Production Analysis
Reserves Estimation in Unconventional Reservoirs — New Rate-Time Relations

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Exponential vs. Hyperbolic Decline in Tight Gas Sands — Understanding the Origin and Implications for Reserve Estimates Using Arps' Decline Curves

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**SPE 116731: Inadequacy of Arps’ Rate Relations**

**ASSUMPTION:** The Arps decline parameter, \( b \), defines the decline behavior...

**REALITY:** Difficult to identify the correct \( b \)-parameter during the early decline period — greatly impacts reserve estimates.

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**SPE 116731 (2008)**

*Exponential vs. Hyperbolic Decline in Tight Gas Sands — Understanding the Origin and Implications for Reserve Estimates Using Arps’ Decline Curves.*


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**Semilog Plot — Hyperbolic Rate Decline Relation**

Effect of the \( b \)-parameter on Production Forecast

Example Case 4 — LWF1 Gas Well (SPE 116731)

**Loglog Plot — Hyperbolic Rate Decline Relation**

Effect of the \( b \)-parameter on Production Forecast

Example Case 4 — LWF1 Gas Well (SPE 116731)

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a. (Semilog plot) Production forecast of a tight gas well.

b. (Log-log plot) Production forecast of a tight gas well.
**SPE 116731: Definitions of Rate Functions**

- **Rate Function Definitions:**
  - **Loss Ratio:**
    \[
    \frac{1}{D} \equiv - \frac{q}{dq/dt}
    \]
    or
    \[
    D \equiv - \frac{dq}{dQ}
    \]
  - **Derivative of Loss Ratio:**
    \[
    b \equiv \frac{d}{dt} \left( \frac{1}{D} \right) \equiv - \frac{d}{dt} \left[ \frac{q}{dq/dt} \right]
    \]
    or
    \[
    b \equiv \frac{d}{dQ} \left[ \frac{1}{D} \right]
    \]
  - **Exponential and Hyperbolic Rate Relations:**
    - **(Exponential Decline)**
      \[
      D = \text{con} \rightarrow q = q_i \exp[-D_i t]; \text{ or } b = \text{con} \rightarrow q = \frac{q_i}{[1 + bD_i t]^{(1/b)}}
      \]
    - **(Hyperbolic Decline)**

- **Cause and Effect:**
  - Hyperbolic relation is mis-applied to transient data.
  - What is the "characteristic behavior" of the $D$ and $b$-parameters? Evaluate continuously using data.
Observed Behavior of the "Decline" Parameter \([D(t)]\):

\[
D \equiv -\frac{1}{q} \frac{dq}{dt} \approx D_\infty + n\hat{D}_i t^{-(1-n)} \quad \left[ \approx D_\infty + At^{-B} \right]
\]

Solving for Flowrate \([q(t)]\) Using the \(D(t)\) Relation:

\[
q = \hat{q}_i \exp\left[-D_\infty t - \hat{D}_i t^n\right]
\]

Solving for the "Hyperbolic" Parameter \([b(t)]\):

\[
b = \frac{n\hat{D}_i (1-n)}{\left[n\hat{D}_i + D_\infty t^{(1-n)}\right]^2} t^{-n}
\]
**Discussion:** Small "Waterfrac" Gas Well

- Liquid loading effects are obvious in the latter portion of the flowrate data.
- The onset of the boundary-dominated flow regime is observed.
- We observe a very good match of the flowrate data using $D_\infty = 0$. 
Discussion: Large "Waterfrac" Gas Well
- Erratic rate behavior caused by liquid loading is seen at late times.
- Outstanding matches of the computed $D$- and $b$-parameters with the power-law exponential model are observed.
We convert the "power-law exponential" rate decline model into a dimensionless form.

\[
q = \hat{q}_i \exp[-D_\infty t - \hat{D} t^n] \quad \rightarrow \quad q_{Dd} = \exp[-\tilde{D}_\infty t_{Dd} - t_{Dd}^n]
\]
We develop type curves using the dimensionless form of the "power-law exponential" rate decline model.

\[ q_{Dd} = \exp\left[-\tilde{D}_\infty t_{Dd} - t_{Dd}^n\right] \]
Discussion: **Tight Gas Well (Bossier)**

