu_g$, $c_g$  
    (correlations require: $T$, $\gamma_{g,sep}$)
  - **Volatile Oil**: Black oil equivalent or compositional formulation.
  - **Gas Condensate**: Dry gas equivalent or compositional formulation.
  - **Water**: $B_w$, $R_{sw}$, $\mu_w$, $c_w$  
    (correlations require: $T$, $\gamma_{g,sep}$, $p_{bw}$, salinity)

● Reservoir Properties:
  - **Porosity** ($\phi$)  
    (core and/or well logs)
  - **Net pay thickness** ($h$)  
    (core and/or well logs)
  - **Wellbore radius** ($r_w$)  
    (well completion history (bit diameter))
  - **Formation Compressibility** ($c_f$)  
    ($c_f=3\times10^{-6}$ psia$^{-1}$ or correlation)

● Well Completion History:
  - **Drilling records**  
    (initial pressures, production tests)
  - **Well files**  
    (well logs, core, PVT, recompletion, workover records)
  - **Annotated production records**  
    (records of activities — very useful)
Allocated Rate Data:
- Common in mature producing environments (e.g., Texas).
- Common in some offshore operations (manifold rates).
- "Allocation" depends on records — and consistency checks.

Poor/Incomplete (or Erroneous) Pressure Data:
- Virtually all production pressure measurements taken at surface.
- Completion changes often not reflected in surface pressures.
- Some pressure data are just wrong (poor gauge, poor timing, etc.).

Well Completion Issues:
- Equipment changes, poor practices, failed equipment, etc.
- UNREPORTED activities (recompletions, workovers, treatments).

Permanent DOWNHOLE Pressure Measurements:
- Expense is justified.
- Provides continuous evaluation of well performance.
- Data volume/sampling is an issue, but not a major problem.
Discussion: Production History Plot — East TX Gas Well

- Good rate and pressure histories.
- Unique case — bottomhole and "production" pressure data.
Discussion: "Log-Log" Plot (Well Test Analysis) — East TX Gas Well

- Used high frequency bottomhole pressure measurements ($p_{ws}$).
- Consistent match of bottomhole and "production" pressure data.
Well Deliverability:

- The first efforts to analyze well performance were an attempt to quantify well potential — *not to estimate reservoir properties*.

- The original well deliverability relation was *completely empirical* (derived from observations), and is given as:

\[
q = C(\bar{p}^2 - p_{wf}^2)^n
\]

- *This relationship is rigorous for low pressure gas reservoirs, \(n=1\) for laminar flow.*
Q. Can the "gas deliverability" or "AOF" be derived?
A. Sort of, see steps below — assume \((\mu_g z)\) product is constant.

Darcy's Law:

\[
v_r = \frac{q_g B_g}{A_r} = + \frac{k}{\mu_g} \left[ \frac{dp}{dr} \right] \quad [A_r = 2\pi rh] \quad \text{or} \quad q_g = \frac{k}{\mu_g B_g} \left( 2\pi h \right) \left[ \frac{dp}{dr} \right]
\]

Separating and Integrating:

\[
\frac{q_g}{2\pi kh} \int_{r_w}^r \frac{1}{r} \, dr = \int_{p_w}^{p_e} \frac{1}{p \mu_g B_g} \, dp \quad \left[ B_g = \frac{p_{sc}}{P_{sc}} \frac{T}{z} \right]
\]

Which reduces to: \([\mu_g z] = \text{constant}\)

\[
\frac{q_g}{2\pi kh} \left[ \ln\left(\frac{r_e}{r_w}\right) \right] = \frac{T_{sc} z_{sc}}{T P_{sc} (\mu_g z)_{c}} \int_{p_w}^{p_e} \frac{1}{p} \, dp
\]

Performing the Pressure Integration:

\[
q_g = 2\pi \frac{kh}{\ln\left(\frac{r_e}{r_w}\right)} \frac{T_{sc} z_{sc}}{T P_{sc} (\mu_g z)_{c}} \frac{1}{2} \left( p_e^2 - p_w^2 \right) \rightarrow q_g = C \left( p_e^2 - p_w^2 \right)
\]

Discussion: Derivation of Well Deliverability Relation

- Actually an empirical result (see Rawlins and Schellhardt (1935)).
- Derivation from steady-state flow (above) is useful for illustration.
- Derivation for pseudosteady-state is similar (variety of results).
Discussion: Well Deliverability (4-point test)

- Probably oldest "reservoir engineering" technique.
- Assumption of pseudosteady-state flow is the weakest link in analysis.
- Does not directly relate time, rate, and pressure performance.
Our focus is the reservoir... but, we also need to consider:

- The well completion.
- The tubulars.
- The surface facilities.
- The reservoir fluid(s).

Discussion:

- Overall flow system (after Fonseca).
- Blasingame axiom: "if there is a problem with the analysis/interpretation of well test and/or production data — the issue most likely stems from the well completion."
Orientation: What Advances do We Need?

● **Pressure Transient Analysis: PTA**
  - Data processing (permanent gauges) *(obvious, but...)*.
  - Numerical modelling *(advise caution in applications)*.
  - Variable-rate analysis *(deconvolution)*.
  - Better data analysis functions *(We can always hope...)*.
  - **Continuous Measurement = Continuous Assessment**

● **Production Analysis: PA**
  - More consistent measurement of \( q \) and \( p_{wf} \).
  - Pressure conversion *(surface \( \rightarrow \) bottomhole)*.
  - Further implementation of semi-analytical solutions.
  - Diagnostic methods for defining pressure transient behavior in production data *(model identification)*.
  - **Continuous Measurement = Productivity Optimization**
Orientation: Questions to Consider

Q1. Practical applications of Pressure Transient Analysis (PTA)?
A1. Estimate/evaluate the following:
   ● Reservoir properties (e.g., $k$, $S$, $x_f$, $\lambda$, $\omega$, etc.). (rarely volume)
   ● Productivity efficiency (damage or stimulation). (direct assessment)
   ● Reservoir pressure (initial or average pressure). ($p_{avg} \rightarrow$ long shut-in)

Q2. What are the major issues or complications in PTA?
A2. Major issues/complications in PTA:
   ● Planning of pressure transient test. (always model prior to testing)
   ● Preparation of well for testing. (execution failures)
   ● Production history is not trivial. (can corrupt interpretation)
   ● Well completion — have records at hand. (leaks, tubulars, placement)
   ● WHY YOU ARE TESTING THE WELL — WHAT IS THE OBJECTIVE?

Q3. Comparison with of PTA with Production Analysis (PA)?
A3. Comparison of PTA and PA:
   ● PTA:
     — HIGH resolution/HIGH frequency (pressure) data. (quality/quantity)
     — Gives SNAPSHOT of the well performance at that time. (stress test)
   ● PA:
     — LOW resolution/LOW frequency (pressure) data. (quality/quantity)
     — LUMPS entire life of well into analysis. (passive monitoring)
Pressure Transient Analysis

Basic Concepts/Processes — Pressure Transient Analysis

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Overview: Tubular System Schematics

- Bottomhole shut-in is preferred — but requires operations, time, expense.
- Surface shut-in for gas wells is common, but must take care in evaluation and interpretation of data — data are often corrupted by wellbore effects.
Overview: Example DST — Flow/Buildup Sequences

- Early flow/shut-in sequences are used to estimate initial reservoir pressure.
- Extended flow sequence used to "sample" near-well reservoir volume.
- Extended shut-in sequence used to estimate near-well reservoir properties.

