Pipe viscometry of foams

C. Enzendorfer

Institute of Drilling and Production, Mining University Leoben, A-8700, Austria

R. A. Harris, P. Valkó, and M. J. Economides

Department of Petroleum Engineering, Texas A&M University, College Station, Texas 77843-3116

P. A. Fokker and D. D. Davies

Shell Research B.V., 2280 AB Rijswijk, The Netherlands

(Received 3 August 1994; accepted 29 November 1994)

Synopsis

This paper describes a method for extracting useful information from small-scale pipe viscometer measurements of foam rheology. The rheology of a foamed polymer solution at a given temperature, pressure, and quality was determined in pipes of five diameters. The flow curves showed a marked dependence on the diameter of the pipe. The concept of apparent slip could be used to explain the phenomenon. The classical slip correction of Mooney was not applicable, but the method developed by Jastrzebski (based on the previous work of Oldroyd) provided a consistent means of apparent slip correction. The geometric interpretation of the two slip correction methods revealed the possible reason for the difference of their performance. The slip corrected measurements were interpreted in the framework of the volume equalization principle. © 1995 Society of Rheology.

I. INTRODUCTION

Foams are two-phase gas–liquid dispersions with the liquid as the continuous phase and the gas as the dispersed phase. In petroleum operations foamed polymer solutions are used widely. They are especially attractive as proppant carrying fluids in hydraulic fracturing of hydrocarbon wells (Cameron and Prud’homme, 1989).

The in situ volumetric phase relation of foams is usually characterized by the quality, \( \Gamma \), defined as the ratio of the gas volume to the total foam volume. High-quality foams, above 93%–97% have the tendency to invert into mist. In a mist the liquid is the internal phase and the gas is the external one.

Several attempts have been made to characterize foam behavior with laboratory scale equipment (Raza and Marsden, 1967; David and Marsden, 1969; Blauer et al., 1974; Calvert and Nezhati, 1986; Reidenbach et al., 1986; Heller and Kuntamukkula, 1987; Khan et al., 1988). According to a recent review by Assar and Burley (1986) most of the studies agree that the apparent viscosity of foam increases with the gas content and
decreases with the shear rate. Unfortunately some other phenomena, first of all geometric effects, make it difficult to characterize the exact form of the variation of the rheological properties.

In determining the rheology of foams, it is convenient to make an explicit distinction between microflow, where the characteristic size of the space confining the flow is commensurable with the bubble size (e.g., flow in porous media and in traditional laboratory viscometers) and macroflow, where the bubble sizes are negligible compared to the characteristic diameter of the flow path (e.g., flow in a fracture or well). In this work we study the macroflow of foams.

For the purpose of flow description the foam can be considered a homogeneous fluid with rheology depending not only on the shear rate but on additional parameters, such as foam quality. Quality based correlations are used widely in the petroleum industry. An alternative approach has been adopted in our previous works (Valkó and Economides, 1992; Winkler et al., 1994). Starting from an invariance requirement for compressible non-Newtonian flow a new class of constitutive equations has been introduced. The specific volume expansion ratio was introduced as the additional parameter, defined by the ratio of the liquid density, \( \rho_l \), to the foam density, \( \rho \),

\[
\epsilon = \frac{\rho_l}{\rho}.
\]

This parameter enables a procedure, called volume equalization, which stands for division by \( \epsilon \). It was shown that the flow curves of foams with different qualities and pressures are converted into one master curve using volume equalization. In other words there exists a unique function, \( f_{\text{VE}} \) relating the volume equalized stress to the volume equalized shear rate

\[
\frac{\tau}{\epsilon} = f_{\text{VE}}\left(\frac{\gamma}{\epsilon}\right).
\]

Industrial-scale experiments (with pipe diameters from 2 to 5 cm and pipe lengths from 30 to 200 m) supported the underlying hypotheses but it was felt that laboratory-scale data should be generated not only to obtain further evidence but also to test their applicability in providing consistently interpretable data. Experiments are reported covering the pressure range \( p = 3-7 \, \text{MPa} \), quality range \( \Gamma = 0.48-0.7 \), and density range \( \rho = 324-539 \, \text{kg/m}^3 \) with a pipe viscometer that utilizes five pipe diameters in series, of range \( D = 0.405-1.2 \, \text{cm} \).

