System Design for the Measurement of Downhole Dynamic Rheology for Foam Fracturing Fluids
H.F. Spörker, P. Trepp, P. Valkó, and M.J. Economides, Mining U. Leoben

ABSTRACT

To investigate the rheology of multi-phase fluids and especially foams under realistic field conditions a downhole flow loop has been constructed and tested. The interactions between gravitational, frictional and other (e.g. gas solubility) factors have been investigated and isolated in a series of experiments. Here the system design of the multi-phase fluid flow loop and the special considerations of the design and their justification are described. To evaluate the data a new two-phase flow equation and a methodology of interpretation are outlined. Applications of the methodology in the treatment of sample data are also presented. The choice of a vertical instead of a horizontal apparatus is justified from the fundamental treatment in this work. It shows that the presence of the gravitational effects magnifies the frictional effects, allowing their more accurate estimation.

INTRODUCTION

Multiphase fluids have been used extensively in several petroleum engineering applications. In recent years, fracturing fluids, with added gas, have captured an ever increasing share of hydraulic reservoirs and underpressured or water-sensitive oil reservoirs. The gas content is obviously useful in the posttreatment "cleanup". In certain cases this cleanup plays a critical role in the overall success of a fracture treatment.

Traditionally, these gas and liquid mixtures have been divided into energized fluids (less than 52% quality - - volume percentage of gas), foamed fluids (quality between 52 and 96%) and mists (more than 96% gas). Of these, foams have found the largest application.

Early generations of foamed fracturing fluids, while fulfilling the cleanup function, suffered from an inability to transport large proppant concentrations, resulting in small fracture conductivities. This problem has been remedied by the addition of crosslinked polymers in the liquid phase, which complicated their nature further. It became apparent that to predict the rheological properties and estimate friction pressure drops in the tubing and model and monitor the generated hydraulic fractures, it would be necessary to conduct substantial experimental work. A downhole rheology apparatus was constructed to allow realistic measurements under field-like conditions.

Below, the necessity for such an apparatus is justified and, in later sections, the process design and operating conditions, emphasizing innovations and difficulties, are outlined.

Foams are virtually the only fluids exhibiting shear rate-dependent viscosity, compressibility and solid-like properties such as bulk modulus, simultaneously. In addition, they are shear history-dependent, inherently unstable and with a special structure. Pressure and temperature changes affect quality and the resulting rheological properties.

Early investigators tried to apply traditional non-Newtonian continuum rheological approaches to describe the flow behavior of foams. Blauer et al. found a pseudoplastic behavior. Sanghani and Ikoku arrived at the same conclusion. The geometry dependence of the obtained rheological parameters led David and Marsden and Beyer et al. to incorporate the concept of wall slippage.

In a critical review, Heller and Kuntamikkula questioned the value of many of the reported rheological measurements because the results appear to have been strongly influenced by the usually overlooked thickness of a liquid film, which formed along the pipe walls or the instrument solid surfaces. Their suggestion was to consider the flow as a plug of foam moving with constant velocity where shear occurs only at the pipe wall within a thin film. Note that the concepts of wall slippage and the Newtonian layer at the pipe wall are completely different. While in the case of slippage, the energy dissipation takes place in the bulk of the foam and only the boundary condition is different from the traditional rheological approach, in the case of the Newtonian layer the dissipation takes place in the thin layer at the wall and hence the shear-stress relationship inside the foam has little effect on the measured pressure losses.

Carvell and Nezhat evaluated their experimental results, obtained from cone and plate rheometer and pipe flow, by applying a yield pseudoplastic model and a Newtonian layer concept simultaneously. Assuming that the three rheological parameters
are the same in the different geometric conditions but the layer thickness is different. They could describe their experimental results with very good correlation coefficients. According to their findings, the flow rate is primarily controlled by the layer thickness and only secondarily by the shear in the bulk foam.

Researchers agree on the importance of texture effects but attempts to characterize Newtonian layer thickness in terms of average bubble size and/or bubble size distribution parameters appear to be extremely difficult (Colvert and Nezhat). Similarly, Harris could not find definitive relationship between observed rheological parameters and texture.

