Streamlines, ray tracing and production tomography: generalization to compressible flow

Akhil Datta-Gupta1, K. Noradaas Kulkarni1, S. Yoon1 and D. W. Vasco2
1Department of Petroleum Engineering, Texas A&M University, College Station, TX 77843, USA
2Earth Sciences Division, Berkeley Laboratory, Berkeley, CA 94720, USA

ABSTRACT: We exploit an analogy between streamlines and seismic ray tracing to develop an efficient formalism for integrating dynamic data into high-resolution reservoir models. Utilizing concepts from asymptotic ray theory in seismic and diffusive electromagnetic imaging, we generalize the streamline approach to compressible flow. A 'diffusive' streamline time-of-flight is introduced for transient pressure calculations. Production data integration is carried out in a manner analogous to seismic tomography and waveform imaging. The power and versatility of our approach is illustrated using synthetic examples that utilize transient pressure, tracer and multiphase production history. A field example from a heterogeneous carbonate reservoir demonstrates the practical feasibility of our approach.

KEYWORDS: inverse problem, travel time, ray tracing, streamlines

INTRODUCTION
Integrating dynamic data into high-resolution reservoir models typically requires the solution of an inverse problem. Such inverse modelling is computationally intensive, often requiring orders of magnitude more computational effort compared to forward modelling or flow simulation. Streamline models have shown significant potential in this respect (Datta-Gupta et al. 1998; Vasco et al. 1999; Vasco & Datta-Gupta 1999). Streamline models are advantageous primarily in two ways. First, streamline simulators offer an efficient approach to fluid flow modelling in the reservoir. Second and more importantly, sensitivities of the production response with respect to reservoir parameters such as porosity and permeability can be computed analytically using a single streamline simulation. These sensitivities quantify the change in production response because of a small perturbation in reservoir parameters and constitute an integral part of most inverse modelling algorithms (Mclaughlin & Townley 1996).

Previous efforts towards dynamic data integration using streamline models have been limited to tracer concentration history and multiphase production response such as water-cut data at the wells. This is because current streamline models are particularly well-suited for modelling tracer transport and waterflooding as the velocity field remains relatively static and the streamlines need to be updated only infrequently. Under such conditions streamline models can be orders of magnitude faster than conventional finite difference simulators (Datta-Gupta & King 1995; Batycky et al. 1997). A key development in streamline modelling has been the introduction of the concept of time-of-flight that has trivialized generalization to three-dimensional flows (Datta-Gupta & King 1995). The time-of-flight formulation effectively decouples pressure from saturation and concentration calculations during flow simulations. Furthermore, mapping of solutions to one-dimensional transport equations onto streamlines is also considerably simplified because we do not need to keep track of the geometry of streamtubes.

In this paper we describe how streamline modelling may be generalized to compressible flow by introducing a 'diffusive' time-of-flight. We then utilize developments in seismic tomography and waveform imaging to formulate an efficient method for integrating production data into high-resolution reservoir models (Vasco & Datta-Gupta 1999, 2000; Vasco et al. 2000). Our approach is very general and can handle transient pressure data, tracer data and multiphase production history. The method is based on an analogy between streamlines and seismic ray tracing. In particular, we adopt an asymptotic approach to develop the streamline time-of-flight equations for compressible and incompressible flows using concepts from geometric optics and seismology (Kline & Kay 1965; Cerveny et al. 1978). In subsurface flow and transport there are several investigations of asymptotic solutions describing solute transport in the limit of long times (van Duijn & Knabner 1992; Grundy et al. 1994; Jackel et al. 1996). Until recently, no systematic attempt has been made to develop an asymptotic series representation of the solution for flow and transport and to relate the orders of the expansion to attributes such as breakthrough or arrival times (Vasco & Datta-Gupta 1999, 2000; Vasco et al. 2000). In this paper we outline a general framework for production data integration into high-resolution reservoir models utilizing concepts from the asymptotic ray theory. A key advantage of the asymptotic approach is that parameter sensitivities required for solving inverse problems related to production data integration can be obtained using very few forward simulations. Thus, such algorithms can be orders of magnitude faster than current techniques that can require multiple flow simulations.

