Performance of Fractured Horizontal Wells in High-Permeability Reservoirs

P. Valkó, SPE and M. J. Economides, SPE, Texas A&M University

Abstract

Two of the most important recent developments in petroleum production are horizontal wells and high-permeability fracturing. This work combines them with potentially considerable incremental production economics. Hydraulic fractures have a distinctly defined azimuth, in almost all cases of interest; they are vertical and normal to the minimum horizontal stress direction. Horizontal wells can thus be drilled either normal or longitudinal to the fracture azimuth. The first configuration has been already considered in the literature and is applicable for relatively low-permeability formations. The emergence of high-permeability fracturing, also apparently successful in a number of fields, often results in low dimensionless conductivity hydraulic fractures. The possibility of fracturing horizontal wells longitudinally may have the net effect of installing a high-conductivity streak in an otherwise limited conductivity flow conduit. A rigorous model was constructed to describe this configuration. This paper presents comparative production rates and cumulative productions for longitudinally fractured horizontal wells vis-à-vis vertical fractured wells and unfractured horizontal wells. The range of attractiveness of the individual options is presented in the framework of discounted revenues.

Introduction

Horizontal wells and high-permeability fracturing are perhaps the most important recent development in petroleum engineering. The productivity index of a horizontal well has been well studied in the literature. The issue of areal permeability anisotropy and its economic impact have also been considered.

Hydraulic fractures are, in almost all cases of interest, vertical and normal to the minimum horizontal stress direction. Horizontal wells can thus be drilled either normal or longitudinal to the fracture azimuth. The first configuration results in transverse fractures and has been found to be applicable for relatively low-permeability formations. For the case of transverse hydraulic fractures a skin effect, introduced by Mukherjee and Economides, accounts for the extra pressure drop when fluid converges from linear to radial flow within the fracture. This calculation points to a substantial reduction in the performance of such transverse fractures, compared to the performance from a fracture completed from a vertical well. There is much less information available on the other configuration, involving longitudinally fractured horizontal wells. The study by Larsen and Hegre, a first step taken in that direction, contains several assumptions restricting the generality of the results. In particular, those authors neglect the horizontal flow and the vertical distribution of pressure in the fracture in the fracture.

The longitudinally fractured horizontal well deserves further attention. The reason is connected with the specific limitations involved in high-permeability fracturing. Indeed, high-permeability fracturing, also apparently successful in a number of fields, often results in low dimensionless conductivity hydraulic fractures. The possibility of fracturing horizontal wells longitudinally may have the net effect of installing a high-conductivity streak in an otherwise limited conductivity flow conduit. A rigorous model was constructed to describe this configuration.

This paper uses the concept of discounted revenue (DR) to compare the economic attractiveness of the individual options. Discounted revenue is the economic indicator fully representing both the cumulative production differences and their distribution in time. It can be used to obtain such economic indicators as net present value (NPV), discounted revenue-to-investment ratio (DRI) and incremental discounted revenue-to-investment ratio (IDRI). The latter indicators contain the information on the costs of realizing an individual option. Costs are important from the point of view of any real life economic decision but they may vary from location to location. The discounted revenue indicator is, however, characteristic for the given configuration and reflects that...
aspect of the economic attractiveness which depends only on the productivity of an individual configuration.

**Overview of Individual Configurations**

In the following comparison we consider three different isotropic formations with permeability \( k_1 = 1 \text{ md} \), \( k_2 = 10 \text{ md} \) and \( k_3 = 100 \text{ md} \). The formation and production characteristics are selected according to Table 1.

### Table 1— Formation and Production Characteristics

<table>
<thead>
<tr>
<th>Symb</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>reservoir permeability</td>
<td>( k_1 = 1 \text{ md} )</td>
<td></td>
</tr>
<tr>
<td>( k_2 = 10 \text{ md} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_3 = 100 \text{ md} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h )</td>
<td>reservoir thickness</td>
<td>75</td>
<td>ft</td>
</tr>
<tr>
<td>( \mu )</td>
<td>fluid viscosity</td>
<td>1</td>
<td>cp</td>
</tr>
<tr>
<td>( B )</td>
<td>formation volume factor</td>
<td>1.1</td>
<td>res. bbl/STB</td>
</tr>
<tr>
<td>( \phi )</td>
<td>reservoir porosity</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>( c_t )</td>
<td>reservoir total compress.</td>
<td>( 10^{-4} )</td>
<td>psi(^{-1} )</td>
</tr>
<tr>
<td>( r_w )</td>
<td>well radius</td>
<td>0.328</td>
<td>ft</td>
</tr>
<tr>
<td>( \Delta p )</td>
<td>drawdown</td>
<td>1000</td>
<td>psi</td>
</tr>
</tbody>
</table>