- Excellent match of the data with the type curve for $n=0.2$ — this yields an upper bound for the reserves ($\approx 5.34$ BSCF).
- The lower bound for the reserves ($G_{p,max}$) is estimated by the second type curve match. $\bar{D}_\infty = 10^{-3.75}$
A Simple Methodology for Direct Estimation of Gas-in-place and Reserves Using Rate-Time Data

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Development: $q_g$-$G_p$ Relation

- Quadratic rate-cumulative production relation can be rearranged to yield a plotting function as:

$$\frac{q_{gi} - q_g}{G_p} = D_i - \frac{1}{2} \frac{D_i}{G} G_p$$

- The plotting function $(q_{gi} - q_g)/G_p$ versus $G_p$ yields an intercept in the $x$-axis of $2G$ — i.e., use to estimate $G$. 

![Graph showing quadratic rate-cumulative production relation](image)
Boundary-dominated flow regime can be identified using the $\alpha$-parameter through the modification of the rate-cumulative production relation:

$$\alpha = \left[ \frac{G_P}{G} \right] - \frac{1}{2} \left[ \frac{G_P}{G} \right]^2$$

The plotting function, $\alpha$ versus $G_P/G$ has a diagnostic value in establishing the boundary-dominated flow regime (i.e., $\alpha = 2$ as $q_g \rightarrow 0$ and $G_P \rightarrow G$).
The plotting functions $q_g/q_{gi}$ versus $G_p/G$ and $q_g$ versus $G_p$ are used in conjunction with the previous plotting functions to yield the best estimate for $G$.

- $q_{gi}$, $D_i$, $G$ parameters are calibrated using the plotting functions.
- We iterate on all plots until the best match is obtained.
Field Example: **Tight Gas Well**

a. Plotting Function 1: *(Tight Gas Well)* \( (q_{gi} - q_g)/G_p \) vs \( G_p \) Plot (Cartesian scale).

b. Plotting Function 2: *(Tight Gas Well)* "\( \alpha \)" Diagnostic Plot — reverse solution for the \( \alpha \)-parameter (Cartesian scale).

c. Plotting Function 3: *(Tight Gas Well)* Model Validation Plot — \( q_g/q_{gi} \) versus \( G_p/G \) (Cartesian scale).

d. Plotting Function 4: *(Tight Gas Well)* Model Validation Plot — \( q_g \) (data and model) versus \( G_p \) (log-log format).
SPE 125031

Decline Curve Analysis for HP/HT Gas Wells: Theory and Applications

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Discussion: Rate-Time Gas Flow Relation (Knowles et al.)

- Basis is the linearization of the nonlinear $\mu_g c_g$ term (Ansah, et al.).
- $D$-function and $b$-function are formulated using the definitions for loss-ratio and the derivative of the loss-ratio.

Rate-Time Relation:

$$q_{Dd} = \frac{4P_{WD}^2 \exp[-P_{WD} t_{Dd}]}{((1 + P_{WD}) - (1 - P_{WD}) \exp[-P_{WD} t_{Dd}])^2}$$

Dimensionless $D$-function $["Loss-Ratio"]$

$$(D_D): \quad D_D = \frac{1}{q_{Dd}} \frac{dq_{Dd}}{dt_{Dd}}$$

$$D_D = \frac{P_{WD}(1 - P_{WD} + (1 + P_{WD}) \exp[P_{WD} t_{Dd}])}{(P_{WD} - 1 + (1 + P_{WD}) \exp[P_{WD} t_{Dd}])}$$

$b$-function $["Derivative of Loss-Ratio"]$

$$b = -\frac{d}{dt_{Dd}} \left[ \frac{q_{Dd}}{(dq_{Dd}/dt_{Dd})} \right]$$

$$b = \frac{2\exp[P_{WD} t_{Dd}] (1 - P_{WD}^2)}{(1 - P_{WD} + (1 + P_{WD}) \exp[P_{WD} t_{Dd}])^2}$$


**Orientation: Rate-Cumulative Production Relation**

Rate-Cumulative Production Relation:

\[ q_{Dd} = 1 - \alpha G_{pD} + \frac{\alpha}{2} G_{pD}^2 \]

\[ \alpha = \frac{2}{1 - p_{WD}^2} \]