Overview: Example Semilog Drawdown Test Plot

- Schematic of the separate and combined influences of skin and WBS.
- Skin effect and wellbore storage (WBS) yield a unique combined influence.
- End of WBS effects can be difficult to distinguish (need $\Delta p'$ function).
Overview: Example Log-Log Drawdown Test Plot

- Can see effect of separate and combined influences of skin and WBS.
- $\Delta p'$ function is horizontal for infinite-acting radial flow (IARF) flow regime.
- $\Delta p$ and $\Delta p'$ functions are "unit slope" for WBS domination.

Distance is related to the SKIN EFFECT.
Overview: Example Semilog Buildup Test Plot

- Similar to drawdown test, BUT pressure buildup is NOT affected by skin.
- Reversed axis is due to "superposition" to account for rate history.
- Pressure buildup data are generally better quality than drawdown data.

### Overview: Summary Table — Flow Regimes

#### Wellbore Storage (WBS): *Universal* phenomena, easy to distinguish.

#### Infinite-Acting Radial Flow (IARF): Traditional regime ($k>0.01$ md).

#### Fractured Wells: Linear and Bi-Linear flow regimes possible.

<table>
<thead>
<tr>
<th>Flow Regime</th>
<th>Cartesian</th>
<th>$\sqrt{\Delta t}$</th>
<th>$\frac{\Delta t}{\Delta x}$</th>
<th>Log-log</th>
<th>Semilog</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wellbore storage</td>
<td>Straight line</td>
<td>Slope $\rightarrow$ $C$</td>
<td>Intercept $\rightarrow$ $\Delta G$</td>
<td>Unit slope on $\Delta p$ and $p'$ $\Delta p$ and $p'$ coincide</td>
<td>Positive $s$ $\Delta p$ and $p'$ coincide</td>
</tr>
<tr>
<td>Linear flow</td>
<td>Straight line</td>
<td>Slope $= m_f+L_f$</td>
<td>Intercept $\rightarrow$ fracture damage</td>
<td>$\frac{1}{2}$ on $p'$ and on $\Delta p$ if $s=0$ $\Delta p&lt;\frac{1}{2}$ on $p'$ if $s \neq 0$ $p'$ at half the level of $\Delta p$</td>
<td>$\Delta p_{fracture}^s$</td>
</tr>
<tr>
<td>Bilinear flow</td>
<td>Straight line</td>
<td>Slope $= m_f+C_{id}$</td>
<td>$\frac{1}{4}$</td>
<td>$p'$ at $\frac{1}{4}$ level of $\Delta p$</td>
<td>$\Delta p_{fracture}^s$</td>
</tr>
<tr>
<td>First IARF (high-$k$ layer, fractures)</td>
<td>Decreasing slope</td>
<td>$p'$ horizontal at $p'_D = 0.5$</td>
<td>Straight line</td>
<td>Slope $= m \rightarrow kh$ $\Delta p_{fracture}^s$</td>
<td>$\Delta p_{fracture}^s$</td>
</tr>
<tr>
<td>Transition</td>
<td>More decreasing slope</td>
<td>$\Delta p = \lambda e^{-2s}$ or $B'$</td>
<td>$p'_D = 0.25$ (transition) $&lt; 0.25$ (pseudosteady state)</td>
<td>Straight line</td>
<td>Slope $= m_2$ (transition) $= 0$ (pseudosteady state)</td>
</tr>
<tr>
<td>Second IARF (total system)</td>
<td>Similar slope to first IARF</td>
<td>$p'$ horizontal at $p'_D = 0.5$</td>
<td>Straight line</td>
<td>Slope $= m \rightarrow kh_p$ $\Delta p_{fracture}^s$</td>
<td>$\Delta p_{fracture}^s$</td>
</tr>
<tr>
<td>Single no-flow boundary</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outer no-flow boundaries (drawdown tests only)</td>
<td>Straight line</td>
<td>Slope $= m \rightarrow \phi A_h$</td>
<td>$p_{ref} = C_A$</td>
<td>Unit slope for $\Delta p$ and $p'$</td>
<td>Increasing slope</td>
</tr>
</tbody>
</table>

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Overview: Properties Obtained from PTA

- **Drawdown Tests (DD):** Fine in theory, difficult in practice (rates!).
- **Buildup Tests (BU):** Most common PTA, several advantages (rates, skin, ...).
- **Falloff Tests (FO):** Same as buildup tests, for injection wells.

Overview: Common Plots/Flow Regimes

- **Wellbore Storage (WBS):** "Unit Slope" trend (straight line on log-log plot).
- **Infinite-Acting Radial Flow (IARF):** Semilog plot relation.
- **Formation Linear Flow (FLF):** Fractured wells, high conductivity fracture.

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## Overview: PTA — Summary of Diagnostics

### Pressure Transient Analysis — Summary of Diagnostics

<table>
<thead>
<tr>
<th>Model</th>
<th>Unfractured well in an infinite-acting homogeneous reservoir</th>
<th>Unfractured well in a bounded homogeneous reservoir</th>
<th>Unfractured well in an infinite-acting homogeneous reservoir with various sealing faults</th>
<th>Fractured well in an infinite-acting homogeneous reservoir</th>
<th>Horizontal well in an infinite-acting homogeneous (isotropic) reservoir</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log(p_D) )</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
<td><img src="image5" alt="Graph" /></td>
</tr>
<tr>
<td>( \log(p_Dt) )</td>
<td><img src="image6" alt="Graph" /></td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
<tr>
<td>( \log(p_D\beta t) )</td>
<td><img src="image11" alt="Graph" /></td>
<td><img src="image12" alt="Graph" /></td>
<td><img src="image13" alt="Graph" /></td>
<td><img src="image14" alt="Graph" /></td>
<td><img src="image15" alt="Graph" /></td>
</tr>
</tbody>
</table>

### Discussion: Pressure Transient Analysis — Summary of Diagnostics

- \( \Delta p \) function (pressure drop)?
- \( \Delta p_d \) function (Bourdet derivative)?
- \( \Delta p_{\beta d} \) function (\( \beta \)-derivative)?

Overview: Questions to Consider

Q1. What are the practical challenges for PTA?
A1. The practical challenges for PTA include:
   ● Identification of appropriate reservoir model. (interpretation)
   ● Distinguishing "artifacts" from data. (bad rate history, etc.)
   ● Estimation of reservoir properties. (data quality/quantity)

Q2. What are the diagnostics for PTA?
A2. The diagnostics for PTA include:
   ● "Conventional" Plots: Log-log, semilog, Cartesian, root-time, etc.
   ● "Diagnostic" Plots: Derivative, Integral, etc. functions
   ● Probably the best approach is to rely mostly on the derivative for diagnostics, and the "conventional" plots for validation.
Pressure Transient Analysis
Pressure-Distance Plots — Pressure Transient Analysis

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Pressure Distributions: Solutions

All relations given in FIELD units.

