A Streamline Approach for Integrating Transient Pressure Data into High Resolution Reservoir Models
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Abstract
Streamline models have shown significant potential in integrating dynamic data into high-resolution reservoir models in a computationally efficient manner. However, previous efforts towards production data integration using streamline models have been limited to tracer data and multiphase production history such as water-cut at the wells. In this paper we generalize the streamline approach to transient pressure applications by introducing a 'diffusive' time of flight along streamlines. We show that the 'diffusive' time of flight allows us to define drainage areas or volumes associated with primary recovery and compressible flow under the most general conditions. We then utilize developments in seismic tomography and waveform imaging to formulate an efficient approach to integrating transient pressure data into high-resolution reservoir models.

Our proposed approach exploits an analogy between a propagating wave and a propagating 'pressure front'. In particular, we adopt a high frequency asymptotic solution to the transient pressure equation to compute travel times associated with a propagating 'pressure front'. The asymptotic approach has been widely used in modeling wave propagation phenomena. A key advantage of the asymptotic approach is that parameter sensitivities required for solving inverse problems related to production data integration can be obtained analytically using a single streamline simulation. Thus, the approach can be orders of magnitude faster than current techniques that can require multiple flow simulations.

We have applied our proposed approach to both synthetic and field examples. The synthetic example utilizes transient pressure response from an interference test in a nine-spot pattern. The spatial distribution of permeability is estimated by matching arrival times of the 'pressure front' in each of the observation wells. The field example is from the Conoco Borehole Test Facility in Kay County, Oklahoma. A series of pressure interference tests were performed in a skewed five-spot pattern to identify the distribution and orientation of the natural fracture system at the Fort Riley formation. We have inverted the pressure drawdowns at the observation wells to create a conceptual model for the Fort Riley formation. The predominant fracture patterns emerging from the inversion are shown to be consistent with outcrop mapping and crosswell seismic imaging.

Introduction
Proper characterization of reservoir heterogeneity is a crucial aspect of any optimal reservoir development and management strategy. In this respect, it is important to reconcile geologic models with the dynamic response of the reservoir. One important source of dynamic data are pressure interference tests that are performed by injecting or producing fluid from one well while observing the pressure response in one or more surrounding wells located several hundred feet away. The observed transient pressure response can be used to estimate the permeability distribution in the reservoir. Transient pressure information is an important dynamic data type because of its wide prevalence as well as the rapid response. Pressure response in observing wells can be obtained hours or days after starting injection or production while it might take months or years to obtain sufficient fluid or tracer production for reliable reservoir characterization.

Previous efforts towards integrating transient pressure data into reservoir models have mostly utilized inverse modeling techniques in conjunction with finite-difference or finite-element based flow simulators. Such inverse modeling is computationally intensive, often requiring orders of magnitude more computational efforts compared to forward modeling or flow simulation. Streamline models have shown significant potential in this respect because of their computational efficiency compared to finite-difference models. Furthermore sensitivities of the production response with respect reservoir parameters such as porosity and permeability can be computed analytically using a single streamline simulation. These sensitivities quantify the
change in production response because of a small perturbation in reservoir parameters and constitute an integral part of most inverse modeling algorithms.\(^7\)

Previous efforts towards dynamic data integration using streamline models have been limited to tracer concentration history and multiphase production response such as water-cut data at the wells. This is because current streamline models are particularly well-suited for modeling tracer transport and waterflooding as the velocity field remains relatively static and the streamlines need to be updated only infrequently. Under such conditions streamline models can be orders of magnitude faster than conventional finite difference simulators.\(^6,10\) A key development in streamline modeling has been the introduction of the concept of time of flight that has trivialized generalization to three-dimensional flows.\(^8\) The time of flight formulation effectively decouples pressure from saturation and concentration calculations during flow simulations. Furthermore, mapping of solutions to one-dimensional transport equations onto streamlines is also considerably simplified because we do not need to keep track of the geometry of streamtubes.