The discounted revenue is calculated from

\[
DR = \frac{Price \sum_{j=1}^{n} N_j}{(1 + i)^j} \tag{1}
\]

where \( N_j \) is the production in year \( j \) from the given configuration and the economic parameters are given in Table 2.

### Table 2— Economic Parameters

<table>
<thead>
<tr>
<th>Symb</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>discount rate</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>time horizon</td>
<td>3</td>
<td>year</td>
</tr>
<tr>
<td>Price</td>
<td>price of oil</td>
<td>18</td>
<td>$/bbl</td>
</tr>
</tbody>
</table>

An overview of the different configurations considered are given in Table 3.

### Table 3— Individual Configurations in an Infinite Acting Reservoir

<table>
<thead>
<tr>
<th>C1</th>
<th>Vertical well</th>
<th>( h ) given</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2</td>
<td>Horizontal well</td>
<td>( + L ) given</td>
</tr>
<tr>
<td>C3</td>
<td>Vertical well</td>
<td>( + h = h, + x_f ) given</td>
</tr>
<tr>
<td>C4</td>
<td>Horizontal well</td>
<td>( + h = h, + x_f = L/2 ) given</td>
</tr>
<tr>
<td>C5</td>
<td>Infinite conductivity fracture</td>
<td>( + h = h, + x_f ) given</td>
</tr>
</tbody>
</table>

Figures 1 and 2 illustrate the relative geometry of the fracture and the well.

**Figure 1** - Vertical well intersected by a longitudinal vertical fracture

**Figure 2** - Horizontal well intersected by a longitudinal vertical fracture
To deal with comparable situations several assumptions are made and several parameters are fixed. Some of these assumptions are quite reasonable others are necessary in order to make the treatment as simple as possible. Because of the nature of the assumptions the results should be considered as guidelines for the real situations. For all configurations infinite acting reservoir (no boundary effects) is assumed. Near wellbore effects are neglected. The vertical well is fully penetrating and so is the fracture. In the case of fractured horizontal well the fracture total length equals the well length. The flow in the entire system obeys Darcy’s law, the pressure gradients are “small”, and gravity effects are negligible. In accordance with similar studies, it is assumed that flow enters the fracture only through the fracture faces and not through the tips, the flow inside the fracture is incompressible, the flow entering the intersected wellbore comes totally from the fracture, and there is no pressure drop within the wellbore.

As far as the fracture extent is considered, we assume that the total propped volume and the fracture permeability are given as shown in Table 4.

| Table 4 — Fixed Fracture Parameters |
|-----------------------------|----------------|----------------|
| Symb | Parameter              | Value | Unit |
| $V_f$ | total propped volume | 2,000 | ft$^3$ |
| $k_f$ | fracture permeability | 50,000 | md |

There is only one way to place the given amount of proppant into the formation if we wish to maximize production from a vertical well. The optimal fracture extent is determined from the condition that the dimensionless fracture conductivity

$$F_{CD} = \frac{k_f x_{f}}{k x_f}$$

should be equal to 1.2. This result was first obtained by Pratts$^{14}$ for pseudo-radial flow. Strictly speaking, the optimal fracture extent is somewhat higher for transient flow and depends on the time horizon considered, but fixing $F_{CD}=1.2$ is a good approximation of the optimum, as illustrated by Fig. 3.

In the following for each permeability we select the half-length of the fracture intersected by a vertical well so that $F_{CD}=1.2$ be obtained.

If $h=h_f$ and $F_{CD}=1.2$, it follows$^{15}$ that

$$x_f = \sqrt{\frac{V_f k_f}{2.4k h}}$$

Figure 3 - Discounted revenue divided by discounted revenue at $F_{CD}=1.2$. Vertical well, fixed propped volume of fracture.