Steady-State Solution:

\[ p_r = p_w + 141.2 \frac{q_{sc} B \mu}{kh} \ln(r/r_w) \quad [p_r \rightarrow p_{wf} \text{ form}] \]

\[ p_r = p_e - 141.2 \frac{q_{sc} B \mu}{kh} \ln(r_e/r) \quad [p_r \rightarrow p_e \text{ form}] \]

Full Solution:

(q_{sc} = \text{constant})

\[ p_D = \frac{1}{141.2 q B \mu} (p_i - p_r) \]

\[ \approx \frac{1}{2} E_1 \left[ \frac{r_D^2}{4t_D} \right] - \frac{1}{2} E_1 \left[ \frac{r_{eD}^2}{4t_D} \right] + 2 \frac{t_D}{r_{eD}^2} \exp \left[ -\frac{r_{eD}^2}{4t_D} \right] + \left[ \frac{r_D^2}{2r_{eD}^2} - \frac{1}{4} \right] \exp \left[ -\frac{r_{eD}^2}{4t_D} \right] \]

Radius of Investigation:

\[ r_{inv} = 2.434 \times 10^{-2} \sqrt{\frac{k}{\phi \mu c_t} t} \]
Pressure Distributions: Transient Flow

- Note the effect of the drawdown.
- Note that the buildup pressure trends retrace last drawdown trend.
- Recall that all measurements are at the wellbore, we cannot "see" in the reservoir — our analyses are inferred from wellbore measurements.
Pressure Distributions: Pseudosteady-State

The physical concept of the PSEUDOSTEADY-STATE FLOW condition is defined as the condition where the pressure at all points in the reservoir changes at the same rate. Mathematically, this condition is given by:

$$\frac{d}{dt} [p(r,t)]_r = \text{constant}$$
Pressure Distributions: Pseudosteady-State

**Concept:** (pressure changes at the same rate at all points in the reservoir)

\[
\left[ \frac{dp}{dr} \right]_r = \text{constant}
\]

**Reservoir Pressure Schematic:**

[Graph showing pressure response for a single well in a closed circular reservoir produced at a constant flowrate (liquid case). The graph has a logarithmic scale on the x-axis for radial distance and a linear scale on the y-axis for reservoir pressure. Key points marked include:
- \( p_j = 4800 \text{ psia} \)
- \( t = 1.77 \text{ hr} \)
- \( t = 291.7 \text{ hr} \)
- \( t = 1298 \text{ hr} \)
- \( t = 5759 \text{ hr} \)
- \( t = 12,125 \text{ hr} \) for transient flow and pseudosteady-state flow.]
Pseudosteady-State Flow: Summary of Relations

\((p_r - p_{wf})\) Flow Relations: (Circular Reservoir)

\[
p_r - p_{wf} = 141.2 \frac{q B \mu}{k h} \left[ \frac{r_e^2}{(r_e^2 - r_w^2)} \ln \left( \frac{r}{r_w} \right) - \frac{1}{2} \frac{(r_e^2 - r_w^2)}{(r_e^2 - r_w^2)} + s \right]
\]

\((\overline{p} - p_{wf})\) Flow Relations: (\(\gamma = 0.577216\) Euler's constant)

\[
\overline{p} = p_{wf} + 141.2 \frac{q B \mu}{k h} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right] \quad \text{(Circular Reservoir)}
\]

\[
\overline{p} = p_{wf} + 141.2 \frac{q B \mu}{k h} \left[ \frac{1}{2} \ln \left( \frac{4}{e^\gamma} \frac{A}{r_w^2} \frac{1}{C_A} \right) + s \right] \quad \text{(General Formulation)}
\]

Time-Dependent Pseudosteady-State Flow Relations:

\[
p_r = p_i - 141.2 \frac{q B \mu}{k h} \left[ \ln \left( \frac{r_e}{r} \right) + \frac{1}{2} \frac{(r_e^2 - r_w^2)}{(r_e^2 - r_w^2)} - \frac{3}{4} \right] - 5.615 \frac{q B}{V_p c_t} t
\]

\[
p_{wf} = p_i - 141.2 \frac{q B \mu}{k h} \left[ \ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4} + s \right] - 5.615 \frac{q B}{V_p c_t} t
\]
Figure 2: Reservoir Pressure Distribution — Constant Rate Transient Flow Drawdown.

\[ r_{inv} = 2.434 \times 10^2 \sqrt{\frac{k}{\phi \mu c_t}} t \]
Pseudosteady-State Flow: *Illustrative Behavior*

Figure 4: Reservoir Pressure Distribution — Log Linear Rate Transient Flow Drawdown.

Pseudosteady-State Flow: *Illustrative Behavior*

Figure 7: Reservoir Pressure Distribution — Constant Wellbore Pressure Transient Flow Drawdown.

The figure illustrates the pressure distribution in a reservoir during transient flow, with constant wellbore pressure. The pressure distribution is shown for different times and radii, with the following key points:

- **(1)** $t_1 = 1.77$ hr, $r_1 = 32.2$ ft
- **(2)** $t_2 = 13.3$ hr, $r_2 = 88.4$ ft
- **(3)** $t_3 = 64.6$ hr, $r_3 = 195$ ft
- **(4)** $t_4 = 291.7$ hr

The pressure distribution is given by the equation:

$$ r_{inv} = 2.434 \times 10^{-2} \sqrt{\frac{k}{\phi \mu c_t}} t $$

where $r_{inv}$ is the radius at which the pressure starts to decrease due to flow, $k$ is the permeability, $\phi$ is the porosity, $\mu$ is the viscosity, and $c_t$ is the compressibility factor.

Pseudosteady-State Flow: *Illustrative Behavior*

**Figure 52:** Reservoir Pressure Distribution — Constant Rate Post-Transient Flow Drawdown, Homogeneous Reservoirs.

Pseudosteady-State Flow: *Illustrative Behavior*

Figure 57: Reservoir Pressure Distribution — Constant Wellbore Pressure Post-Transient Flow Drawdown, Homogeneous Reservoirs.

- \( t_1 = 1298 \text{ hr} \)
- \( t_2 = 5759 \text{ hr} \)
- \( t_3 = 12125 \text{ hr} \)

Reservoir Pressure Distribution During Constant Wellbore Pressure Post-Transient Flow Drawdown, Homogeneous Reservoirs.

Q1. Why study "reservoir pressure trends?"
A1. We can not measure pressure in the reservoir — only at the wellbore (or sandface). In order to estimate the behavior in the reservoir, we must use "model-based" pressure distributions.

Q2. Isn't the use of a simple model too limiting?
A2. Actually, no. Simple models are extremely consistent, and as such, even when "wrong," the "trend" behavior is typically quite representative.