The organization of this paper is as follows. First, we briefly review the fundamentals of streamline simulation. Second, we introduce the necessary concepts from asymptotic ray theory and utilize them to develop equations for streamline time-of-flight for tracer transport. Third, we generalize the approach to compressible flow by introducing a 'diffusive' time-of-flight based upon an asymptotic solution of the diffusion equation. Fourth, we outline an efficient formalism for production data.
STREAMLINE SIMULATION: BACKGROUND

We start by briefly reviewing the underlying concepts of streamline simulation for single and multiphase incompressible flow in non-deformable permeable media. The details of streamline simulation can be found elsewhere (King & Datta-Gupta 1998; Crane & Blunt 1999). In this approach we decouple flow (pressure) and transport (saturation and concentration) calculations by a coordinate transformation from the physical space to one following flow directions, namely the tracer time-of-flight along streamlines. The time-of-flight is defined as

$$\tau(x) = \int \frac{ds}{v(x)}.$$  \hspace{1cm} (1)

In the above expression, $v(x)$ denotes the interstitial velocity and $\tau(x)$ is the travel time of a neutral tracer along streamlines, $\psi$. We rewrite Equation (1) in a differential form as follows

$$v(x) \cdot \nabla \tau(x) = 1,$$  \hspace{1cm} (2)

where the $\nabla$-gradients are defined along streamlines. We can now utilize the operator identity (Datta-Gupta & King 1995),

$$\nabla \cdot v = \frac{\partial}{\partial t},$$  \hspace{1cm} (3)

to reduce multidimensional transport equations into a series of one-dimensional equations along streamlines.

As an illustration, consider the convective transport of a neutral tracer. The governing equation is given by

$$\frac{\partial C(x,t)}{\partial t} + v(x) \cdot \nabla C(x,t) = 0,$$  \hspace{1cm} (4)

where $C$ represents the tracer concentration. Rewriting Equation (4) in time-of-flight coordinates we obtain

$$\frac{\partial C(x,t)}{\partial \tau} + \frac{\partial C(x,t)}{\partial \tau} = 0.$$  \hspace{1cm} (5)

Physically, we have moved to a coordinate system in which all streamlines are straight lines but the length of the domain, measured in units of $\tau$, is now variable. The tracer response at a producing well can be obtained by simply integrating the contributions of individual streamlines reaching the producer

$$C(\tau) = \int C_0(\tau - \tau(\psi)) d\psi,$$  \hspace{1cm} (6)

where $C_0$ is tracer concentration at the injection well.

For multiphase flow, the transport equation for saturation along streamlines is the familiar Buckley–Leverett equation in time-of-flight coordinates

$$\frac{\partial S_w(x,t)}{\partial \tau} + \frac{\partial F_w(x,t)}{\partial \tau} = 0,$$  \hspace{1cm} (7)

where $S_w$ denotes water saturation and $F_w$ is the fractional flow of water. If we assume uniform initial fluid saturation and constant injection flux, then we obtain the following self-similar solution in terms of $\tau/t$,

$$\frac{dF_w(S_w)}{dS_w} = \frac{\tau}{t},$$  \hspace{1cm} (8)

where the fractional flow $F_w$ is a single-valued function of water saturation. Thus, once we compute $\tau$, Equation (8) can be used to predict waterflood performance.

To summarize, the basic steps involved in streamline simulation are: (i) tracing streamlines based on a velocity field, typically derived numerically using finite difference or finite element methods; (ii) computing tracer travel time or time-of-flight along streamlines; (iii) decoupling the transport equations (concentration and saturation equations) using a coordinate transformation from physical space to the time-of-flight coordinates following flow directions; (iv) solving the transport equations along streamlines; and (v) occasionally updating the streamlines to account for mobility effects or changing field conditions. These steps are illustrated in Figure 1 for a heterogeneous quarter five-spot pattern. The computational advantage of the streamline approach can be attributed to the fact that streamlines need to be updated only infrequently and the transport equations along streamlines are decoupled from the underlying grid, thus allowing for faster solution. Furthermore, the self-similarity of the solution along streamlines may allow us to compute the solution only once and map it directly to the time of interest.

STREAMLINES AND ASYMPTOTIC RAY THEORY

In this section we draw upon the analogy between a propagating fluid front and a propagating wave and derive the streamline time-of-flight equations by utilizing concepts from asymptotic ray theory in geometric optics and seismology. The asymptotic approach provides us with an efficient formalism for production data integration into reservoir models using techniques from geophysical inverse theory. Furthermore, we can generalize the streamline time-of-flight to compressible flow utilizing concepts from the asymptotic theory for diffusive electromagnetic imaging.