**Dimensionless D-function**

\[ D_D = \frac{-dq_{Dd}}{dG_{pD}} \]

\[ D_D = \alpha (1 - G_{pD}) \]

**b-function (b):**

\[ b = -q_{Dd} \frac{d}{dG_{pD}} \left[ \frac{1}{(dq_{Dd}/dG_{pD})} \right] \]

\[ b = \frac{2 - 2\alpha G_{pD} + \alpha G_{pD}^2}{2\alpha(G_{pD} - 1)^2} \]

**Discussion: Rate-Cumulative Gas Flow Relation**

- The definition of the loss-ratio can be re-cast in terms of rate and cumulative production.
- A quadratic relationship exists between rate and cumulative production.

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Orientation: **Analysis Methodology**

**Discussion: Methodology**

- The main goal is to match the data with the model using the definitions for the \( q-D-b \) functions during the *boundary-dominated flow regime*.
- \( b\)-function \(\rightarrow 0.5 \) for high drawdown cases (almost constant behavior).
Field Example: **HP/HT Tight Gas Well**

- 3.5 years of daily data are available for a hydraulically fractured well completed in a HP/HT gas reservoir.
- Well clean-up effects, liquid-loading, and operational changes are observed in the data trends.
- The flowrate data are reviewed prior to analysis; and any erroneous/redundant data points are removed.
- The half-slope trend is evident in the rate-integral derivative function.

\[ p_i = 14000 \text{ psia and } T_R = 260^\circ \text{F} \]
Field Example: HP/HT Tight Gas Well

- **a.** $q_g$ versus $G_p$ (Cartesian plot).
- **b.** $D$-function versus $t$ (Cartesian plot).
- **c.** $b$-function versus $t$ (Cartesian plot).
- **d.** $q_g$ versus $t$ (Semilog plot).
- **e.** $D$-function versus $t$ (Semilog plot).
- **f.** $b$-function versus $t$ (Semilog plot).

**Field Example: Application of the Methodology**

- For the computation of $D$- and $b$-parameter data functions we remove the outlying data points; then we perform the numerical differentiation.
- Our analysis using the proposed semi-analytical relation provides a gas-in-place estimate of approximately 8.0 BSCF.
Field Example: **HP/HT Tight Gas Well**

- **Field Example: Application of the Methodology**
  - Reasonable matches of the $D$-function with the data using the semi-analytical model is achieved (post-transient flow only).
  - The matches of the $b$-function data with the semi-analytical model are problematic — data indicate no unique characteristic behavior.
  - Computation of the $b$-parameter data function is severely affected by factors such as liquid loading.
Field Example: **HP/HT Tight Gas Well**

- We observe a good match of the flowrate data with the model (except for the early time data affected by "cleanup").
- The "power-law exponential" model yields $G_{p,max} \approx 8.0$ BSCF.
- Gas-in-place estimates are consistent comparing the methods we used.
SPE 135616

Hybrid Rate-Decline Models for the Analysis of Production Performance in Unconventional Reservoirs

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Stretched Exponential Function: Kohlrausch (1854)

- **Observed Behavior of Decline Parameter (D):**

\[
D \equiv -\frac{1}{q} \frac{dq}{dt} \approx n \hat{D}_i t^{-(1-n)}
\]

- **Solving for Flowrate:**

\[
q = \hat{q}_i \exp[-\hat{D}_i t^n]
\]

- **Literature:**
  - Kohlrausch (1854).
  - Kisslinger (1993)
  - Decays in randomly disordered, chaotic, heterogeneous systems (e.g. relaxation, aftershock decay rates, etc.).