Q3. What is the "radius of investigation?"
A3. For the infinite-acting radial flow case, the radius of investigation is the point in the reservoir where the logarithm of radius equation (straight line) intersects the initial reservoir pressure. It is a fictitious point, but it represents the "theoretical" location of the front of the pressure distribution front.

\[ r_{inv} = 2.434 \times 10^{-2} \sqrt{\frac{k}{\phi \mu c_t}} \]
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Pressure Transient Analysis

Basic Analysis Plots — Pressure Transient Analysis

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Wellbore Storage: Base Relations/Concepts

\[(q_{sf} - q)B = 24C_s \left[ \frac{dp_{wf}}{dt} - \frac{dp_{tf}}{dt} \right] \]  
(field units form)

(Definition of \( C_s \) for a fluid filled wellbore)

\[ C_s = c_w V_{wb} \]

(Definition of \( C_s \) for a well with a rising or falling liquid level)

\[ C_s = \frac{144}{5.615} \frac{A_{wb}}{\rho (g / g_c)} \]
Wellbore Storage: Base (Cartesian) Plots

(Drawdown Case)

\[ p_{wf} = p_i - \frac{qB}{24C_S} t \]

(Buildup Case)

\[ p_{ws} = p_{wf} (\Delta t = 0) + \frac{qB}{24C_S} t \]
Wellbore Storage: Approximate Solution

**Dimensionless Pressure Relation:** Constant Approximation for $p_{SD}(t_D)$

$$p_{wCD}(t_D) = p_{SD}(t_D) \left[ 1 - \exp \left( \frac{-t_D}{p_{SD}(t_D) C_D} \right) \right]$$
Wellbore Storage: "Bourdet-Gringarten" Type Curve

- **Legend:**
  - Radial Flow Type Curves
    - $p_D$ Type Curve
    - $p'_D$ Type Curve

- **Type Curve for an Unfractured Well in an Infinite-Acting Homogeneous Reservoir with Wellbore Storage and Skin Effects**

- **Wellbore Storage Domination Region**
  - $p'_D = 1/2$

- **Radial Flow Region**
  - $p_D = 1/2$, $p_{Drt} = p_D$

- **Wellbore Storage Distortion Region**

- **Parameters and Units**
  - $C_D$ = Constant
  - $p_D$, $p'_D$, $p_{Drt}$ = Pressure Parameters
Wellbore Storage: *Cartesian Plot*

**Regression Equations:**

Wellbore Storage Equation:

\[ p_{WS}(\Delta t) = 3534 + 973.33 \Delta t \]

**Legend:** Lee Text Example 2.2

- Pressure Data

**Data for Lee Example 2.2:**

Reservoir Properties:
- \( c_f = 17.0 \times 10^6 \) psia\(^{-1} \)
- \( r_w = 0.198 \) ft
- \( h = 69 \) ft
- \( \phi = 0.039 \) (fraction)
- \( k = 7.85 \) md
- \( s = 5.79 \)

Oil Properties:
- \( B_o = 1.136 \) RB/STB
- \( \mu_o = 0.8 \) cp

Production Parameters:
- \( q_o = 250 \) STB/D
- \( t_f = 13,630 \) hrs
- \( p_{WF}(\Delta t = 0) = 3534 \) psia

**Linear Portion Indicates Wellbore Storage Domination**

\[
p_{WS}(\Delta t = 0) = 3534 \text{ psia}
\]

\[
\begin{align*}
p_{WS} &= p_{WF}(\Delta t = 0) + m_{WS} \Delta t \\
p_{WS} &= 3534 + 973.33 \Delta t
\end{align*}
\]
Wellbore Storage: Log-Log Plot

\[ \Delta p = p_{WS} - p_{wf} (\Delta t = 0) = +m_{WBSS} \Delta t \]
Infinite-Acting Radial Flow (IARF): Semilog Plot

\[ p_{WS} - p_{wf}(\Delta t=0) = 162.6 \frac{qB\mu}{kh} \log(\Delta t) + 162.6 \frac{qB\mu}{kh} \left[ \log \left( \frac{k}{\phi \mu c r_w^2} \right) - 3.2275 + 0.8686s \right] \]

Semilog Plot -- Lee Text Example 2.2
(Summary of Average Reservoir Pressure Methods)

Regression Equations:
- RHM Equation: Regression Method
  \[ P_{WS} = 4422.8 - 1225.3(8.671 + \Delta t) \]
- Muskat Equations: Regression Method
  \[ P_{WS} = 4408.5 - 83.798 \exp(-0.052809\Delta t) \]
  \[ P_{WS} = 4408.9 - 79.331 \exp(-0.050129\Delta t) \]
  \[ - 58.845 \exp(-0.28359\Delta t) \]

Data for Lee Example 2.2:
- Reservoir Properties:
  \( c_f = 17.0 \times 10^{-6} \) psia
  \( r_w = 0.198 \) ft
  \( h_b = 69 \) ft
  \( \phi = 0.039 \) (fraction)
  \( k = 7.65 \) md
  \( s = 5.79 \)
- Oil Properties:
  \( B_o = 1.136 \) RB/STB
  \( \mu_o = 0.8 \) cp

Average Pressure Estimates:
- \( P_{avg} \) Muskat (1 term) = 4408.5 psia
- \( P_{avg} \) Muskat (2 term) = 4408.9 psia
- \( P_{avg} \) RHM (reg) = 4422.8 psia
- \( P_{avg} \) MBH (Lee text) = 4411.0 psia

\[ P_{WS} = P_{wf,1hr} + m_{sl} \log(\Delta t) \]
Infinite-Acting Radial Flow (IARF): Log-Log Plot

\[ \Delta p' = \Delta t \frac{dp_{WS}}{d\Delta t} \] and for radial flow only, \( (\Delta p')_{rf} = \frac{1}{\ln(10)} m_{sl} \)
Average Reservoir Pressure: *Cartesian Plot*

**Muskat Plotting Function Approach**

*Lee Text Example 2.2*

**Muskat Equations: (Time Format)**

1-Term Muskat Equation:

\[ p_{WS} = 4408.5 - 83.798 \exp(-0.052809\Delta t) \]

2-Term Muskat Equation:

\[ p_{WS} = 4408.9 - 79.331 \exp(-0.050129\Delta t) - 58.845 \exp(-0.28359\Delta t) \]

**Data for Lee Example 2.2:**

- **Reservoir Properties:**
  - \( c = 17.0 \times 10^{-6} \text{ psia}^{-1} \)
  - \( r_w = 0.198 \text{ ft} \)
  - \( h = 69 \text{ ft} \)
  - \( \phi = 0.039 \) (fraction)
  - \( k = 7.65 \text{ md} \)
  - \( s = 5.79 \)

- **Oil Properties:**
  - \( B_o = 1.136 \text{ RB/STB} \)
  - \( \mu_o = 0.8 \text{ cp} \)

- **Production Parameters:**
  - \( q_o = 250 \text{ STB/D} \)
  - \( t_o = 13,630 \text{ hrs} \)
  - \( p_{WS(\Delta t=0)} = 3534 \text{ psia} \)

**Symbol Derivative**

- \( L = 0.05 \)
- \( L = 0.10 \)
- \( L = 0.15 \)
- Moving Least Squares

**Muskat Straight Line: \( dp_{WS}/d\Delta t \) Format**

\[ p_{WS} = 4408.5 - 20 \ dp_{WS}/d\Delta t \]

\[ p_{WS} = \bar{p} - a \exp\left[-b\Delta t\right] \text{ and } p_{WS} = \bar{p} - \frac{1}{b} \frac{d}{d\Delta t}\left[p_{WS}\right] \]
**Conventional PTA Plots: Questions to Consider**

**Q1. What are the "conventional" PTA plots?**

**A1. Listing of plots:**
- Log-log (diagnostic) plot ($\Delta p_w$ and $\Delta p_w'$) *(reservoir boundaries, WBS, $k$)*
- Semilog plot ($p_w$ vs. log$[t]$) *(k,s)*
- Early-time Cartesian plot ($p_w$ vs. $t$) *(WBS)*
- Late-time Cartesian plot ($\Delta p_w$ vs. $d\Delta p_w/dt$) *(average reservoir pressure)*

**Q2. Strengths of "conventional" PTA plots?**

**A2. Sampling:**
- Observation of straight-line or constant behavior.
- Simplified "flow regime" relations that can be used to estimate reservoir and/or well properties.