In this paper we first generalize the concept of streamline time of flight to compressible flow by introducing a ‘diffusive’ or ‘pressure’ time of flight. We then utilize developments in seismic tomography and waveform imaging to formulate an efficient methodology for integrating transient pressure data into high-resolution reservoir models. Our proposed method is based on an analogy between streamlines and seismic ray tracing. In particular, we adopt an asymptotic approach to develop the streamline time of flight equations for compressible flows using concepts from geometric optics and seismology.\(^11,12\) In subsurface flow and transport there are several investigations of asymptotic solutions describing solute transport in the limit of long times.\(^13,14\) However, only recently attempts have been made to develop an asymptotic series representation of the solution for flow and transport and to relate the orders of the expansion to attributes such as breakthrough or arrival times.\(^5,6\) In this paper we utilize asymptotic solutions to develop a general framework for production data integration into high resolution reservoir models. A key advantage of the asymptotic approach is that parameter sensitivities required for solving inverse problems related to production data integration can be obtained using very few forward simulations. Thus, such algorithms can be orders of magnitude faster than current techniques that can require multiple flow simulations.

**The Asymptotic Approach**

Asymptotic ray theory (also known as ‘Debye series method’ or ‘ray series method’) forms the mathematical basis for geometrical ray theory and has been extensively used in electromagnetic and seismic wave propagation.\(^11,12\) The method has also proved valuable in the analysis of front propagation in general.\(^15\) Many of the concepts such as rays and propagating interfaces or discontinuities have direct counterparts in hydrology and petroleum engineering.\(^16,17\) The method involves properties of the wave front and ray paths of the wave equation that have been studied for over a century.

Our goal here is to find a solution to the diffusive pressure equation that mimics the one found in wave propagation phenomena. The underlying idea is to look for a solution in terms of an asymptotic series. For transient pressure response, we can utilize concepts from diffusive electromagnetic imaging to examine frequency domain solutions in inverse powers of \(\sqrt{-i\omega}\) as follows\(^18\)

\[
e^{-\sqrt{-i\omega} \tau(x)} \sum_{k=0}^{\infty} A_k(x) \left( \sqrt{-i\omega} \right)^k
\]

In these expansions, \(\tau(x)\) represents the phase of a propagating wave and thus corresponds to the geometry of a propagating front, \(\omega\) is the frequency of the wave and \(A_k(x)\) are real functions that relate to the amplitude of the wave. A solution of the above form can be interpreted on physical grounds based on the scaling behavior of diffusive flow. Such asymptotic solutions have been applied to electromagnetic imaging, for example, to the ‘telegraph equation’ which can be considered as an extension of the wave equation with an extra diffusive term.\(^11\) The advantage of this form of expansion is that the initial terms of the series represent rapidly varying (high frequency, large \(\omega\)) components of the solution and successive terms are associated with lower frequency behavior. Hence the propagation of a sharp front is described by the initial terms of the series. We will specifically focus on the high frequency solution that corresponds to the propagation of a sharp front. This is called the geometric ray approximation or the \(0^{th}\) order term in the asymptotic ray theory.\(^5,6\)

The transient pressure response in a heterogeneous permeable medium is governed by the well known diffusivity equation\(^19\)

\[
\phi(x) \mu c_i \frac{\partial P(x,t)}{\partial t} - \nabla \cdot (k(x) \nabla P(x,t)) = 0
\]

where \(P(x,t)\) represents pressure, \(\phi(x)\) denotes porosity, \(k(x)\) denotes permeability, \(\mu\) and \(c_i\) represent fluid viscosity and total compressibility, respectively. We now consider an asymptotic solution to the diffusivity equation in the frequency domain by applying a Fourier transform

\[
\tilde{P}(x, \omega) = \int_{-\infty}^{\infty} P(x,t) e^{-i\omega t} dt
\]