In a series of recent publications, the two-dimensional hexagonal model of Princen and Prud'homme was developed further. The aim of these works was to obtain rheological properties such as yield stress and mean viscosity in terms of intrinsic properties of the bulk fluid phases and of interfacial tension, quality, and texture. Khan and Armstrong arrived at the conclusion that foams are Bingham Plastics where the rate-dependent contribution to the shear stress and the effect of fluid viscosity are almost negligible at practical flow conditions. In other words, the effective viscosity is inversely proportional to the shear rate and the proportionality coefficient is the yield stress. A qualitatively similar conclusion was reached by Kraynik and Hansen. On the contrary, Schwartz and Princen obtained an expression for effective viscosity, showing that fluid viscosity has a larger impact than shear rate. While these attempts helped in understanding the unusual rheological behavior of foams of high quality, the resulting models are frequently of little engineering use.

The flow of foams in vertical pipes and annuli necessitates the consideration of the compressibility of the gas phase and the shear stress and the effect of fluid viscosity are almost negligible at practical flow conditions. In other words, the effective viscosity is inversely proportional to the shear rate and the proportionality coefficient is the yield stress. A qualitatively similar conclusion was reached by Kraynik and Hansen. On the contrary, Schwartz and Princen obtained an expression for effective viscosity, showing that fluid viscosity has a larger impact than shear rate. While these attempts helped in understanding the unusual rheological behavior of foams of high quality, the resulting models are frequently of little engineering use.

The flow loop layout, as shown in Fig. 2, consisted of four main parts: a) fluid storage tanks, b) pumping and measuring equipment, c) gas storage, injection, and measuring equipment, and d) test well. These are described below.

**Fluid Storage**

Fluid storage capacity was 33 m³ (210 bbl) in four independent tank compartments. Two large (12 m³ (75 bbl)) compartments were designed to handle the circulation fluids with the other two pits (4.5 m³ (30 bbl)) to accommodate chemicals, base fluids for gel mixing and corrosion inhibitors. The size of the large tank compartments was designed to handle a 20 minute circulation run at a maximum circulation rate of 8x10⁻³ m³/s (3 BPM). All tanks were covered, the large circulation tanks were also fitted with ventilation fans to remove excess Nitrogen and CO₂ gas coming out of the returning foam, but did not impair the operation.

**Pumping and Measuring Equipment**

To achieve the desired flexibility in flow rates and pressures, centrifugal pumps were ideally suited for the expected pressure ranges and the solid-free fluids. In addition, pressure pulses that could be created by positive displacement duplex or triplex pumps were avoided. These pulses could influence the pressure measurement electronics. Instead of one large pump, three identical multiple-stage centrifugal pumps were installed, again for flexibility. The pumps were equipped with electric motors running off the 380 V public net and rated at 15 kW each.

With these motors each pump could deliver 2.65x10⁻³ m³/s (1 BPM) against 3x10⁵ Pa (440 psi) operating pressure and could easily be chocked by a throttle valve in the discharge line to 0.53x10⁻³ m³/s (0.2 BPM) under continuous operation.

The flow system was designed so that each of the three circulating pumps had access to each tank compartment, with an additional suction line from the two large tanks to the third pump. This allowed fluid transfer between the individual tanks while circulating the well with the remaining two pumps. Ball valves were selected and equipped with pneumatic actuators and electronic controls. This allowed the desired...
Gas Storage and Injection Equipment

A Nitrogen trailer unit complete with tank, pump and atmospheric vaporizer was used. The comparatively low pressures in the system (max. 2.5x10^6 Pa (360 psi)) allowed gas injection directly from the storage tanks without high-pressure pumps. Supply tank volume was 5 m^3 (1300 gal) liquid for Nitrogen and CO_2 each. Exact planning for injection programs was necessary to minimize the tank losses due to normal vaporization. An average loss of 2 % per day at ambient temperatures of 20° C (68° F) was observed when not draining liquid from the tank for injection.

Liquid Nitrogen was charged with a high-pressure pump into four bundles of high-pressure gas bottles through an atmospheric vaporizer. This accounted for a gas storage buffer volume of 500 standard m^3. The use of buffer bundles was necessary, because the pump/vaporizer combination could only handle less than 400 standard m^3 per hour, whereas the experiments could need more than 600 standard m^3 per hour in extreme cases. All gas supply lines were designed with one inch diameter and proved to be large enough to handle even the highest injection rates.