**Q3. Weaknesses of "conventional" PTA plots?**

**A3. Sampling:**
- Can observe "artifact" regimes (e.g., linear flow) that are not real.
- Observe only part of the overall behavior in time, can be limiting in a diagnostic sense.
Pressure Transient Analysis
Reservoir Models — Pressure Transient Analysis

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Orientation: *PTA Model-based Analysis*

- **Reservoir Models:**
  - Unfractured Well
  - Fractured Well
  - Naturally Fractured Reservoir

- **Type Curve Library:**
  - Unfractured Well: \( WBS + IARF \) \("Bourdet-Gringarten"\)
  - Pressure Buildup in a Rectangle (Unfractured Well) \("Ansah"\)
  - Linear (Sealing) Reservoir Boundaries \("Stewart"\)
  - Fractured Well: no \( WBS \) \("Cinco and Samaniego"\)
  - Fractured Well: \( WBS \) \("Economides"\)
  - Naturally Fractured Reservoir: Unfractured well \("Angel"\)

- **Field Examples:** (from SPE 103204)
  - Unfractured oil wells.
  - Hydraulically fractured gas wells.
  - Hydraulically fractured water injection wells.
Unfractured Well: Flow Regimes

Discussion: Flow Regimes (Unfractured Wells)

- **INFINITE-ACTING RADIAL FLOW (IARF)** is the most "popular" regime.
- **PSEUDOSTEADY-STATE (PSS)** flow → CLOSED BOUNDARIES.
- **STEADY-STATE (SS)** flow → CONSTANT PRESSURE (not realistic).

Schematic drawing of geometry and boundary conditions for radial flow, constant-rate cases.
Discussion: Orientation and Solutions (Unfractured Wells)

- Pressure profile propagates radially away from well (homogeneous).
- Cylindrical source solution → finite wellbore.
- Line source solution → infinitesimal wellbore (i.e., a line).

Dimensionless pressure function at various dimensionless distances from a well located in an infinite system.

Calculated pressure response for a well in an infinite-acting system.
**Discussion: Skin Factor Concept (Unfractured Wells)**

- Finite skin concept → zone of "altered" permeability near the well.
- Infinitesimal skin concept → mathematical convenience.
- Negative skin has mathematical (and physical) limitations.
**Discussion: Flow Regimes**

- **FORMATION LINEAR** flow **DOES NOT EXIST** (a few seconds at most).
- **FORMATION** linear flow $\rightarrow$ High fracture conductivity.
- **BILINEAR** flow $\rightarrow$ Low fracture conductivity.
Discussion: Fracture Flux Distributions

- Discretized fracture must be solved numerically.
- High-conductivity fracture → flux distribution IS NOT significant at well.
- Finite-conductivity fracture → flux distribution IS significant at well.
Fractured Well: Fracture Damage Comparison

Discussion: Fracture Damage Comparison

- Argument: Finite conductivity can be modeled as damage...  \((false!\)\)
- "Fluid loss" damage is no referred to as "fracture face" skin.  \((not \ correct\)\)
- "Choked fracture" damage is just a constant skin factor.
Fractured Well: Analytical Solution (Uniform Flux)

General (Uniform Flux) Solution: (Infinite Conductivity Solution; $x_D \approx 0.732$)

$$p_D(t_{Dxf},|x_D|<1) = \frac{\sqrt{\pi} t_{Dxf}}{2} \left[ \text{erf} \left( \frac{1-x_D}{\sqrt{2 t_{Dxf}}} \right) + \text{erf} \left( \frac{1+x_D}{\sqrt{2 t_{Dxf}}} \right) \right]$$

$$+ \frac{(1-x_D)}{4} E_1 \left( \frac{(1-x_D)^2}{4 t_{Dxf}} \right) + \frac{(1+x_D)}{4} E_1 \left( \frac{(1+x_D)^2}{4 t_{Dxf}} \right)$$

Short-Time Solution: Linear Flow

$$p_{wD}(t_{Dxf}) = \sqrt{\pi} t_{Dxf}$$

Long-Time Solution: Pseudoradial Flow (Infinite Conductivity Fracture)

$$p_{wD}(t_{Dxf}) = \frac{1}{2} [\ln(t_{Dxf}) + 2.20000]$$ (pseudoradial flow: <1% error, $t_{Dxf} > 10$)

Identities:

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) \, dt$$

$$[\text{erf}(0) = 0; \text{erf}(\infty) = 1; \text{erf}(-\infty) = -1]$$

$$E_1(z) = \int_z^{\infty} \frac{e^{-t}}{t} \, dt$$

$$[E_1(z < 0.01) \approx \ln \left( \frac{1}{ze^{\gamma}} \right) ; E_1(\infty) = 0]$$

($\gamma = 0.577216...$ Euler's constant)
Fractured Well: Finite- Conductivity Type Curve

Discussion: Finite-Conductivity Type Curve

- **FORMATION LINEAR** flow → High fracture conductivity.
- **BILINEAR** flow → Low fracture conductivity.
- Linear and bilinear flow end due to "tip effect" — flow around fracture tip.
Fractured Well: Skin Factor Correlation

**Discussion: Skin Factor Correlation**

- Developed to relate *Pseudoradial* flow skin factor and fracture cases.
- Useful in 1980's to generate a skin factor that managers could understand for cases of fractured wells (still used for that purpose ...).
Naturally Fractured Reservoirs: Fracture Patterns

Fracture patterns are due to stress orientation.
- Large-scale fractures can yield tremendous productivity.
- Stress state changes during production (depletion) — re-fracture?

Discussion: Fracture Patterns

a. Natural fracture dependence on stress state and orientation.

b. Schematic of compressional and tensional fracturing in-situ.


Discussion: Fracture Models

- Kazemi initially produced "slab" model using numerical simulator.
- De Swaan developed the solution for transient interporosity flow.
- Najurieta developed Laplace domain form of De Swaan result.
Discussion: Warren and Root Model

- "Borrowed" (i.e., stolen) from Barenblatt and Zheltov.
- By far the most popular "heterogeneous" reservoir model.
- Some physical limitations, but its simplicity provides unique flexibility.
Laplace Domain Solution:

$$\bar{p}_D(u, r_D, \omega, \lambda, s) = \frac{1}{u} K_0(\sqrt{uf(u)}r_D) + \frac{s}{u} \quad \text{(Line Source Solution)}$$

$$\approx \frac{1}{2u} \ln \left[ \frac{4}{e^\gamma} \frac{1}{r_D^2} \frac{1}{uf(u)} \right] + \frac{s}{u} \quad \text{("Log" Approximation)}$$

$$f(u) = \frac{\lambda + \omega(1-\omega)u}{\lambda + (1-\omega)u}$$

Real Domain Solution: (Derived from the Log Approximation Solution)

$$p_D(t_D, r_D, \omega, \lambda, s) \approx \frac{1}{2} \ln \left[ \frac{4}{e^\gamma} \frac{t_D}{r_D^2} \right] - \frac{1}{2} E_1 \left[ \frac{\lambda}{\omega(1-\omega)} t_D \right] + \frac{1}{2} E_1 \left[ \frac{\lambda}{(1-\omega)} t_D \right] + s$$

$$p'D(t_D, r_D, \omega, \lambda, s) \approx \frac{1}{2} + \frac{1}{2} \exp \left[ \frac{-\lambda}{\omega(1-\omega)} t_D \right] - \frac{1}{2} \exp \left[ \frac{-\lambda}{(1-\omega)} t_D \right]$$
Reservoir Models: Questions to Consider