Therefore, we consider the fracture half-lengths for the individual permeabilities as depicted in Table 5.

| Table 5 — (Nominal-) Optimal Fracture Extent ($F_{CD}=1.2$) |
|-----------------------------|----------------|----------------|
| $k$, md | $x_f$, ft | $w$, in. |
| 1 | 745 | 0.21 |
| 10 | 236 | 0.68 |
| 100 | 75 | 2.15 |

For the sake of comparison we consider the same fracture in both the case of vertical well and that of the horizontal well. To maintain consistency, we select the length of the horizontal well according to $L=2x_f$, even for the case where fracture is not present.

The above specifications determine all the necessary input parameters to calculate $DR$ for all the five configurations in the three formations, i.e. altogether 15 cases.

**Calculation Methods**

The details of the calculation procedures differ from configuration to configuration.

Configuration 1 (C1: vertical well) is calculated by standard reservoir engineering methods.

Configuration 2 (C2: horizontal well) is calculated either from the uniform flux horizontal well solution$^{18}$ using the Gringarten et al.$^{17}$ or from the Kuchuk$^{18}$ approximation or by the Wilkinson and Hammon$^{19}$ method. From our point of view (cumulative production) these methods are equivalent.

Configuration 3 (C3: vertical well intersected by a finite conductivity fracture) is calculated from the Cinco-Ley and Meng$^{20}$ method.

Configuration 4 (C4: horizontal well intersected by a finite conductivity fracture) is calculated by the method described by us elsewhere$^{13}$. 
Configuration 5 (C5: infinite conductivity fracture) is a hypothetical configuration. It is included into the comparison because it provides an upper bound on the production from a fracture of given extent, regardless of the way the well intersects the fracture. Infinite conductivity fracture production is calculated from the uniform flux fracture solution using the Gringarten et al. approximation. Virtually the same results are obtained using one of the above finite conductivity methods but with a very large dimensionless fracture conductivity. Note that the infinite conductivity fracture idealization refers to both the vertical well and the horizontal well cases.

In all these calculations the dimensionless cumulative production

\[ N_D = \int_0^t q_{\text{wD}}(t')dt' \] ..............................................(4)

is calculated using the integration rule of Laplace-transformation:

\[ N_D = \frac{1}{s^3 \overline{p}_{\text{wD}}} \] ..............................................(5).

where \( s \) is the Laplace variable and \( \overline{p}_{\text{wD}} \) is the Laplace transform of the dimensionless constant rate solution. The dimensionless cumulative production is defined as

\[ N_D = \left[ \frac{\alpha_1 B \mu}{2 \pi k h(p_i - p_{\text{wf, const}})} \right] \left\{ \frac{\alpha_2 k}{\phi \mu c x_f^2} \right\} N \] .......(6)

for the fractured wells; and

\[ N_D = \left[ \frac{\alpha_1 B \mu}{2 \pi k h(p_i - p_{\text{wf, const}})} \right] \left\{ \frac{\alpha_2 k}{\phi \mu c L^2 / 4} \right\} N \] .......(7)

for the unfractured horizontal well.

For the horizontal well intersecting a finite conductivity fracture (case C5) the solution depends on the dimensionless thickness \( h_D \) (or fracture height) defined by the ratio of the formation thickness to the fracture half-length:

\[ h_D = \frac{h}{x_f} \] ...................................................(8)

In our calculations the dimensionless thickness (fracture height) is not an independent variable, because for every permeability it is determined by the “optimal” half-length of the fracture.

Results and Discussion

In Table 6 shown are the discounted revenues for the 15 cases. For a given permeability all values lie between the vertical well (C1) and the infinite conductivity fracture (C5) cases. One can consider that interval as a window of opportunity. Figure 4 shows how the different configurations make use of this window of opportunity.
The plot shows that for small dimensionless thickness, (i.e., where the fractured horizontal well extends far enough) the dimensionless pressure needed to obtain a given flowrate is much less than for a vertical well intersected by the same fracture. As a consequence, at small dimensionless thicknesses the cumulative production curve is almost indistinguishable from the infinite conductivity curve already at $F_{CD}=1$ as seen from Fig. 6.

The influence of the dimensionless fracture conductivity shows only at larger dimensionless thicknesses, as illustrated by Fig. 7.