Asymptotic ray theory (also known as ‘Debye series method’ or ‘ray series method’) forms the mathematical basis for geometrical ray theory and has been extensively used in both electromagnetic (Kline & Kay 1965; Luneburg 1966) and seismic (Cerveny et al. 1978) wave propagation. The method has also proved valuable in the analysis of front propagation in general (Sethian 1996), and many of the concepts such as rays and propagating interfaces or discontinuities have direct counterparts in hydrology (Bear 1972) and petroleum engineering (Bratvedt et al., 1992). We will start with a brief outline of the asymptotic approach for the scalar wave equation to point out some of its salient features. The method involves properties of the wavefront and ray paths of the wave equation that have been studied for over a century. The underlying idea is to look for a frequency domain solution to the wave equation in the form of an asymptotic series in inverse powers of $\omega$,

$$u(x,\omega) = e^{-i\omega t} \sum_{k=0}^{\infty} \frac{A_k(x)}{(\omega)^k}.$$  \hspace{1cm} (9)

In these expansions, $\tau(x)$ represents the phase of a propagating wave and thus describes the geometry of a propagating front. Also, $A_k(x)$ are real functions that relate to the amplitude of the wave. The advantage of this form of expansion is that the initial terms of the series represent rapidly varying (high frequency, large $\omega$) components of the solution and successive terms are associated with lower frequency behaviour. Hence, the propagation of a sharp front is described
Fig. 1. A step-wise illustration of streamline modelling in a heterogeneous quarter five-spot pattern: (a) permeability field; (b) streamlines; (c) time-of-flights or tracer fronts; (d) tracer response at the producer; (e) water-cut response at the producer.

Fig. 2. An illustration of ‘pressure fronts’ or ‘diffusive’ time-of-flight for a heterogeneous media: (a) heterogeneous permeability field; (b) streamlines; (c) pressure fronts; (d) tracer fronts.
by the initial terms of the summation (Vasco & Datta-Gupta 1999).

As an example, we consider an asymptotic solution to the tracer transport Equation (4) in the frequency domain (after applying a Fourier transform):

\[ \hat{C}(x, \omega) = \int_{-\infty}^{\infty} C(x', \omega) e^{i\omega t'} \, dt'. \]

The transport equation now reduces to the following

\[ (-i\omega) \hat{C}(x, \omega) + v(x) \cdot \nabla \hat{C}(x, \omega) = 0. \]

Asymptotic solution to the above equation will take the form

\[ \hat{C}(x, \omega) = e^{-i\omega t} \sum_{k=0}^{\infty} \frac{A_k(x)}{(i\omega)^k}. \]

We will specifically focus on the high frequency solution that corresponds to the propagation of a sharp front. This is called the geometric ray approximation or the zero-th order term in asymptotic ray theory

\[ \hat{C}(x, \omega) = e^{-i\omega t} A_0(x). \]

In general, we may substitute Equation (12) in (11) and equate the coefficients of different powers of \( \omega \) to obtain differential equations for \( \tau(x) \) and \( A_0(x) \) (Chapman & Drummond 1982; Fatemi et al. 1995). However, in practice one only solves for the evolution of the phase function, \( \tau(x) \) and the amplitude, \( A_0(x) \).

For production data integration purposes, we will mainly concern ourselves with the matching of breakthrough response or ‘first arrivals’ at the producing wells (Vasco & Datta-Gupta 1999, 2000). Thus, we will limit our consideration to the phase function that governs the geometry of the propagating tracer front. It is worth mentioning, however, that we can account for the amplitudes of the tracer response by incorporating additional terms in the asymptotic expansion (Vasco et al. 1999). Collecting the terms of the order of \( \omega \) we obtain an equation for the phase function \( \tau(x) \) as follows (Vasco & Datta-Gupta 1999)

\[ v(x) \cdot \nabla \tau(x) = 1. \]