Pressure regulators were initially set to 2.5x10^6 Pa (360 psi) for Nitrogen, but had to be opened to 6x10^6 Pa (870 psi) later, because it was shown that against a circulating pressure of 2 to 2.5x10^6 Pa (300-550 psi), the required rates of up to 600 standard m^3 per hour could not be achieved. Ideally, the pressure regulator should be placed as close to the storage outlets as possible to avoid low temperatures through the flowmeter because of gas expansion. While this was not critical for Nitrogen, a CO_2 gas temperature below +5° C would result in some condensation, and therefore would not allow accurate flowmeter readings.

Carbon dioxide could be injected right from the storage tank through normal tank vaporization. Two additional 10 kW hot water heaters were connected to the storage tank, with automatic control as soon as the tank pressure dropped below 3x10^6 Pa (440 psi). Without any other installations, injection rates up to 500 standard m^3 per hour could be achieved.

Initial concerns of standpipe freezing with injection gas temperatures around -20° C (-4 °F) and gas contents up to 80 % could be discarded after the first full-size tests. At maximum total flow rates (8x10^6 m^3/s (3 BPM)) and maximum gas rates (80 %), only 0.4-0.5 m (1-2 ft) behind the gas injection point showed temperatures below 0° C where the gas stream hit the pipe wall. No diameter reduction (pump pressure increase) could be detected.

Measuring the gas flow proved to be critical with both gases, because throttling the gas flow before the flowmeters resulted in low measuring temperatures. A slight adoption of the gas system with a heated water bath right before the flowmeters resulted in adequate gas temperatures. The throttle valve for adjusting the gas injection rate was positioned right behind the flowmeter. Ideally, the supply gas temperature should be at the same level as the injection fluid temperature (15° C (59° F)).

Check valves in the gas lines next to the injection point were necessary. During the first trial runs without these valves, any gas shut-off resulted in fluid pressed back into the gas supply lines resulting in a freeze-up of the gas flow meters.

To induce enough turbulence for creating foam in the standpipe instead of foaming on the way down the tubing, a foam generator was designed and installed (Fig. 3). This foam generator consisted of a 0.114 m (4 1/2 in.) OD, 1.2 m (4 ft) long pipe section filled with curled steel chips. The average size of these steel particles was 0.02 m (3/4 in.) wide, 0.5 m (20 in.) long and 0.001 m (1/32 in.) thick. On both ends the foam generator was equipped
with meshes to hold back any particles larger than 5x10⁻³ m (1/5 in.) in diameter. Separated in the middle by a flange connection, the foam generator could be easily opened to change or remove the scrap steel filling.

**Test Well and Operating Conditions**

The 190 m (623 ft) deep well was cemented and cased with a 0.168 m (6 5/8 in.) casing. For the circulation experiments a sealed casing string inside the cemented casing was required. The maximum diameter that could be safely run inside the previous string and still allow enough annular space to run electric cables to surface was 0.114 m (4 1/2 in.) casing with 0.127 m (5 in.) coupling OD. The available pipe quality resulted in a casing ID of 0.1 m (3.937 in.). For the inner tubing string 0.0935 m (2 3/8 in.) tubing was selected. This would result in adequate pressure losses both inside the tubing and in the 2 3/8 in. x 4 1/2 in. annulus.

Both casing and tubing were sand-blasted on the inside to remove any possible scale or corrosion. However, since the tubulars were not directly from the pipe mill, had already undergone some field service, the condition of the inner pipe wall was more similar to actual field conditions.

The first trial runs were made to calibrate all measurement electronics and to check the maximum continuous gas injection rates that could be reached with the installed gas unit.

First priority was the specification of the long-term gas temperatures through the flow meters. A series of gas-injection tests at various flow rates (from 150 to 600 standard m³/h in 50 standard m³/h intervals) delivered a compensation chart with consolidated flowing gas temperatures for periods of 15-30 minutes.

These temperature readings were taken with temperature transducers attached to the outer surface of the gas lines, immediately before entering the flowmeter and 1.5 m (5 ft) from the heated water bath kept at 30°C (86°F).

The second calibration run was made to validate the flow meter readings against the weight loss in the CO₂ storage tank to produce a second compensation chart. When the first trials with gas temperatures below 0°C (32°F) produced about 10% error between flowmeter reading and actual gas injection due to a certain liquid content in the CO₂ stream, the addition of the hot water heater in the gas supply line running at 30°C (86°F) resulted in very accurate flowmeter readings, compared to the actual storage tank weight loss.