The diffusivity equation is now transformed as follows

\[
\phi(x) \mu c_i (-i\omega) \tilde{P}(x, \omega) = \nabla \cdot \frac{k(x)}{k(x)} \nabla \tilde{P}(x, \omega)
\]
As discussed before, asymptotic solution to the above equation will take the form,
\[ P(x, \omega) = e^{-\sqrt{-i \omega} \tau(x)} \sum_{k=0}^{\infty} \frac{A_k(x)}{\sqrt{-i \omega}} \]  
(5)

The high frequency solution is given by the initial terms of the asymptotic series and will correspond to the propagation of a 'pressure front'
\[ P(x, \omega) = e^{-\sqrt{-i \omega} \tau(x)} A_0(x) \]  
(6)

In general, we may substitute Eq. 5 into Eq. 4 and equate the coefficients of different powers of \( \alpha \) to obtain differential equations for \( \tau(x) \) and \( A_0(x) \). However, in practice one only solves for the evolution of the phase function, \( \tau(x) \) and the amplitude, \( A_0(x) \). Equating the coefficients of the highest order in \( \sqrt{-i \omega} \), that is \( \left( \sqrt{-i \omega} \right)^2 \), we obtain an equation for the phase function \( \tau(x) \) representing a propagating pressure front
\[ \sqrt{\alpha(x)} \left\| \nabla \tau(x) \right\| = 1 \]  
where \( \alpha \) is the diffusivity,
\[ \alpha(x) = \frac{k(x)}{\phi(x) \mu \epsilon_i} \]  
(8)

It is interesting to note that Eq. 7 has the same form as the streamline equation for a propagating tracer front
\[ v(x) \cdot \nabla \bar{\tau}(x) = 1 \]  
(9)
where \( v(x) \) is the interstitial velocity and \( \bar{\tau}(x) \) is the well-known tracer time of flight for incompressible flow.\(^{8-10}\) On comparing Eq. 7 with Eq. 9, we see that for compressible flow, the 'pressure front' propagates with a velocity \( \sqrt{\alpha(x)} \). In the next section we introduce a 'diffusive' time of flight for transient pressure response and relate it to the observed pressure response at the wells.

**A Diffusive (Pressure) Time of Flight**

We derived the following equation for a propagating 'pressure front'
\[ \sqrt{\alpha(x)} \left\| \nabla \tau(x) \right\| = 1 \]  
(10)

Considering streamlines to be perpendicular to the front, we can now define a time of flight for 'diffusive' or 'compressible' flow as follows
\[ \tau(x) = \int_{\psi}^{s} \frac{ds}{\sqrt{\alpha(x)}} \]  
(11)

where \( \psi \) refers to a streamline and \( s \) is distance along the streamline. Notice that the 'diffusive' time of flight will have units of square root of time, which is consistent with the scaling behavior of diffusive flow.

We are now faced with the question about the physical significance of a 'pressure front' or a 'diffusive' time of flight. The answer can be obtained by examining the time domain solution to the 0th order asymptotic expansion for an impulse source. We first look at the solution for a two-dimensional medium\(^{18}\)
\[ P(t) = A_0(x) \frac{\tau(x)}{2 \sqrt{\pi t}} \exp \left( -\frac{\tau^2(x)}{4t} \right) \]  
(12)

For a fixed position, \( x \), the pressure response will be maximized when
\[ \frac{\partial P(t)}{\partial t} = 0 = A_0(x) \frac{\tau(x)}{2 \sqrt{\pi}} \left[ -t^\frac{3}{2} \exp \left( -\frac{\tau^2(x)}{4t} \right) + \frac{\tau^2(x)}{4t^3} \exp \left( -\frac{\tau^2(x)}{4t} \right) \right] \]  
(13)

This results in the following relationship between the observed time and the 'diffusive' time of flight
\[ t_{\max} = \frac{\tau^2(x)}{4} \]  
(14)
where \( t_{\max} \) corresponds to the time at which the pressure response (drawdown or build-up) reaches a maximum at a location. For a three-dimensional medium, the time domain solution for an impulse source is given by\(^{18}\)
\[ P(t) = A_0(x) \frac{\tau(x)}{2 \pi t} \exp \left( -\frac{\tau^2(x)}{4t} \right) \]  
(15)