Equation (14) is the familiar streamline time-of-flight equation for tracer front propagation as given in Equation (2). The perpendiculars to the front are streamlines that are analogous to rays. In fact, Equation (14) is much like the Eikonal equation in travel time tomography

\[ \nabla \tau(x) \cdot \nabla \tau(x) = \frac{1}{V^2(x)}, \]

where \( \tau \) is the travel time and \( V \) is the propagation velocity. Eikonal equations appear in many contexts, such as elastic and electromagnetic wave propagation and its properties are well developed in the literature (Nolet 1987; Anile et al. 1993). Streamline-based production data integration can draw upon such similarities to utilize existing methods from seismology. However, we must point out several underlying differences between seismic and optical rays and streamlines. First, the time-of-flight equation is a first-order partial differential equation (pde), whereas the Eikonal equation is a second-order pde. In fact, Equation (14) can be thought of as the square root of Equation (15), with the negative component of the velocity being ignored. Physically, this implies that particles do not reflect along streamlines. Furthermore, because the Eikonal equation is a second-order pde, conservation of volume occurs in a six-dimensional phase space, whereas the conservation of volume for streamlines is in physical space. Thus, rays may cross, but streamlines do not.

**GENERALIZATION TO COMPRESSIBLE FLOW: A ‘DIFFUSIVE’ TIME-OF-FLIGHT**

The asymptotic approach can be generalized to transient pressure solution by utilizing concepts associated with asymptotic solutions of the diffusion equation (Cohen & Lewis 1967; Virieux et al. 1994; Vasco et al. 2000). The goal here is to find a solution to the diffusive pressure equation that mimics the one found in the wave propagation phenomena as discussed in the previous section. The transient pressure response in a
heterogeneous permeable medium is governed by the well-known diffusivity equation

\[ \frac{\partial P(x,t)}{\partial t} - \nabla \cdot (k(x)\nabla P(x,t)) = 0. \quad (16) \]

In Equation (16) \( P(x,t) \) represents pressure, \( k(x) \) denotes permeability, \( \mu \) and \( c_t \) represent fluid viscosity and total compressibility, respectively. A Fourier transform of Equation (16) results in the following equation in the frequency domain

\[ \Phi(k) = \frac{\mu_0}{k(x)} (-i \omega) \hat{P}(x, \omega) = \nabla^2 \hat{P}(x, \omega) + \nabla k(x) \cdot \nabla \hat{P}(x, \omega). \quad (17) \]

Asymptotic approach follows if we consider a solution in terms of inverse powers of \( \sqrt{k(x)} \)

\[ \hat{P}(x, \omega) = e^{-\sqrt{-i \omega \eta_0} \tau} \sum_{n=0}^{\infty} A_k(x) \left( \sqrt{k(x)} \right)^n. \quad (18) \]

A solution of the above form can be interpreted on physical grounds based on the scaling behaviour of diffusive flow. Such asymptotic solutions have been applied to electromagnetic imaging, for example, to the ‘telegraph equation’ which can be considered an extension of the wave equation with an extra diffusive term (Kline & Kay 1965). Again, the high frequency solution is given by the initial terms of the asymptotic series and will correspond to the propagation of a ‘pressure front’. We, therefore, consider a solution of the form:

\[ \hat{P}(x, \omega) = e^{-\sqrt{-i \omega \eta_0} \tau} A_0(x). \quad (19) \]

Inserting Equation (19) into (17) and collecting terms of the highest order in \( \sqrt{-i \omega \eta_0} \), that is, \( (\sqrt{-i \omega \eta_0})^n \), results in the following equation for the phase function or the propagating front

\[ \sqrt{\alpha(x)} \| \nabla \tau(x) \| = 1, \quad (20) \]

where \( \alpha \) is the diffusivity and is given by

\[ \alpha(x) = \frac{k(x)}{\Phi(x) \mu_0}. \quad (21) \]

Notice that Equation (20) has a similar form as Equation (14), the equation governing the propagation of a tracer front. Considering streamlines to be perpendicular to the front, we can now define a time-of-flight for ‘diffusive’ or ‘compressible’ flow as follows

\[ \tau(x) = \frac{ds}{\sqrt{\alpha(x)}}. \quad (22) \]

The ‘diffusive’ time-of-flight will have units of square root of time and again, this is consistent with the scaling behaviour of diffusive flow.