In the course of data processing it became obvious that the measured flow curve corresponding to a given temperature, pressure, and quality (or density) depends strongly on the diameter of the pipe. Similar phenomena have been reported by several authors (David and Marsden, 1969; Calvert and Nezhati, 1986) but no satisfactory procedure has been proposed to cope with the problem. This paper reports a series of experiments with foamed polymer solutions, proposes a slip correction method that is novel for foams, and interprets the results in the framework of the volume equalized principle.

II. EXPERIMENTAL EQUIPMENT

A diagram of the equipment used is shown in Fig. 1. The flow loop is suitable to generate nitrogen/gelled water foams at temperatures up to 60 °C and pressures up to 9 MPa. Although a modified rotational viscometer was also included in the loop, in this paper we restrict ourselves to reporting the data from the pipe viscometer.
The experiments are conducted using an electropneumatic membrane pump. A membrane accumulator is placed at the pump outlet to damp mechanical pulses of the pump. The gas supply for the foam generator is regulated by a mass flow meter and controller.

The foam generator is shown in Fig. 2. It consists of a cylindrical stainless steel cell of 12 cm length, in which a Teflon cylinder is mounted. A 2 mm capillary hole is drilled in the middle of the inner cylinder. The gas is injected before the inlet of the foam generator. The mixture flows through the hole to a chamber filled with 20/40 mesh sand (main grain...
diameter of 0.8 mm). The foam is generated when the mixture passes this sand pack and the sintered glass disc behind it.

A viewing cell is placed after the foam generator enabling visual inspection of the foam texture. Another viewing cell placed after the viscometer is used to check whether the foam texture has changed during the measurement. The temperature is measured electrically at the pipe viscometer outlet. Three manometers are present in the process line. Their positions are at the pump outlet, the foam generator inlet, and the first outlet of the pipe viscometer.

The pipe viscometer consists of five pipes made of stainless steel. The different sizes are given in Table I. Each pipe is equipped with three differential pressure transducers. The variation in pipe diameter and the application of different flow rates allows measurements in a wide range of shear rates: our data cover a range between 3 and 2500 s⁻¹. At the absolute pressures shown in Table I the effect of pressure drop over pipes on the absolute pressure was negligible. The three pressure transducers in each pipe facilitate the characterization of entrance effects: the pressure difference can be measured over a 1 m interval or over either of the 0.5 m intervals. No entrance effects have been found. It is also possible to reverse the order of the four last pipes to check if there are any thixotropic effects. These have not been found either.

The water based polymer, a 0.48% by mass (40 lb/1000 gal water) Hydroxypropylguar solution was foamed with N₂ gas at room temperature T = 21 °C(68 °F). A more detailed description of the experimental facilities is given by Enzendorfer (1994).

### III. DATA TREATMENT AT A GIVEN TEMPERATURE, PRESSURE, AND FOAM QUALITY

To arrive at an unambiguous flow curve it is necessary to correct the measurement series for apparent slip. This is clear from the data of Table I which contains the observed data for a specified foam [series 9, see Table II, with p = 5 MPa (730 psi), Γ = 0.585, and ε = 2.24]. In Fig. 3 the wall stress is plotted versus the nominal Newtonian shear rate. There is a shift in the flow curves when the diameter is changed. This section describes the procedure for apparent slip correction using one selected series for the purpose of illustration.

#### A. Slip correction methods

The shift of the flow curves to lower stresses with decreasing diameter of the tube can be well treated assuming an apparent slip velocity at the wall. The observed flow rate is written as the sum of two flow rates
TABLE II. Experimental conditions. (Parameters typeset in italics are intentionally specified, other parameters are consequence of the design.)