Again, the solution at a fixed position, \( x \), will be maximized when its derivative is set equal to zero
\[ \frac{\partial P(t)}{\partial t} = 0 = A_0(x) \frac{\tau(x)}{2 \sqrt{\pi}} \left[ -\frac{3}{2} t^\frac{3}{2} \exp \left( -\frac{\tau^2(x)}{4t} \right) + \frac{\tau^2(x)}{4t^{3/2}} \exp \left( -\frac{\tau^2(x)}{4t} \right) \right] \]  
(16)

This results in the following relationship between the observed time and the 'diffusive' time of flight for a three-dimensional medium
\[ t_{\max} = \frac{\tau^2(x)}{6} \]  
(17)

Physically, the 'diffusive' time of flight is a associated with the propagation of a front of maximum drawdown or build up for an impulse source or sink. This concept is closely related to the idea of a drainage radius during primary recovery or compressible flow.\(^{19}\)
**Time of Flight and Radius of Drainage**

We can formally relate the ‘diffusive’ or ‘pressure’ time of flight to the radius of drainage (investigation) utilizing classical well test solutions. This also serves to validate the time of flight formulation by comparison with analytic solutions. We have seen that for an impulsive source, \( \delta(t) \), the ‘diffusive’ time of flight corresponds to the propagation of a front of maximum drawdown or build-up. The pressure solution for a continuous line source in a two-dimensional, infinite reservoir is given by \(^{19}\)

\[
P_D(r_D, t_D) = \frac{1}{2} E_1 \left( \frac{r_D^2}{4 t_D} \right)
\]

(18)

where \( E_1 \) is the exponential integral solution, \( r_D \), \( t_D \), and \( P_D \) are dimensionless radius, time and pressure respectively. This is the solution for a step function (Heaviside) source in which production/injection starts at time zero and continues at a constant rate. Because the derivative of a step function is a Dirac delta function, we will study the derivative of the pressure solution rather than the pressure solution itself. We therefore find the maximum in the derivative of the solution.

\[
\frac{\partial^2 P_D}{\partial t_D^2} = 0 = \frac{1}{2} \left[ \left( \frac{r_D^2}{4 t_D} \right)^2 - 1 \right] \frac{1}{t_D^2} \exp \left( \frac{r_D^2}{4 t_D} \right)
\]

(19)

By setting the second derivative given by Eq. 19 equal to zero and solving for \( r_D \), we obtain the following expression for the radius of drainage

\[
r_D = \sqrt{4 t_D}
\]

(20)

This expression for the radius of drainage is valid for a two-dimensional medium. For a three-dimensional medium, the continuous line-source solution is given by \(^{22}\)

\[
P_D = \frac{1}{2} \text{erfc} \left( \frac{r_D}{\sqrt{4 t_D}} \right)
\]

(21)

The second derivative of the continuous line-source solution for a three-dimensional medium is as follows

\[
\frac{\partial^2 P_D}{\partial t_D^2} = \frac{\exp \left( \frac{r_D^2}{4 t_D} \right) r_D^4}{4 \sqrt{\pi} \left( \frac{r_D^2}{t_D} \right)^{3/2} t_D^4} + \frac{\exp \left( - \frac{r_D^2}{4 t_D} \right) r_D^4}{4 \sqrt{\pi} \left( \frac{r_D^2}{t_D} \right)^{3/2} t_D^4} - \frac{\exp \left( - \frac{r_D^2}{4 t_D} \right) r_D^2}{8 \sqrt{\pi} \left( \frac{r_D^2}{t_D} \right)^2 t_D^3} \exp \left( - \frac{r_D^2}{4 t_D} \right) r_D^2
\]

(22)

By setting the second derivative given by Eq. 22 equal to zero and solving for \( r_D \), we obtain the following expression for the radius of drainage

\[
r_D = \sqrt{6 t_D}
\]

(23)

We can see that from a two-dimensional medium to a three-dimensional medium, the coefficient in the equation for the radius of drainage simply changes from 2 to \( \sqrt{6} \).