We are now faced with the question about the physical significance of a ‘pressure front’. The answer can be obtained by examining the time domain solution to the zero-th order asymptotic expansion for an impulse source

\[ P(t) = A_0(x) \frac{\tau(x)}{2\sqrt{\pi} t^{3}} e^{-\frac{\tau^2(x)}{4t}}. \quad (23) \]

For a fixed position \( x \), the pressure response will be maximized when

\[ \frac{\partial P(t)}{\partial t} = 0 = A_0(x) \tau(x) \left( -\frac{3}{2} t^{\frac{5}{2}} e^{-\frac{\tau^2(x)}{4t}} + t^2 \frac{3}{2} e^{-\frac{\tau^2(x)}{4t}} \right). \quad (24) \]

This results in the following relationship between the observed time and the ‘diffusive’ time-of-flight

\[ t_{\text{max}} = \frac{\tau^2(x)}{6}, \quad (25) \]

where \( t_{\text{max}} \) corresponds to the time at which the pressure response (drawdown or build up) reaches a maximum at a location. Notice that the ‘diffusive’ time-of-flight is actually associated with the propagation of a front of maximum drawdown or build up for an impulse source/sink.
concept is closely related to the idea of a ‘drainage radius’ during primary recovery or compressible flow (Raghavan 1993). An illustration of the drainage radius computed based on the ‘diffusive’ time-of-flight is shown in Figure 2. The results correspond to a single producer in a heterogeneous permeable media. We simply compute the ‘diffusive’ time-of-flight along streamlines and then relate it to the arrival time of the pressure front using Equation (25). For comparison purposes we have also shown the tracer time-of-flight based on Equation (14). The pressure front propagates much faster compared to the tracer front, as might be expected for diffusive transport.

**PRODUCTION TOMOGRAPHY**

Our goal is to reconcile high-resolution geostatistical reservoir models to field production history. The asymptotic approach discussed here provides us with a powerful and efficient formalism for integration of production data using streamline methods. A key advantage of the asymptotic approach is that parameter sensitivities required for solving inverse problems related to production data integration can be obtained using very few forward simulations (Vasco & Datta-Gupta 1999, 2000; Vasco et al., 2000). In addition, the approach is very versatile and the production data can be in the form of transient pressure response, tracer response or multiphase production history. Because the high frequency asymptotic solutions specifically deal with the propagation of a sharp front, we will focus mainly on incorporating the arrival time or breakthrough response at the wells. Our previous experience has shown that matching the breakthrough time at the wells recovers most of the significant features of the heterogeneity embedded in the production response (Datta-Gupta et al., 1998). It is worth mentioning here that it is possible to incorporate the entire production response into reservoir models by considering additional terms in the asymptotic expansion (Vasco & Datta-Gupta 1999).

Integration of production data will typically require the solution of an inverse problem. The mathematical formulation
behind solving such inverse problems using the streamline approach has been discussed elsewhere (Vasco et al. 1999; Yoon et al. 1999). Briefly, in this approach we start with a geostatistical model that already incorporates static reservoir information, such as well logs and seismic data. We then minimize a penalized misfit function consisting of three terms, as follows:

$$\| \mathbf{d} - S \mathbf{R} \| + \beta_1 \| \mathbf{R} \| + \beta_2 \| L \mathbf{R} \|. \quad (26)$$

In the above expression, $$\mathbf{d}$$ is the data residual vector, that is, the difference between the observed and calculated production response, $$S$$ is the sensitivity matrix that accounts for the change in production response because of a small change in reservoir properties, such as permeability or porosity. Also, $$\mathbf{R}$$ corresponds to the change in the reservoir property and $$L$$ is a second spatial difference operator. The first term is called the ‘data misfit’ term that minimizes the difference between the observed and calculated production response. The second term, ‘norm constraint’, ensures that the final model is not significantly different from the initial model. This is justified because our initial or prior model already contains sufficient geological and static information related to the reservoir. Finally, the third term, ‘roughness penalty’ simply recognizes the fact that production data are an integrated response and are, thus, best suited to resolve large-scale structures rather than small-scale property variations.

A critical aspect of any inverse modelling is computing the elements of the sensitivity matrix, $$S$$. Such sensitivity computation can be broadly classified into three categories: direct or perturbation approach (Yeh 1986), sensitivity equation method (Anterion et al. 1989) and adjoint state methods (Sun & Yeh 1990). For high-resolution reservoir models, the direct method or the sensitivity equation method can be computationally prohibitive. The adjoint state method requires solution of the adjoint equations that are fewer in number. However, the method can be complex to implement, particularly for multiphase flow because of the coupling between the flow and transport equations. The asymptotic approach offers a unique advantage in this respect because the sensitivity of the production response with respect to all reservoir parameters can be computed analytically using a single streamline simulation (Vasco & Datta-Gupta 1999). For example, if we assume that streamlines or ‘ray paths’ do not shift because of small perturbations in reservoir properties, then the sensitivity of the ‘diffusive’ time-of-flight with respect to reservoir permeability and porosity will be given by (Nordaas 2000)