<table>
<thead>
<tr>
<th>Data series</th>
<th>Pressure, ( P ) (MPa)</th>
<th>Quality, ( r )</th>
<th>Density, ( \rho ) (kg/m(^3))</th>
<th>Specific volume expansion ratio, ( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>0.7</td>
<td>355</td>
<td>2.82</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.7</td>
<td>340</td>
<td>2.94</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.7</td>
<td>324</td>
<td>3.09</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.685</td>
<td>355</td>
<td>2.82</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.51</td>
<td>355</td>
<td>2.82</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0.6</td>
<td>447</td>
<td>2.24</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>0.6</td>
<td>435</td>
<td>2.30</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>0.6</td>
<td>421</td>
<td>2.38</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>0.585</td>
<td>447</td>
<td>2.24</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>0.57</td>
<td>447</td>
<td>2.74</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
<td>0.5</td>
<td>539</td>
<td>1.86</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>0.5</td>
<td>529</td>
<td>1.89</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>0.5</td>
<td>517</td>
<td>1.93</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>0.49</td>
<td>539</td>
<td>1.86</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>0.48</td>
<td>539</td>
<td>1.86</td>
</tr>
</tbody>
</table>

\[ q_{\text{obs}} = q_{\text{true}} + q_{\text{slip}} \]  
(3)

The first term is related to shear flow and the second term is due to slip at the wall. By dividing by the cross-sectional area and multiplying by \((8/D)\), Eq. (3) can be written in the form

\[ \left( \frac{8 \mu_{\text{avg}}}{D} \right)_{\text{obs}} = \left( \frac{8 \mu_{\text{avg}}}{D} \right)_{\text{true}} + \left( \frac{8 \mu_{\text{slip}}}{D} \right), \]  
(4)

where \( \mu_{\text{slip}} \) is the slip velocity. The first term on the right-hand side of Eq. (4) represents the part of the nominal Newtonian shear rate that is dependent only on the wall shear stress, not on the tube diameter. The knowledge of this "true," but still nominal, New-

![FIG. 3. Wall stress vs observed nominal Newtonian flow rate in different pipes. \( p = 5 \) MPa, \( \Gamma = 0.585 \), \( \rho = 447 \) kg/m\(^3\), \( \varepsilon = 2.24 \).](image-url)
tonian shear rate is necessary to obtain the flow curve of the fluid. This requires a procedure called slip correction that is intended to identify and deduct the second term on the right-hand side, i.e., the slip constituent of the nominal Newtonian shear rate.

Of course, any correction method requires assumptions on the slip velocity. We applied two known approaches: the method introduced by Mooney (1931) and the Oldroyd-Jastrzebski method described by Jastrzebski (1967).

**B. Mooney method**

Mooney’s key assumption is that the slip velocity depends only on the wall stress

\[ u_{\text{slip}} = \beta \tau_w, \]  

where \( \beta \) is the slip coefficient (which itself might depend on the wall stress). This is a reasonable assumption, at least for fluids without macroscopically observable structures. Rewriting Eq. (4) in terms of the slip coefficient yields

\[ \frac{8 \mu_{\text{avg}}}{D}_{\text{obs}} = \left( \frac{8 \mu_{\text{avg}}}{D} \right)_{\text{true}} + \left( 8 \beta \tau_w \right) \frac{1}{D}. \]  

Thus, a plot of the observed nominal Newtonian shear rate versus the reciprocal of the pipe diameter at a fixed wall stress should yield a straight line. The slope divided by \( (8 \beta \tau_w) \) is the slip coefficient, \( \beta \) (characteristic for the given wall stress). Note that for the given wall stress an extrapolation to the infinite pipe diameter (where the distortion caused by slip is negligible) yields the best estimate of the true nominal Newtonian shear rate. This estimate is given by the intercept of the straight line with the vertical axis. A negative intercept would show that the basic assumption concerning the dependence of the slip velocity does not apply.