Q1. What are the "traditional" reservoir models?
A1. Listing:

- Infinite-Acting Radial Flow (IARF) model (unfractured well)
- Vertically Fractured Wells:
  - Infinite-Conductivity Vertical Fracture
  - Finite-Conductivity Vertical Fracture
- Naturally-Fractured/Dual Porosity Reservoirs:
  - Pseudosteady-State Interporosity Flow (Warren and Root)
  - Transient Interporosity Flow (Kazemi-De Swaan-Najurieta)
- Horizontal Wells (problematic, requires interactive model (computer))
Pressure Transient Analysis
Type Curves — Pressure Transient Analysis

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Type Curves: \( WBS + IARF \) ("Bourdet-Gringarten")

- "Starting point" for virtually all pressure transient test analysis.
- \( \Delta p' \): \( WBS \) domination = "unit slope;" Infinite-acting radial flow (IARF) = 1/2.
**Type Curves: Late-Time Buildup ("Ansah")**

- A "correlation" of late-time cases of pressure buildup in a rectangle.
- Helps to distinguish the Muskat (late-time) pressure buildup behavior.

Type Curves: Sealing Faults ("Stewart")

- Solutions for sealing faults have a distinct (and unique) behavior.
- The radial composite model can be virtually indistinguishable (check geology!).
Type Curves: Fractured Well (No WBS) ("Cinco")

- Distinctive flow regimes — pressure and derivative function diagnostics.
- Somewhat impractical (no wellbore storage effects).

Type Curve for a Well with a Finite Conductivity Vertical Fractured in an Infinite-Acting Homogeneous Reservoir (No Wellbore Storage) ($C_{fD} = (w/k_f)/(k_f x_f) = 0.25, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, 10000$)

Legend:
- $p_D$ Solution
- $p_{Dd}$ Solution
- $p_{D_{fD}}$ Solution

Type Curves: Fractured Well (No WBS) ("Cinco") (β-Derivative Formulation)

- $p_{Dβd}$ is a very strong diagnostic (linear and bi-linear flow).
- $p_{Dβd}$ is less distinctive for infinite-acting radial flow, but still useful.

**Type Curves: Fractured Well (WBS) ("Economides") ($C_{fD}=1$)**

Type curve for a well with finite conductivity vertical fracture in an infinite-acting homogeneous reservoir with wellbore storage effects $C_{fD} = (\frac{w k_f}{k x_f}) = 1$

Legend: $C_{fD} = \frac{(w k_f)}{(k x_f)} = 1$

- $p_D$: Solution
- $p_{Dd}$: Solution
- $p_{D/d}$: Solution

- **Radial Flow Region**
- **Wellbore Storage Distortion Region**
- **Wellbore Storage Domination Region**

**Type Curve: Fractured Well (WBS) ("Economides") ($C_{fD}=1$) [$C_{fD}=F_{cD}$]**

- $C_{fD}=1$: VERY LOW fracture conductivity (similar to damage).
- Very strong bi-linear flow signature.

Type Curves: Fractured Well (WBS) ("Economides") ($C_{fd}=10$)

Type Curve for a Well with Finite Conductivity Vertical Fracture in an Infinite-Acting Homogeneous Reservoir with Wellbore Storage Effects $C_{fd} = (w/k_f)/(k_x) = 10$

Legend: $C_{fd} = (w/k_f)/(k_x) = 10$
- $p_D$ Solution
- $p_{Dd}$ Solution
- $p_{D/d}$ Solution

Wellbore Storage Domination Region

Radial Flow Region

Wellbore Storage Distortion Region

Type Curve: Fractured Well (WBS) ("Economides") ($C_{fd}=10$) [$C_{fd}=F_{cd}$]
- $C_{fd}=10$: MEDIUM fracture conductivity.
- Pressure drop and pressure derivative signatures vary (linear and bi-linear flow).
Type Curves: Fractured Well (WBS) ("Economides") ($C_{fD}=10^3$)

Type Curve for a Well with Finite Conductivity Vertical Fracture in an Infinite-Acting Homogeneous Reservoir with Wellbore Storage Effects $C_{fD} = (wk_f)/(kx_f) = 100$

Legend: $C_{fD} = (wk_f)/(kx_f) = 100$
- $p_D$: Solution
- $p_{Dd}$: Solution
- $p_{D/d}$: Solution

$C_{fD}=1\times10^3$: VERY HIGH fracture conductivity (infinite fracture conductivity).
- Very strong (formation) linear flow signature.

Type Curves: Naturally Fractured Reservoir (No WBS)

"Onur, Satman, and Reynolds" Type Curve: $p_{wD}'$ vs. $t_D \lambda/(1-\omega)$—Various $\lambda$ and $\omega$ Values

**Pseudosteady-State Interporosity Flow**

- "Fracture" System Radial Flow Region, $p_D' = 1/2$
- "Total" System Radial Flow Region, $p_D' = 1/2$
- $\omega = 5 \times 10^{-1}$
- $\omega = 2 \times 10^{-1}$
- $\omega = 1 \times 10^{-3}$
- $\omega = 2 \times 10^{-3}$
- $\omega = 5 \times 10^{-3}$
- $\omega = 1 \times 10^{-2}$

Type Curve for an Unfractured Well in an Infinite-Acting Naturally-Fractured Reservoir with NO Wellbore Storage

Type Curve: Naturally Fractured Reservoir (No Wellbore Storage)

- Pseudosteady-state "interporosity" flow case.
- This is the "cubes" or "Warren and Root" model.

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Type Curves: Naturally Fractured Reservoir (No WBS)

"Onur, Satman, and Reynolds" Type Curve: $p_{wD}'$ vs. $t_D \lambda/(1-\omega)$—Various $\lambda$ and $\omega$ Values

**Transient Interporosity Flow**

- Transient "interporosity" flow case.
- This is the "slabs" or "Kazemi" model.


Type Curve for an Unfractured Well in an Infinite-Acting Naturally-Fractured Reservoir with NO Wellbore Storage or Skin Effects — Plotting Format From: paper SPE 23830, Onur, M., and Satman, A.: "New Type Curves to Determine Naturally Fractured Reservoir Parameters"
Type Curves: Naturally Fractured Reservoir (w/WBS)

Type Curve: Naturally Fractured Reservoir (WITH Wellbore Storage)
- Pseudosteady-state "interporosity" flow case shown for emphasis.
- This is the "Angel" type curve format.

Type Curves: Questions to Consider

Q1. Any advantage of using type curves instead of interactive models?
A1. Type curves are static representations of solutions ... but, this helps the analyst to develop a visual diagnostic for each reservoir model. Both type curves and interactive models are useful for pressure transient analysis (PTA) and production analysis (PA).

Q2. Best diagnostics on type curves?
A2. Sampling:
   ● Pressure derivative function changed diagnostics (1980's).
   ● Pressure integral function(s) never caught on (1990's).
   ● $\beta$-pressure derivative identifies "power law" regimes (2006).