Valkó (2009)

\[
q(t) = \hat{q}_i \exp[-(t / \tau)^n]
\]

Jones (1942) and Arps (1945)

\[
q(t) = q_o \exp \left[ \frac{-D_o t^{m-1}}{100 (m-1)} \right]
\]
**Discussion: Stretched Exponential Function**

- Single, double and four exponentials are used to approximate the data using linear least squares.
- Stretched exponential function can be described as a linear superposition of exponential decays.

\[
q(t) = \hat{q}_i \exp[-\hat{D}_i t^n]
\]

\[
q(t) = \sum_{i=1}^{n} q_i \exp[-a_i t]
\]
**Rate-Time Equations: Theoretical Considerations**

**Discussion: Rate-Time Gas Flow Relation (Knowles et al.)**
- Basis is the linearization of the nonlinear \( \mu g c_g \) term (Ansah et al.).
- \( D \)-function and \( b \)-function are formulated using the definitions for loss-ratio and the derivative of the loss-ratio.
- See Ansah et al. (2000), Knowles et al. (1999), and Ilk et al. (2009) for more details.

**Rate-Time Relation:**

\[
q_{Dd} = \frac{4 p_{wD}^2 \exp[-p_{wD} t_{Dd}]}{((1 + p_{wD}) - (1 - p_{wD}) \exp[-p_{wD} t_{Dd}])^2}
\]

**Conclusions:**
- Theoretical justification for hyperbolic decline relation for gas flow?
- \( b = 0.5 \) for high drawdown cases \( \left( \frac{p_{wf}}{p_i} \leq 0.05 \right) \).
- ONLY valid for **BOUNDARY-DOMINATED FLOW REGIME**.
- Exponential decline at very late times.
**Diagnostics: \( \beta_{q,cp} \)-Derivative**

\( \beta(t) \)-Derivative: Well Test Analysis (Hosseinpour-Zoonozi et al. 2006)

\[
\Delta p \beta_d (t) = \frac{d \ln(\Delta p)}{d \ln(t)} = \frac{1}{\Delta p} \frac{d \Delta p}{dt}
\]

\( \beta(t) \)-Derivative: Modification for this work (for constant pressure)

\[
\beta_{q,cp} (t) = -\frac{d \ln(q)}{d \ln(t)} = -\frac{t}{q} \frac{dq}{dt}
\]

**Discussion:**
- Strong diagnostic character of the \( \beta_{q,cp} \)-derivative function.
- Holly Branch tight gas field production data exhibit similar characteristic behavior.
- Early time data are affected by "non-reservoir" effects.
Diagnostics: $\beta_{q,cp}$-Derivative
Field Example: *Mexico Gas Well*

Discussion:
- Fractured vertical gas well with 43 years of production.
Field Example: *Mexico Gas Well*

Discussion:
- Boundary-dominated flow regime is apparent at late times.
**Field Example:** *Shale Gas Well (Field D)*

- **Discussion:**
  - Horizontal well with multiple fractures with 340 days of production.
Field Example: Shale Gas Well (Field D)

Discussion:
- Outstanding data quality provides remarkable character.
Discussion: Rate-Time Models

- Rate-time models decrease the uncertainty in reserves estimates.
Production Analysis
Reserves Estimation in Unconventional Reservoirs — New Rate-Time Relations (Summary)

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New rate-time relations which utilize power-law, hyperbolic, stretched exponential and exponential components are proposed to model rate-time behavior.

The basis for the proposed relations is data characteristics.

The proposed rate-time models are centered on the stretched exponential function.

We include power-law component for approximating early-time behavior and hyperbolic and exponential components for representing late time behavior.

A variety of field examples using production data acquired from tight and shale gas reservoir systems are presented.

Summary:
Conclusions:

● Continuous evaluation of the $D$-parameter (based on the definition of loss-ratio) indicates power-law behavior for almost all analyzed cases in low to ultra-low permeability reservoirs.

● The only exception is the Mexico well with more than 40 years of production data available where the effects of boundary-dominated flow regime are being established.

● The power-law behavior of the $D$-parameter data trend yields the stretched exponential function.
Conclusions:

- The stretched exponential function is rigorous and effective in modeling the behavior of production data.
- Modeling the late-time behavior with different functional forms might reduce the uncertainty associated with production extrapolation.
- The computed $\beta_{q,cp}$-derivative data functions for wells producing in the same field indicates that the well completion and geology are the primary factors affecting well performance for wells in unconventional reservoirs.
Production Analysis
Reserves Estimation in Unconventional Reservoirs — New Rate-Time Relations
(End of Lecture)

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