In the previous section we established the physical significance of a pressure front by finding the relationship between the observed time and the ‘diffusive’ time of flight. We now rearrange Eq. 14 to get an expression for time of flight in terms of the arrival time of the pressure front in a two-dimensional medium

\[
\tau(x) = \sqrt{4 t_D}
\]

(24)

Similarly, for a three-dimensional medium, we rearrange Eq. 17 to obtain the following expression

\[
\tau(x) = \sqrt{6 t_D}
\]

(25)

Clearly, the expressions for time of flight are analogous to those of radius of drainage for both two and three dimensional media.

For comparison purposes, we use Eq. 23 to calculate the radius of drainage at four different times, 6 hours, 21 hours, 36 hours and 51 hours after start of water-injection. In Fig. 1 the four circles correspond to the analytical radius of drainage for these times. We also computed the diffusive time of flight using Eq. 11 and relate the time of flight to the arrival time of the pressure front using Eq. 17. These arrival times are contoured in Fig. 1. We can see that the circles and the contour levels in Fig. 1 are identical indicating the validity of the time of flight formulation. The primary advantage of the time of flight approach in defining the radius of investigation is its generality. We can now easily define radius of investigation for arbitrary heterogeneous media under the most general conditions. We will illustrate these concepts later in the paper.

**Integration of Transient Pressure Data**

Our final goal is to reconcile high-resolution geostatistical reservoir models to field production history. The asymptotic approach discussed here provides us with a powerful and efficient formalism for integration of transient pressure data using streamline methods. A key advantage of the asymptotic approach is that parameter sensitivities required for solving inverse problems related to production data integration can be obtained using very few forward simulations. Because the high frequency asymptotic solutions specifically deal with the propagation of a sharp front, we will focus mainly on incorporating the arrival time or breakthrough response at the wells. Our previous experience has shown that matching the breakthrough time at the wells recovers most of the significant features of the heterogeneity embedded in the production response.\(^5\)\(^6\) It is worth mentioning here that it is possible to incorporate the entire production response into reservoir models by considering additional terms in the asymptotic expansion.\(^5\) However, matching ‘amplitudes’ of the
production response can be time consuming and very often
does not result in any significant improvement over matching
‘arrival’ times only.21

Integration of production data will typically require the
solution of an inverse problem. The mathematical formulation
behind solving such inverse problems using the streamline
approach has been discussed elsewhere.23 Briefly, in this
approach we start with a geostatistical model that already
incorporates static reservoir information such as well logs and
seismic data. We then minimize a penalized misfit function
consisting of three terms as follows

$$\|\delta d - S \delta R\| + \beta_1 \|\delta R\| + \beta_2 \|L \delta R\|$$  \hspace{1cm} (26)

In the above expression, $\delta d$ is the data residual vector, that is,
the difference between the observed and calculated production
response, $S$ is the sensitivity matrix that accounts for the
change in production response because of a small change in
reservoir properties such as permeability or porosity. Also,$\delta R$ correspond to the change in the reservoir property and $L$ is
a second spatial difference operator. The first term is called
the ‘data misfit’ term that minimizes the difference between
the observed and calculated production response. The second
term, ‘norm constraint’, ensures that the final model is not
significantly different from the initial model. This is justified
because our initial or prior model already contains sufficient
globular and static information related to the reservoir.
Finally, the third term, ‘roughness penalty’ simply recognizes
the fact that production data is an integrated response and is
thus, best suited to resolve large-scale structures rather than
small-scale property variations.