Q3. Advice/cautions?
A3. Sampling:
   ● Fractured wells can be extremely difficult to analyze, should have an estimate of permeability to "lock" that aspect of the analysis.
   ● The dual porosity model is often abused — i.e., used where it is not warranted.
   ● Be VERY careful with "flexible" models such as the radial composite (reservoir) model and the changing wellbore storage (wellbore) model.
Pressure Transient Analysis
Field Examples — Pressure Transient Analysis

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Field Cases: Infinite-Acting Radial Flow (IARF)

Type Curve Analysis Results — SPE 11463 (Buildup Case)
(Well in an Infinite-Acting Homogeneous Reservoir)

Legend: Radial Flow Type Curve
- $p_D$ Solution
- $p_{Dd}$ Solution
- $p_{Dβd}$ Solution

Match Results and Parameter Estimates:

- $[p_D/\Delta p]_{\text{match}} = 0.02 \text{ psi}^{-1}$, $C_D e^{2s} = 10^6$ (dim-less)
- $[(t_D/C_D)/f]_{\text{match}} = 38 \text{ hours}^{-1}$, $k = 399.481 \text{ md}$
- $C_s = 0.25 \text{ bbl/psi}$, $s = 1.91$ (dim-less)

Reservoir and Fluid Properties:
- $r_w = 0.3 \text{ ft}$, $h = 100 \text{ ft}$
- $c_t = 1.1 \times 10^{-5} \text{ psi}^{-1}$, $\phi = 0.27$ (fraction)
- $\mu_o = 1.24 \text{ cp}$, $B_o = 1.002 \text{ RB/STB}$
- $q_{\text{ref}} = 9200 \text{ STB/D}$, $p_{\text{wf}}(\Delta t=0) = 1844.65 \text{ psia}$

Discussion: Unfractured oil well (SPE 11463)

- Strong wellbore storage signature ($p_{Dβd} = 1$).
- Transition region from wellbore storage to infinite-acting radial flow.
**Field Cases: Infinite-Acting Radial Flow (IARF)**

**Type Curve Analysis — SPE 12777 (Buildup Case)**
(Well in an Infinite-Acting Homogeneous Reservoir)

**Legend: Radial Flow Type Curve**
- $p_D$ Solution
- $p_{Dd}$ Solution
- $p_{D\beta d}$ Solution

**Reservoir and Fluid Properties:**
- $r_w = 0.29$ ft, $h = 107$ ft
- $c_t = 4.2 \times 10^{-6} \text{ psi}^{-1}$, $\phi = 0.25 \text{ (fraction)}$
- $\mu_o = 2.5 \text{ cp}$, $B_o = 1.06 \text{ RB/STB}$

**Production Parameters:**
- $q_{ref} = 174 \text{ STB/D}$

**Match Results and Parameter Estimates:**
- $[p_D/\Delta p]_{match} = 0.018 \text{ psi}^{-1}$, $C_{De} = 10^{10} \text{ (dim-less)}$
- $[(tD/C_D)/f]_{match} = 15 \text{ hours}^{-1}$, $k = 10.95 \text{ md}$
- $C_s = 0.0092 \text{ bbl/psi}$, $s = 8.13 \text{ (dim-less)}$

**Discussion: Unfractured oil well (SPE 12777)**
- This result is an excellent match of all functions.
- $\beta$-derivative function is an excellent diagnostic for the wellbore storage and transition flow regimes.
**Discussion:** Unfractured oil well in dual porosity system: (SPE 13054)

- Derivative functions indicate dual porosity signature — good match.
- "Less-than-perfect" late time data match may be due to rate history effects.
Field Cases: *Dual Porosity, Infinite-Acting Radial Flow*  

**Type Curve Analysis — SPE 18160 (Buildup Case)**  
(Well in an Infinite-Acting Dual-Porosity Reservoir \( trn \) — \( \omega = 0.237, \ \alpha = 1 \times 10^{-3} \))

Legend:
- \( p_{D} \) Solution
- \( p_{Dd} \) Solution
- \( p_{D,\beta d} \) Solution
- \( p_{D} \) Data
- \( p_{Dd} \) Data
- \( p_{D,\beta d} \) Data

**Legend:** \( p_{D} \) Data

**Match Results and Parameter Estimates:**
- \( [p_{D}/\Delta p]_{\text{match}} = 0.09 \text{ psi}^{-1} \), \( C_{D} e^{2s} = 1 \) (dim-less)
- \( [(t_{D}/C_{D})/t]_{\text{match}} = 150 \text{ hours}^{-1} \), \( k = 678 \text{ md} \)
- \( C_{s} = 0.0311 \text{ bbl/psi} \), \( s = -1.93 \) (dim-less)
- \( \omega = 0.237 \) (dim-less), \( \alpha = C_{D} \times \lambda = 0.001 \) (dim-less)
- \( \lambda = 2.13 \times 10^{-8} \) (dim-less)

**Reservoir and Fluid Properties:**
- \( r_{w} = 0.29 \text{ ft} \), \( h = 7 \text{ ft} \)
- \( c_{i} = 2 \times 10^{-5} \text{ psi}^{-1} \), \( \phi = 0.05 \) (fraction)
- \( \mu_{o} = 0.3 \text{ cp} \), \( B_{o} = 1.5 \text{ RB/STB} \)
- **Production Parameters:**
  - \( q_{\text{ref}} = 830 \text{ Mscf/D} \)

**Discussion:** Unfractured oil well in dual porosity system: *(SPE 18160)*

- Strong performance of the \( \beta \)-derivative function — particularly in the region defined by transition from wellbore storage to transient interporosity flow.
Field Cases: *Hydraulically Fractured Wells*

Type Curve Analysis — SPE 9975 Well 5 (Buildup Case)
(Well with Infinite Conductivity Hydraulic Fractured)

Reservoir and Fluid Properties:
- \( r_w = 0.33 \, \text{ft} \), \( h = 30 \, \text{ft} \),
- \( c_t = 6.37 \times 10^{-5} \, \text{psi}^{-1} \), \( \phi = 0.05 \) (fraction)
- \( \mu_{gi} = 0.0297 \, \text{cp} \), \( B_{gi} = 0.5755 \, \text{RB/Mscf} \)

Production Parameters:
- \( q_{ref} = 1500 \, \text{Mscf/D} \)

Match Results and Parameter Estimates:
- \( \frac{[p_D/\Delta p]_{match}}{C_{Dr}} = 0.000021 \) psi\(^{-1} \), \( C_{Dr} = 0.01 \) (dim-less)
- \( \left[ \frac{[t_{Dxf}/C_{Dxf}]}{f_{match}} \right] = 0.15 \) hours\(^{-1} \), \( k = 0.0253 \, \text{md} \)
- \( C_{Df} = 1000 \) (dim-less), \( x_f = 279.96 \, \text{ft} \)

Discussion: Fractured gas well: buildup test (SPE 9975 — Well 5)
- Wellbore storage effects \( (p_{D\beta d} = 1) \).
- Linear flow regime could be diagnosed clearly \( (p_{D\beta d} = 1/2) \) — very good match.
**Field Cases: Hydraulically Fractured Wells**

Type Curve Analysis — SPE 9975 Well 10 (Buildup Case)
(Well with Finite Conductivity Hydraulic Fracture — $C_{Df} = 2$)

**Match Results and Parameter Estimates:**

- $[p_D/\Delta p]_{match} = 0.0012$ psi$^{-1}$, $C_{Df} = 100$ (dim-less)
- $[t_{Dxf}/C_{Df}]_{match} = 7.5$ hours$^{-1}$, $k = 0.137$ md
- $C_{Df} = 2$ (dim-less), $x_f = 0.732$ ft