The above results show clearly, that a dimensionless fracture conductivity of order one is no more necessary if the fracture is intersected by a horizontal well instead of a vertical well.

In the following we consider configuration C4: horizontal well with a longitudinal fracture; but reduce the fracture conductivity by a factor of ten (without changing the length or any other parameter.) One can consider this calculation as placing only one tenth of the proppant into the fracture (assuming that the proppant bed permeability is maintained in the narrower fracture as well.) Figure 8 shows the discounted revenues, not only for a ten-fold reduction of the $F_{CD}$, but also for a one hundred-fold reduction. (For comparison the fractured vertical well case is also shown.)
The important result is that the horizontal well fractured with tenfold less proppant still outperforms the fractured vertical well for \( k = 1 \) and 10 md, and it is competitive at 100 md. In fact for the 1 and 10 millidarcies range even the hundred times reduced fracture width is more than enough. The “competitive” widths (providing nearly equivalent productivity as the optimal fracture for a vertical well) are shown in Table 7.

<table>
<thead>
<tr>
<th>( k ), md</th>
<th>( w ), in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.002</td>
</tr>
<tr>
<td>10</td>
<td>0.007</td>
</tr>
<tr>
<td>100</td>
<td>0.2</td>
</tr>
</tbody>
</table>

It is no longer true, that high-permeability fracturing only means wide fractures which can be obtained only by tip screen-out technique. The combination of horizontal wells with moderate-width fracturing may provide the optimal configuration.

Conclusions
The possibility of fracturing horizontal wells longitudinally may have the net effect of installing a high-conductivity streak in an otherwise limited conductivity flow conduit. A rigorous model predicts almost infinite conductivity fracture behavior in contrast to the finite conductivity behaviour of a vertical well intersected by a similar fracture. Comparative discounted revenue calculations for longitudinally fractured horizontal well intersected by a similar fracture. Comparative discounted revenue calculations for the fractured vertical well vis a vis the optimal fracture for a vertical well) might be sufficient to achieve a certain production goal.

Nomenclature

- \( B \): formation volume factor, \( - \), \( - \), res. bbl/STB
- \( c_i \): reservoir total compressibility, \( \text{Lt}^2/\text{m}, \text{Pa}-1, \text{psi}-1 \)
- \( DR \): discounted revenue, \( \$ \)
- \( h \): reservoir thickness, \( L, m, \text{ft} \)
- \( h_f \): fracture height, \( L, m, \text{ft} \)
- \( k \): reservoir permeability, \( L^2, \text{m}^2, \text{md} \)
- \( k_f \): fracture permeability, \( L^2, \text{m}^2, \text{md} \)
- \( F_{CD} \): dimensionless frac. conductivity
- \( h_D \): dimensionless height (\( h/x_i \)),
- \( p \): pressure, \( \text{m}/\text{Lt}^2, \text{Pa}, \text{psi} \)
- \( \Delta p \): pressure drawdown, \( \text{m}/\text{Lt}^2, \text{Pa}, \text{psi} \)
- \( q \): flow rate, \( L^3/t, \text{m}^3/s, \text{STB}/\text{D} \)
- \( i \): discount rate
- \( n \): number of years considered
- \( x_f \): fracture (half-) length, \( L, m, \text{ft} \)
- \( w \): fracture width, \( L, m, \text{in} \)
- \( \mu \): fluid viscosity, \( \text{m}/\text{Lt}, \text{Pa} \cdot \text{s}, \text{cp} \)
- \( \phi \): reservoir porosity

Subscripts

- \( w \): wellbore
- \( D \): dimensionless
- \( f \): fracture

References


**SI Metric Conversion Factors**

- \( \text{cp} \times 1.0^* \times E-03 = \text{Pa} \cdot \text{s} \)
- \( \text{ft} \times 3.048^* \times E-01 = \text{m} \)
- \( \text{in} \times 2.54^* \times E+00 = \text{m} \)
- \( \text{md} \times 9.869233 \times E-04 = \mu \text{m}^2 \)
- \( \text{psi} \times 6.894757 \times E+00 = \text{kPa} \)
- \( \text{bbl} \times 1.589873 \times E-01 = \text{m}^3 \)

*conversion factor is exact