$$\delta \tau(x) = \int \frac{\delta f(x)}{\phi(x)} ds, \quad (27)$$

where

$$f(x) = \frac{1}{\sqrt{z(x)}} \sqrt{\frac{\phi(x) \mu_i}{k(x)}}$$

$$\frac{\partial f(x)}{\partial k(x)} = -\frac{1}{2k(x) \sqrt{z(x)}}$$

$$\frac{\partial f(x)}{\partial \phi(x)} = \frac{1}{2\phi(x) \sqrt{z(x)}}.$$
eight surrounding producers (Fig. 3). We will treat each grid block permeability as an unknown parameter and, thus, the total number of parameters is 441. In all the following cases our starting model is a homogeneous permeability field that is conditioned at the wells.

**Transient pressure response**

Figure 4 shows the transient pressure response at the eight producing wells for the reference permeability field. We have normalized the pressure response at each well by its peak value for ease of display. The pressure response corresponds to an interference test whereby water is injected into the central well and the producers act as observation wells. Also, superimposed on Figure 4, is the pressure response from our initial model. The first step in the streamline approach is to compute the ‘pressure front’ arrival times at the wells. As discussed before, for an impulsive source, $\delta(t)$ these arrival times are related to the time at which the pressure response reaches a maximum at the observation wells. However, in field situations we have a finite source, that is, the source function corresponds to a Heaviside function, $\theta(t-t_0)$ as opposed to an impulse function. However, the impulse function is simply the time derivative of the Heaviside function. Hence, for a finite source, the behaviour of the time derivative of pressure will be analogous to the pressure response corresponding to an impulsive source. Thus, the travel time analysis should be carried out with respect to the time derivatives of pressure at the observation wells (Fig. 5). The arrival times corresponding to the peak slopes at the wells will be related to the ‘diffusive’ time-of-flight by Equation (25) as follows

$$\tau_{\text{obs}} = \sqrt{6\tau_{\text{max}}}. \quad (30)$$

In our approach, we first compute $\tau_{\text{obs}}$ at each well by identifying the time at which the slope of pressure response reaches a maximum. The results are shown in Figure 6 for the reference and initial permeability fields. The streamline model is then used to match the $\tau_{\text{obs}}$ at the wells through inverse modelling. This match is facilitated by the fact that the sensitivity of the diffusive time-of-flight, $\tau_{\text{obs}}$, with respect to all
reservoir parameters can be computed analytically using a single streamline simulation using Equations (27) and (28). The arrival time match at the eight observation wells is shown in Figure 7a. The permeability field obtained by the inverse modelling is shown in Figure 7b. On comparison with the reference permeability field, we can see that the dominant features of the reference model are reproduced by simply matching the arrival times of the ‘pressure front’ at the wells. Finally, the pressure derivatives at the observation wells after the arrival time match are shown in Figure 7c. As expected, the peaks line-up, although the amplitudes of the response are off in some of the wells. Matching of the amplitudes, in general, can be time consuming and does not seem to yield any significant improvement in the result (Datta-Gupta et al. 1998).

Tracer response

We now consider reproducing the heterogeneity pattern in the reference permeability field using a tracer test, whereby a tracer is injected into the central injector and the tracer concentration response is sampled at the producing wells. Figure 8 shows the tracer response at the eight producers. For comparison purposes we have again superimposed the tracer response from our initial model. Figure 9a shows the arrival time match at the producing wells using the streamline approach. For this case, we have chosen the arrival times to be the time of arrival of the primary peak of the tracer response at the producers. Thus, during inverse modelling we are simply ‘lining up’ the peaks for the observed and calculated tracer response. It is worth mentioning here that any consistent choice of arrival times will suffice for inversion purposes. For example, we might choose to time-shift the observed tracer response at each well using small increments over a time interval and compute a cross-correlation between observed and computed tracer response at each increment. One possible choice of arrival time would be the time shift for which the cross-correlation is maximized (Luo & Schuster 1991). Figure 9b shows the permeability field derived after matching the peak arrival times at the producers. Again, we are able to reproduce most of the large-scale features in the reference permeability field by utilizing the tracer response at the wells. The tracer concentration response at the wells after the arrival time match is shown in Figure 9c. Although the ‘amplitudes’ of the response are still off for a few wells, the overall match appears to improve significantly by simply matching the arrival times at the wells.