A critical aspect of any inverse modeling is computing the
elements of the sensitivity matrix, $S$. The asymptotic approach
offers unique advantage in this respect because the sensitivity
of the transient pressure response with respect to all reservoir
parameters can be computed analytically using a single
streamline simulation. For example, if we assume that
streamlines or ‘ray paths’ do not shift because of small
perturbations in reservoir properties, then the sensitivity of the
‘diffusive’ time of flight with respect to reservoir permeability
and porosity will be given by

$$\delta \tau(x) = \int_{\nu} \delta f(x) \, ds$$  \hspace{1cm} (27)

where

$$f(x) = \frac{1}{\sqrt{\alpha(x)}} = \frac{\phi(x) \mu c_i}{k(x)}$$

$$\frac{\delta f(x)}{\delta k(x)} = -\frac{1}{2k(x)\sqrt{\alpha(x)}}$$

$$\frac{\delta f(x)}{\delta \phi(x)} = \frac{1}{2\phi(x)\sqrt{\alpha(x)}}$$

The integral in Eq. 27 can be evaluated after a single
streamline simulation. The analytic calculations of sensitivities
leads to substantial savings in computation time during data
integration using streamline methods.

The minimum in Eq. 26 can be obtained by an iterative
least-squares solution to the augmented linear system

$$\begin{bmatrix} S & \beta_1 I \end{bmatrix} \begin{bmatrix} \delta d \\ \beta_2 L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (29)

The weights $\beta_1$ and $\beta_2$ determine the relative strengths of the
prior model and the roughness term. The selection of these
weights can be somewhat subjective although there are
guidelines in the literature.24 In general, the inversion results
will be sensitive to the choice of these weights. An iterative
sparse matrix solver, LSQR is used for solving this augmented
linear system efficiently.25 The LSQR algorithm is well suited
for highly ill-conditioned systems and has been widely used
for large-scale tomographic problems in seismology.26

Applications

In this section we first illustrate how the concept of a
diffusive time of flight can be utilized to determine the radius
of drainage in an arbitrary heterogeneous medium. Next, we
demonstrate the applicability of the streamline-based
approach for integrating transient pressure response into
reservoir models using a synthetic and a field example.

Radius of Drainage Computations. Figures 2a and 2b show
two heterogeneous permeability fields generated using
Sequential Gaussian simulation.22 We will compute the radius
of drainage at various times for a single injection well located
in the center. The streamlines for these permeability fields are
shown in Figs. 2c and 2d which also show the location of the
injector. We can see that the streamlines are more densely
distributed in the high permeability regions as one might
expect. In order to compute the radius of drainage, we first
calculate the diffusive time of flight along the streamlines
using Eq. 11. We then compute the arrival time of the pressure
front based on Eq. 17. Figures 3a and 3b display the contours
of the arrival times which correspond to the drainage radius at
various times. The effect of heterogeneity on the radius of
drainage is quite apparent in these figures. We can see that for
the heterogeneous cases, the pressure time of flight forms
ellipsoids elongated in the direction of high permeability. For
comparison purposes, the tracer times of flight for the
heterogeneous permeability fields are shown in Figs. 3c and
3d, respectively. We can see that the maximum contour levels
are 0.8 hours and 4.1 hours for the pressure time of flight
compared to 7000 days and 10000 days for the tracer time of
flight. Thus, the pressure front propagates orders of magnitude
faster compared to the tracer front.
Integration of Transient Pressure Data. In this section we will discuss integration of transient pressure response during reservoir characterization using streamline models. We will first verify our approach using a synthetic example. We then apply the proposed method to a field example, a fractured limestone formation in the Conoco Borehole Test Facility, Kay County, Oklahoma.

Synthetic Example. The reference permeability field used for the synthetic example is the same as shown in Fig. 2b. The well configuration used is an inverted 9-spot pattern (Fig. 4) with a central injection well and eight surrounding observation wells. Figure 5 shows the transient pressure response at the eight observing wells for the reference permeability field. We have normalized the pressure response at each well by its peak value for ease of display. Also, superimposed in Fig. 5 is the pressure response from our initial model, a homogeneous permeability field that is conditioned at the wells. The first step in the streamline approach is to compute the arrival times of the ‘pressure front’ at the wells. As discussed before, for an impulsive source, \( \delta(t) \) these arrival times are related to the time at which the pressure response reaches a maximum at the observation wells. However, in field situations we have a finite source, that is, the source function corresponds to a Heaviside function, \( \theta(t - t_0) \) as opposed to an impulse function. Recognizing that the impulse function is simply the time derivative of the Heaviside function, the travel time analysis should be carried out with respect to the time derivatives of pressure at the observation wells. This is illustrated in Fig. 6. The arrival times corresponding to the peak slopes at the wells will be related to the ‘diffusive’ time of flight by Eq. 17 as follows

\[
\tau_{\text{obs}} = \sqrt{6t_{\text{max}}}
\]