The third calibration run was performed at maximum gas injection rates for Nitrogen (650 standard m³/h [2293 scf/h]) and CO₂ (500 standard m³/h [17644 scf/h]) to check the maximum possible tank drainage rate on a continuous basis.

Standard operating procedure for foam circulation runs started with the definition of the gas injection rate in standard m³/h for a given total flow rate (2.65x10⁵, 5.3x10⁵ or 8x10⁵ m³/s respectively (1.2 or 3 BPM)) and given standpipe (= gas injection) pressure. These calculations were performed taking into account gas compressibility for expected circulation and bottomhole pressures as well as gas (e.g., CO₂) solubilities. Then the appropriate fluid flow rate was regulated with the pump choke valves, with the return line choke valve kept wide open.

Once the desired fluid flow rate was constant, the gas injection lines were opened, and the gas throttle valves were set to the appropriate position. Should the standpipe pressure be below 2x10¹⁰ Pa (290 psi), the return line choke valve was closed further, and the pump choke and gas throttle valve were opened until the standpipe pressure read 2x10¹⁰ Pa (290 psi). Fluid and gas flow rates were fine-tuned to the calculated settings, and the well was circulated for 3-10 minutes depending on the total flow rate to ensure a homogenous fluid distribution.

Keeping a constant pressure proved to be difficult because of wellbore storage effects from a high gas content. To ensure constant foam quality throughout the well, all choke, flow rates and pressures needed to remain constant for the time it took to circulate one well volume (1.3 m³ (8 bbl)). Otherwise, the well either was loading or unloading gas through the circulation cycle, and therefore did not show the correct gas/fluid ratio. Since a steady-state gas-fluid ratio throughout the well is crucial for compensating the hydrostatic pressure difference between surface and bottomhole pressure sensors, this point was emphasized.

**TWO-PHASE FLOW EQUATIONS**

For two phase downward flow, an improved version of Lord's17 pressure drop equation is used. The details of the development are given in Appendix A. The main assumptions, i.e. isothermal flow, constant friction factor, no phase hold-up and no change in solubility, are the same as in Lord's work. There are, however, two important differences between his equation and the one presented here. Firstly, Lord used the "engineering gas law" i.e. a constant compressibility factor while the gas behavior is described by the vitral equation. Interestingly enough, this improves the accuracy of the equation without affecting the form of the analytical solution; only the expressions for some constants (a and b) are different. Nevertheless, the final form of the present equation differs considerably from that, presented by Lord. The reason is that he used the specific volume in the differential mechanical energy balance while pressure is used here. As a result Lord obtained a complicated pressure drop equation, whose coefficients can be determined by solving a 4 by 4 system of linear algebraic equations. Explicit expressions for the coefficients were not given. Moreover, in the limiting case, when the flowing fluid is a purely incompressible liquid, Lord's equation does not reduce to the well known Fanning pressure drop equation. The new solution, devoid of these problems, is:

\[ K_1(p_1 - p_2) + K_2 \ln \frac{p_1}{p_2} + K_3 \ln \frac{p_1 - a_1}{p_2 - a_1} + K_4 \ln \frac{p_1 - a_2}{p_2 - a_2} = \frac{L}{D} \]

where

\[ a_1 = \frac{ac}{sd} \left( 2f_1 b c + \sqrt{2f_1 Dg} \right) \]

\[ a_2 = \frac{ac}{sd} \left( 2f_1 b c - \sqrt{2f_1 Dg} \right) \]

\[ K_1 = \frac{b}{s_d} \]

\[ K_2 = -\frac{1}{2f_1} \]

\[ K_3 = -\frac{bc - 2f_1 abc + b a_1 \left( 4f_1 abc^2 + s_d \right)}{s_d} \]

\[ K_4 = -\frac{2\sqrt{2f_1 Dg}}{2f_1 Dg} \]

where the constants a, b, c and s_d are defined in Appendix A. It is supposed that s_d is not zero. When s_d is zero, a balance of frictional and gravitational effects results in another form of the analytical solution. This singular case is, however, of little practical value and hence not discussed here.
The counterpart of Eq. 1 for upward flow is given in Appendix B. In order to show the importance of the pressure drop equation an illustrative example is given in Appendix C.

The calculations in Appendix C show that the total pressure profile changes substantially with the friction factor and therefore the bottomhole pressure is an extremely sensitive indicator of the friction phenomenon, therefore augmenting its determination.