While the intercept is a good indicator of the validity of the key assumption, in practice it is not used to develop a corrected flow curve. Instead, the slip coefficient is determined for several different wall stresses. Then, this coefficient is plotted against the wall shear stress and a suitable relationship is established which is then used to correct the measurements for all the available data. Once the slip coefficient is known as a function of the wall stress, the original measurements can be corrected by subtracting the second term in Eq. (6). Since usually the data points do not correspond exactly to the same wall stress, interpolation may be necessary.

**C. Oldroyd–Jastrzebski method**

For fluids with macroscopic structure such as slurries or foams, the apparent slip is the result of a more complex interaction between the wall and the fluid and hence, the diameter of the pipe affects the apparent slip. Jastrzebski (1967) has shown that for slurries the apparent slip velocity depends not only on the wall stress but also on the pipe diameter according to

\[ u_{\text{slip}} = \beta_c \tau_w, \]  

where \( \beta_c \) is the modified slip coefficient (itself perhaps depending slightly on the wall stress). Rewriting Eq. (4) in terms of the modified slip coefficient leads to

\[ \left( \frac{8 \mu_{\text{avg}}}{D} \right)_{\text{obs}} = \left( \frac{8 \mu_{\text{avg}}}{D} \right)_{\text{true}} + \left( 8 \beta_c \tau_w \right) \frac{1}{D^2}. \]
According to Eq. (8), a plot of observed Newtonian shear rate versus the reciprocal of the squared pipe diameter at a fixed wall stress should yield a straight line. The slope divided by \((8\tau_w)\) is the modified slip coefficient, \(\beta_c\) (characteristic for the given wall stress).

Once the modified slip coefficient is determined for several wall stresses, the correction of the measurements follows a similar pattern to the one applied in the Mooney method, but using Eq. (8) rather than Eq. (6).

The Oldroyd–Jastrzebski method is the preferred apparent slip correction procedure for slurries (Hanks, 1986).

**D. Application of the slip correction methods**

The Mooney method [Eq. (6)] implies that the intercept of a plot of apparent shear rate versus the reciprocal diameter should yield the true shear rate for a given wall stress. Figure 4 is the Mooney plot of the data of Table I. The data points at a specified wall stress were obtained from linear interpolation in logarithmic coordinates. A straight line was fit to the data points corresponding to a given wall stress. The method of least squares was used if the number of data points available was greater than two. Many of the intercepts in Fig. 4 are negative and show a chaotic dependence on the wall stress. For certain wall stresses (15, 20, 22 Pa) the absolute value of the negative intercept is two to four times greater than its standard deviation obtained from the least-squares fit. Similar pictures were obtained for the other data sets (corresponding to other pressures and/or in situ gas-to-liquid volume relation). Since negative true shear rates are nonphysical, it is obvious that the Mooney correction is not applicable for foamed polymer solutions. This has been noted by other researchers as well (Heller and Kuntamukkula, 1987).

The application of the Oldroyd–Jastrzebski method to the same data is shown in Fig. 5. The intercepts of the straight lines with the vertical axis are all positive and vary consistently with the wall stress. For each selected wall stress the intercept is at least three times greater than its standard deviation obtained from the least-squares fit. Clearly, this method appears to be more appropriate for correcting the flow data of foam for slip.
Equation (8) also shows that the modified slip coefficients, $\beta_c$, are equal to the slope of each line divided by $(8\tau_w)$. In Fig. 6 the modified slip coefficient, $\beta_c$, is plotted as a function of the wall stress, $\tau_w$. This illustration suggests that a simple linear model is adequate to describe the relationship. Using the least-squares straight line model, the

$$\beta_c = a + b \tau_w$$

Where:

- $a = 7.6 \times 10^6 \, m^2/(Pa \, s)$
- $b = 7.0 \times 10^7 \, m^2/(Pa^2 \, s)$
modified slip coefficient can be expressed at any wall stress. Rearranging Eq. (8) and substituting the equation of the straight line gives the corrected or “true” shear rate

$$\left( \frac{8u_{\text{avg}}}{D} \right)_{\text{true}} = \left( \frac{8u_{\text{avg}}}{D} \right)_{\text{obs}} - \frac{8 \tau_w (a + b \tau_w)}{D^5},$$  \hspace{1cm} (9)

where the parameters $a$ and $b$ are shown in Fig. 6.