**Legend:**
- $p_D$ Data
- $p_{Dd}$ Data
- $p_{D\beta d}$ Data

**Legend:**
- $p_D$ Solution
- $p_{Dd}$ Solution
- $p_{D\beta d}$ Solution

**Reservoir and Fluid Properties:**
- $r_w = 0.33$ ft, $h = 27$ ft,
- $c_t = 5.10 \times 10^{-5}$ psi$^{-1}$, $\phi = 0.057$ (fraction)
- $\mu_{gi} = 0.0317$ cp, $B_{gi} = 0.5282$ RB/Mscf

**Production Parameters:**
- $q_{ref} = 1300$ Mscf/D

**Discussion:** Fractured gas well: buildup test (SPE 9975 — Well 10)

- $p_{D\beta d} = 1$ indicates wellbore storage effect.
- The well is either poorly fracture-stimulated, or a "skin effect" has obscured any evidence of a fracture treatment.
Field Cases: *Hydraulically Fractured Wells*

**Type Curve Analysis — SPE 9975 Well 12 (Buildup Case)**
(Well with Infinite Conductivity Hydraulic Fracture)

**Reservoir and Fluid Properties:**
- \( r_w = 0.33 \text{ ft} \), \( h = 45 \text{ ft} \),
- \( c_t = 4.64 \times 10^{-4} \text{ psi}^{-1} \), \( \phi = 0.057 \) (fraction)
- \( \mu_{gi} = 0.0174 \text{ cp} \), \( B_{gi} = 1.2601 \text{ RB/Mscf} \)

**Production Parameters:**
- \( q_{ref} = 325 \text{ Mscf/D} \)

**Match Results and Parameter Estimates:**
- \( [p_D/\Delta p]_{\text{match}} = 0.0034 \text{ psi}^{-1} \), \( C_{Df} = 0.1 \) (dim-less)
- \( [(t_{Dx}/C_{Df})/t]_{\text{match}} = 37 \text{ hours}^{-1} \), \( k = 0.076 \text{ md} \)
- \( C_{ID} = 1000 \) (dim-less), \( x_f = 3.681 \text{ ft} \)

**Legend:**
- Infinite Conductivity Fracture
  - \( p_D \) Solution
  - \( p_{Dd} \) Solution
  - \( p_{D\beta d} \) Solution

**Discussion:** Fractured gas well: buildup test (SPE 9975 — Well 12)

- Wellbore storage domination regime (\( p_{D\beta d} = 1 \)).
- The \( p_{Dd} \) and \( p_{D\beta d} \) signatures during mid-to-late times confirm the well is highly stimulated.
Field Cases: 

**Hydraulically Fractured Wells**

**Type Curve Analysis — Well 207 (Pressure Falloff Case)**
(Well with Infinite Conductivity Hydraulic Fracture)

**Legend:**
- Infinite Conductivity Fracture
- $p_D$ Solution
- $p_{Dd}$ Solution
- $p_{Dβd}$ Solution

**Match Results and Parameter Estimates:**

\[
[p_D/Δp]_{match} = 0.009 \text{ psi}^{-1}, \quad C_{Df} = 0.001 \text{ (dim-less)}
\]

\[
[(t_{Dxf}/C_{Df})/t]_{match} = 150 \text{ hours}^{-1}, \quad k = 11.95 \text{ md}
\]

\[
C_{ID} = 1000 \text{ (dim-less)}, \quad x_f = 164.22 \text{ ft}
\]

**Reservoir and Fluid Properties:**
- $r_w = 0.3 \text{ ft}, \quad h = 103 \text{ ft}$
- $c_t = 7.7 \times 10^{-6} \text{ psi}^{-1}$, $ϕ = 0.11$ (fraction)
- $μ_w = 0.92 \text{ cp}$, $B_w = 1 \text{ RB/STB}$

**Production Parameters:**
- $q_{ref} = 1053 \text{ STB/D}, \quad p_{wfi(Δt=0)} = 3119.41 \text{ psia}$

**Discussion:** Fractured water injection well (Samad thesis — Well 207)

- $β$-derivative function confirms the existence of an infinite conductivity vertical fracture for this case ($p_{Dβd} = 1/2$).
**Field Cases: Hydraulically Fractured Wells**

**Type Curve Analysis — Well 5408 (Pressure Falloff Case)**
(Well with Infinite Conductivity Hydraulic Fracture)

**Legend:**
- Infinite Conductivity Fracture
- \( p_D \) Solution
- \( p_{Dd} \) Solution
- \( p_{D\beta d} \) Solution

**Reservoir and Fluid Properties:**
- \( r_w = 0.198 \) ft, \( h = 196 \) ft,
- \( c_t = 6.53 \times 10^{-6} \) psi\(^{-1}\), \( \phi = 0.18 \) (fraction)
- \( \mu_w = 0.9344 \) cp, \( B_w = 1.002 \) RB/STB

**Production Parameters:**
- \( q_{ref} = 350 \) STB/D, \( p_w(\Delta t=0) = 2518.1 \) psia

**Match Results and Parameter Estimates:**
- \( [p_D/\Delta p]_{\text{match}} = 0.0045 \) psi\(^{-1}\), \( C_{Df} = 0.1 \) (dim-less)
- \( [(t_{Dxf}/C_{Df})/t]_{\text{match}} = 3 \) hours\(^{-1}\), \( k = 1.06 \) md
- \( C_{ID} = 1000 \) (dim-less), \( x_f = 29.13 \) ft

**Discussion:** Fractured water injection well (Samad thesis — Well 5408)

- Wellbore storage domination (\( p_{D\beta d} = 1 \)) and infinite-acting radial flow (\( p_{Dd} = 1/2 \))
  — good match with infinite conductivity fracture type curve.
Field Cases: **Hydraulically Fractured Wells**

Type Curve Analysis — Well 2403 (Pressure Falloff Case)  
(Well with Infinite Conductivity Hydraulic Fracture)

Reservoir and Fluid Properties:
\[ r_w = 0.3 \text{ ft}, \quad h = 102 \text{ ft}, \]
\[ c_t = 7.21 \times 10^{-6} \text{ psi}^{-1}, \quad \phi = 0.11 \text{ (fraction)} \]
\[ \mu_w = 0.85 \text{ cp}, \quad B_w = 1.002 \text{ RB/STB} \]

Production Parameters:
\[ q_{ref} = 73 \text{ STB/D}, \quad p_{wH(t=0)} = 2630.89 \text{ psia} \]

Match Results and Parameter Estimates:
\[ \left[ p_D / \Delta p \right]_{match} = 0.18 \text{ psi}^{-1}, \quad C_{Df} = 1 \text{ (dim-less)} \]
\[ \left[ (t_{Dxf}/C_{Df})/q \right]_{match} = 2 \text{ hours}^{-1}, \quad k = 12.85 \text{ md} \]
\[ C_{ID} = 1000 \text{ (dim-less)}, \quad x_f = 50.136 \text{ ft} \]

Discussion: Fractured water injection well (**Samad thesis — Well 2403**)

- From these data we can observe the flow regimes for wellbore storage domination \( p_{D\beta_d} = 1 \), and the infinite-acting radial \( p_{Dd} = 1/2 \).
Pressure Transient Analysis
(End of Lecture)