Water-cut response

In general, transient pressure response from interference tests and inter-well tracer data are not commonly available in field situations. However, water-cut response at the producing wells is routinely measured and is, thus, the single most prevalent source of dynamic data. We now consider reproducing the heterogeneity in the reference permeability field using water-cut response at the producing wells in the nine-spot pattern. These water-cut responses are shown in Figure 10 along with the responses corresponding to our initial model. The results after arrival time match are shown in Figure 11a. For this case, we have defined the arrival times simply as the time to reach a 10% water-cut at the producing wells. The permeability field derived by utilizing the water-cut response at the wells is shown in Figure 11b. Again, we are able to reproduce the large-scale features of the reference permeability field reasonably well. The water-cut response at the wells after the arrival time match is shown in Figure 11c. A significant improvement in the overall match is observed by simply matching the arrival times at the wells.

Production data integration: a field example

The North Robertson Clearfork Unit (NRU) is a heterogeneous, low permeability carbonate reservoir located in the Permian Basin of West Texas (Douillet et al. 1995). The reservoirs in NRU are typically characterized by a high degree of vertical and lateral heterogeneity with low porosity (7.5% limestone matrix), low permeability (0.1–10 mD), poor water-flood sweep efficiency and low oil recovery factor. The non-reservoir rock types are relatively impermeable and form vertical barriers contributing to reservoir heterogeneity and compartmentalization. Identification of the location and distribution of these barriers is critical to the success of secondary and tertiary recovery efforts. The presence of fractures also contributes to heterogeneity, further complicating the production response.

Two sections of NRU, sections 326 and 327, are selected as detailed study area (Fig. 12). The reservoir model consists of a mesh size of 100 x 50 x 12 or a total of 60 000 cells. Altogether, 42 wells, including 27 producing wells, and 15 injection wells from the study area are used to characterize heterogeneity based on the water-cut response from the producing wells. An initial reservoir model was constructed based on well log data from 30 wells using geostatistical methods. Figure 13 compares the water-breakthrough response at the producing wells based on the initial model with the field data. We have also superimposed on Figure 13 the calculated and observed water-breakthrough times at the producing wells after 20 arrival time iterations. Clearly, the water-breakthrough times are now in complete agreement with the field data. The computation time for the arrival time matching was less than 2 hours of cpu time on a workstation. The permeability variations after incorporating the water-breakthrough response are displayed on Figure 14. The permeability field derived from this inversion appears to be consistent with two earlier studies: one based on decline curve analysis (Douillet et al. 1995) and the other based on inversion of water-cut data using a coarser model (Xue &
Datta-Gupta 1997). The streamline pattern for the final model is shown in Figure 15.

CONCLUSIONS

In this paper we have described a general framework for production data integration into high resolution reservoir models using streamline methods. We have exploited the analogy between streamlines and seismic ray tracing and utilized concepts from asymptotic ray theory to derive the streamline time-of-flight equations. We have then generalized the streamline approach for modelling transient pressure behaviour in the reservoir by introducing a 'diffusive' time-of-flight. The production data integration problem has been posed in a manner analogous to seismic tomography and solved using efficient techniques from geophysical inverse methods. The primary advantage of the asymptotic approach is that the sensitivity of the production response with respect to all reservoir parameters can be obtained analytically using a single streamline simulation. This, in conjunction with the speed of computation of streamline methods, makes our approach ideally suited for production data integration into reservoir models consisting of hundreds of thousands of grid blocks with reasonable computational efforts.

We have illustrated the power and versatility of our approach using synthetic examples that involve integration of transient pressure data, tracer data and multiphase production history. Finally, a field example demonstrates the viability of the method for practical applications. The examples presented in this paper assume a steady velocity field. Changing field conditions, such as infill drilling, zone isolations, etc., can be handled by a sequence of steady state calculations. This will

![Fig. 11. Results from integration of water-cut response: (a) arrival time match at the observation wells; (b) permeability field derived from the arrival time match; (c) water-cut response after the arrival time match.](image)

![Fig. 12. Well configuration for the study area (sections 326 and 327) in the NRU.](image)
require mapping of saturations as well as sensitivities between streamlines and will generally lead to increased computational costs.

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