The corrected data are plotted in Fig. 7. The corrected measurements lie on one curve that can be considered as the true flow curve corresponding to the given temperature, pressure, and quality. The solid line in Fig. 7 represents the best fit. Also shown in Fig. 7 are two predictions from rheological models published for the same foam system, one by Reidenbach et al. (1986) using a quality based correlation, and one from the volume equalized power law with parameters published in Valkó and Economides (1992). Both model predictions give the correct order of magnitude. A better fit with published model coefficients cannot be anticipated because there may have been several minor differences in the materials used (polymer molecular weight, foamer) and differences in experimental details (temperature variations, foam texture, shear rate range) may have affected the results.

IV. DATA TREATMENT AS A FUNCTION OF PRESSURE AND FOAM QUALITY

A total of 15 sets of data were obtained in this project. We used the same liquid and gas, only changing the pressure and the mass ratio of gas to liquid in each set. The experimental conditions are listed in Table II. Three levels of the quality and three levels of the density were studied at three pressure levels.

The Oldroyd–Jastrzebski slip correction was applied to every data set. The parameters in the equation of the modified slip coefficient as a function of the wall stress depended on the experimental conditions. At high quality (large specific volume expansion ratio)
The modified slip correction factor showed almost no dependence on the wall stress. At lower qualities (specific volume expansion ratios) the dependence was similar to the one shown in Fig. 6. The dependence was only slightly affected by the system pressure, showing a parallel downward shift with increasing pressure.

The measured data were analyzed using composite plots. Figure 8 shows all the experiments with specified quality. Open symbols represent measurements with the highest quality, symbols with dots correspond to intermediate quality, and filled symbols are used for low quality. Figure 8 confirms the well-known fact that foams become more viscous with increasing quality. Figure 9 is a similar plot for the experiments with specified density. Here open, dotted, and filled symbols represent high intermediate and low specific volume expansion ratio, respectively. A comparison of Figs. 8 and 9 indicates that the increase in apparent viscosity can be equally well interpreted as a consequence of the increasing specific volume expansion ratio and not only as the consequence of increasing quality. A comparison of the scatter in the constant-quality and in the constant-density measurements reveals that there is no particular advantage of selecting the quality as an additional parameter when describing foam rheology.

From Figs. 8 and 9 it is apparent that, although a single flow curve can be constructed for a specified quality (density) and pressure, the flow curves are shifted with changing quality (density). To have a useful rheological model, a means of collapsing the flow curves of any quality (density) and pressure into a single flow curve is required. Volume equalization provides this means. Figure 10 shows a volume equalized plot of all the available data. The volume equalized measurements form one master curve, irrespective of the pressure, quality, or density characteristics of the specified data sets. The remaining scatter can be attributed to experimental errors magnified somewhat at lower pressures, where the foams were less uniform.

A yield stress could not be quantified within the experimental error, although the higher quality/lower density foams give somewhat more evidence for a yield stress than the lower quality/higher density foams. The latter observation is in accord with the work of Kraynik (1988).
The volume equalized power law is a special case of Eq. (2)

$$\frac{\tau}{\epsilon} = K_{VE} \left( \frac{\dot{\gamma}}{\epsilon} \right)^n$$

(10)

and the two parameters characterizing the given gas/liquid pair at the given temperature can be obtained from Fig. 10 by fitting the least-squares straight line through the data as

FIG. 9. Wall stress vs corrected nominal Newtonian shear rate, over a range of densities (specific volume expansion ratios).