In our approach, we first compute \( \tau_{\text{obs}} \) at each well by identifying the time at which the time derivative of pressure response reaches a maximum. The results are shown in Fig. 7 for the reference and initial permeability fields. The streamline model is then used to match the \( \tau_{\text{obs}} \) at the wells through inverse modeling. This match is facilitated by the fact that the sensitivity of the diffusive time of flight \( \tau_{\text{obs}} \) with respect to all reservoir parameters can be computed analytically by a single streamline simulation using Eqs. 27 and 28. The arrival time match at the eight observation wells is now shown in Fig. 8. The permeability field obtained by the inverse modeling is shown in Fig. 9. On comparison with the reference permeability field (Fig. 2b), we can see that the dominant features of the reference model are reproduced by simply matching the arrival times of the ‘pressure front’ at the wells.

Field Example. We will now proceed to a field example, which is an interference test in a skewed 5-spot pattern at the Conoco Borehole Test Facility in Kay County, Oklahoma (Fig. 10). The wells GW-1 through GW-5 penetrate the Fort Riley formation, a fractured limestone formation. The Fort Riley Limestone is part of the Lower Permian Chase Group which consists primarily of limestones and shales. Two roughly orthogonal, near vertical, fractures have been mapped in the area, a dominant fracture in east-northeast and a less striking fracture in north-northwest. The average permeability in the Fort Riley Limestone itself is about 1.2 mD. A series of field experiments were carried out to characterize the natural fracture system at the Fort Riley formation. The experiments included a set of interference and tracer tests, both cross-well and single-well seismic surveys, and the drilling of a slant well to penetrate a suspected fracture.

The pressure interference test studied here involved groundwater withdrawal at a constant rate of 2.3 L/min from the westernmost well (GW-5). The pressure response was observed in the surrounding wells. For our analysis, we fitted the observed data points with B-splines as shown in Fig. 11. Arrival times were computed for each of the four observing wells, GW-1 to GW-4, based on the peak slope arrival as discussed before. Starting with a homogeneous permeability field, the arrival times were then matched using the iterative procedure described in the paper. The results are shown in Fig. 12. The final estimate of the permeability field is shown in Fig. 13. In Fig. 13a the entire permeability range is shown. We can see that the main feature of the permeability field is a dominant fracture in the east-northeast direction located to the north of well GW-3. This finding is consistent with the results of a previous site characterization of the Conoco Borehole Test Facility. In Fig. 13b we show the heterogeneity in the lower permeability range. Here we can also see that there is a less dominant fracture in north-northwest as noted by D’Onfro et al.

As previously mentioned, independent geophysical experiments were performed to better locate the most dominant fracture with respect to the well GW-3. Cross-well seismic data were used to determine if the fracture was to the north or south of this well. A high-frequency seismic source was placed in well GW-3 and the seismic waves were recorded in wells GW-1 and GW-4. Air was pushed through the fracture pathway by simultaneously injecting air into well GW-5 and pumping water from well GW-2. Seismic data were recorded before and after the air injection. It was hypothesized that if the fracture was indeed located to the north of well GW-3, the presence of air in the fracture would lead to a significant attenuation in seismic amplitudes arriving at the well GW-1 whereas seismic waves arriving at the well GW-4 would not be influenced by the air injection. Figure 14 shows the seismic amplitudes before and after air injection. The large amplitude attenuation in well GW-1 because of the air injection confirms that the fracture is indeed located to the north of the well GW-3. These results are consistent with the permeability distribution obtained from the inverse modeling.