FIG. 10. Volume equalized composite plot of all measurements.
suggested by Winkler et al. (1994). The flow behavior index is equal to the slope of the line, \( n = 0.34 \), and the pipe consistency index is equal to the intercept, \( K_{p, VE} = 2.86 \) Pa s\(^{n}\).

Applying the well-known relation between the pipe consistency index and the true consistency index (Metzner, 1961), we obtain the following volume equalized power law: parameters: \( n = 0.34 \) and \( K_{VE} = \left(\frac{4n}{(1 + 3n)}\right)^{\frac{1}{n}} K_{p, VE} = 2.50 \) Pa s\(^{n}\).

V. DISCUSSION

The physical mechanisms responsible for the apparent slip phenomena are reviewed by Cohen (1986). Without dipping into a mechanical interpretation we give a brief geometric comparison of the Mooney and the Oldroyd–Jastrzebski methods.

Assuming a Newtonian slip layer with (constant) viscosity, \( \mu \) and (variable) thickness, \( \delta \), the slip velocity can be assumed to be equal to the local velocity of the inner boundary of the slip layer:

\[
\frac{\delta(D - \delta)}{\mu D} = \frac{\delta(D - \delta)}{\mu D}.
\]

Comparing Eq. (11) with Eq. (5) we see that

\[
\beta = \frac{\delta(D - \delta)}{\mu D}.
\]

Assuming that the slip layer thickness is negligible compared to the diameter, i.e., \( \delta \ll D \), we obtain

\[
\beta \approx \frac{\delta}{\mu}.
\]

Since the slip coefficient is determined by the wall stress, the Mooney method implies that the slip film thickness is a function of the wall stress and does not vary with the pipe diameter.

A similar analysis of the Oldroyd–Jastrzebski assumption leads to

\[
\beta_{c} = \frac{\delta(D - \delta)}{\mu}.
\]

Equation (14) indicates, that the product \( \delta(D - \delta) \) is a function of the wall stress and does not vary with the pipe diameter. The geometric interpretation is that the length of the chord tangent to the inner core of the flowing foam is determined by the wall stress (see Fig. 11). With decreasing pipe diameter the thickness of the film increases at a fixed wall stress. The chord length, however, remains invariant. Since the length of the chord is the maximum distance between two points in the fluid without crossing a gas bubble, the assumption of a constant \( \beta_{c} \) at given wall stress means that the slip layer thickness varies to provide a constant value of the maximum gas-free distance in the foam. This is consistent with the idea that most of the liquid in a foam is concentrated in so-called Plateau borders, the region where the liquid lamella join together, and in which the gas–liquid interface shows a fixed curvature (Kraynik, 1988). The Plateau borders at the pipe wall will contain more liquid if the curvature of the wall is larger, i.e., the pipe diameter is smaller.
VI. CONCLUSIONS

The rheology of a foam based on a polymer solution was characterized in a pipe viscometer with five pipe diameters. This yielded valuable information, especially on the apparent slip characteristics of the foam. It was shown that the Mooney approach to slip correction is not applicable for foams. The data indicate that the film thickness decreases with increasing pipe diameter, and could be successfully treated using the Oldroyd-Jastrzebski method of slip correction. In this way, a small-scale pipe viscometer can be used to characterize the bulk foam rheology.

The corrected flow curve corresponding to a given pressure and quality (or density) shows approximately power-law behavior with no clear indication of yield stress. The increasing viscosity with increasing quality (or decreasing density) is evident from the composite plots. The fact that the individual flow curves form one master curve when volume equalization is applied means that this scaling gives the right dependence of the rheology of foam on the volumetric relationship of its two constituents: the gas and the liquid.

ACKNOWLEDGMENT

We wish to thank A. de Kruijf for invaluable technical assistance in carrying out the experiments.

References