Conclusions

1. We have generalized the streamline approach to model
transient pressure behavior in the reservoir by introducing a ‘diffusive’ time of flight.

2. An analogy between the radius of drainage and the ‘diffusive’ time of flight is established. We have shown that the diffusive time of flight can be used to define radius of drainage during primary recovery or compressible flow in heterogeneous permeable media under most general conditions.

3. The transient pressure integration problem has been posed in a manner analogous to seismic tomography and solved using efficient techniques from geophysical inversion.

4. We have shown that the sensitivity of the arrival time of pressure with respect to all reservoir parameters can be obtained analytically using a single streamline simulation. This in conjunction with the speed of computation of streamline methods make our approach ideally suited for transient pressure integration into high-resolution reservoir models.

5. We have validated our approach using synthetic examples. A field example from the Conoco Borehole Test Facility (CBTF) in Kay County, Oklahoma demonstrates the viability of our method for practical applications.

6. Two almost orthogonal vertical fracture sets were mapped at CBTF. The inferred fracture locations are shown to be consistent with the results of an independent geophysical experiment.

Nomenclature

- \( A_k \): real functions related to amplitude of wave
- \( c_i \): total compressibility
- \( d \): data vector
- \( K \): absolute permeability
- \( M \): number of data
- \( N \): number of parameters
- \( P \): pressure
- \( r_D \): dimensionless radius
- \( s \): distance along streamline
- \( S \): sensitivity matrix
- \( t \): time
- \( t_D \): dimensionless time
- \( t_{\text{max}} \): time corresponding to maximum pressure change
- \( T \): travel time
- \( v \): interstitial velocity
- \( V \): propagation velocity
- \( w \): weighting factor
- \( x \): location
- \( Y \): parameter vector
- \( \alpha \): diffusivity
- \( \varepsilon \): data misfit
- \( \phi \): porosity
- \( \mu \): fluid viscosity
- \( \tau \): phase of wave/diffusive time of flight
- \( \omega \): frequency of wave
- \( \psi \): refers to a streamline

Acknowledgements

We would like to acknowledge the financial support from the industrial sponsors of the Joint Industry Project on integrated reservoir description and also from the State of Texas Advanced Technology Program (ATP).

References


Fig. 1 Analytic radius of drainage and ‘diffusive’ or ‘pressure’ time of flight.

Fig. 2 Heterogeneous permeability fields (2a and 2b) and the corresponding streamlines (2c and 2d).

Fig. 3 Diffusive (pressure) time of flight (3a and 3b) versus tracer time of flight (3c and 3d) for the permeability fields in Fig. 2.

Fig. 4 Well configuration for transient pressure data integration: synthetic example.

Fig. 5 Transient pressure response from the reference and initial model: synthetic example.

Fig. 6 An illustration of pressure arrival time computation.

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<tr>
<th>Well #</th>
<th>Observed Response</th>
<th>Calculated Response</th>
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Fig. 7 Arrival time calculations for the initial and reference model: synthetic example.

Fig. 8 Initial (8a) and final (8b) 'pressure front' arrival times match at the observation wells: synthetic example.

Fig. 9 Estimated permeability field after arrival time match: synthetic example.

Fig. 10 Area map, Conoco Borehole Test Facility.

Fig. 11 (a)-(d) Drawdown response (diamonds) and B-spline fit (solid line) in well GW-1 to well GW-4 during constant rate water withdrawal from well GW-5, Conoco Borehole Test Facility.

Fig. 12 Initial (12a) and final (12b) arrival time match, Conoco Borehole Test Facility.
Fig. 13 Estimated permeability field after arrival time match. Fig. 13a shows the entire permeability range, indicating the dominant fracture. Fig 13b shows a lower permeability range indicating the presence of an orthogonal fracture.

Fig. 14 Cross-well seismic amplitudes, Conoco Borehole Test Facility.