**CALIBRATION**

Circulation tests with water at flow rates of $2.65\times 10^4$, $5.30\times 10^4$ and $7.95\times 10^4$ m$^3$/s (1.2 and 3 BPM respectively) showed a friction pressure somewhat higher than for new tubulars. Relative roughness was matched with the Moody diagram and was found to be $e/D=0.003$, where $e$ is the absolute roughness and $D$ the pipe diameter. (See Fig.4 where the smooth pipe curve is also shown.) This relative roughness value identifies the condition of the pipe and is to be used throughout the data evaluation if necessary (i.e. if turbulent conditions are encountered). However, the runs described below were conducted at laminar conditions, where the impact of the pipe roughness is negligible.

**SAMPLE FOAM RUNS**

Data were collected for Nitrogen, CO$_2$ and mixed gas foams with the continuous phase being water, gel or crosslinked gel with or without a foaming agent.

For each run 120 sets of data points were obtained and averaged. The mass flow rates, entering the flow loop, were corrected for the solubility of gas applying Henry’s law. The obtained temperature, flow rate and pressure averages were inserted into Eq.1, which was then solved for the friction factor $f_f$. Once the friction factors were known, a non-linear least squares procedure (Gauss-Newton-Marquardt algorithm) was applied to fit several rheological model equations to the friction factor versus flow rate curve.

As an example, consider a 70% quality Nitrogen/water foam $N_H=0.7$. One run where the density was $\rho=315.5$ kg/m$^3$ and the inlet velocity $w=2.478$ m/s, $\tau_f=0.43$ Pa and $\mu_f=0.0397$ Pa s first the Hedstrom number is calculated according to Eq.11: $N_{Hed}=3457$. From Eq.11 the Bingham Reynolds number is $N_{B}=1019$. Applying these values Eq.13 is solved numerically to give the computed Fanning friction factor $f_F=0.0245$, which is a slightly worse estimate of the experimental value derived from Eq.1 ($f_f=0.0257$) than the Power Law estimate.

The above sample calculations are characteristic for the performance of the two models. The standard relative error of the Power Law model was 28% lower than the one for the Bingham Plastic model in this particular case.

**CONCLUSIONS**

The rationalization of constructing a downhole flow loop, based on the inadequacy of small-scale horizontal measurements has been indicated. Downhole pressure measurements, the ability to control and regulate flowrates and pressures and adequate mixing capabilities were necessary. The well depth (190 m (623 ft)) was proven long enough for appropriate data gathering and treatment.

In order to evaluate the data a methodology of interpretation for vertical flow of multiphase fluids was developed. The treatment is
based on a new two-phase flow equation using the Fanning friction factor as the main means of data reduction. This methodology can be used for both Power Law and Bingham Plastic model to predict flow behavior.

NOMENCLATURE

- \(a\) m/s²: constant coefficient
- \(b\) m³/kg: constant coefficient
- \(B\) m³/mol: second virial coefficient
- \(B'\) 1/Pa: modified second virial coefficient \(B' = B / (RT)\)
- \(c\) kg/(s m³): constant coefficient
- \(D\) m: pipe diameter
- \(f\) -  Fanning friction factor
- \(g\) m/s²: gravitational acceleration
- \(K\) Pa s: viscosity
- \(m\) kg/s: flow rate
- \(n\) - flow behavior index
- \(N_a\) - Hedstrom number
- \(N_a\) Bingham Reynolds number
- \(N_{\text{law}}\) - Power Law Reynolds number
- \(K_\text{p}\) - coefficients
- \(L\) m: length
- \(M\) kg/mol: molar mass
- \(p\) Pa: pressure
- \(R\) J/(mol K): universal gas constant
- \(s\) m²/s²: coefficient
- \(s'\) m²/s²: coefficient
- \(T\) K: temperature
- \(u\) m/s: linear velocity
- \(v\) m³/kg: specific volume
- \(w\) - mass fraction
- \(z\) m: depth coordinate
- \(a\) Pa: coefficients
- \(\mu\) Pa s: plastic viscosity
- \(\rho\) kg/m³: density
- \(\tau_w\) Pa: wall shear stress
- \(\tau_y\) Pa: yield stress

Subscripts:

- \(g\) - gas
- \(inc\) - incompressible liquid
- \(1\) - inlet
- \(2\) - outlet

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REFERENCES

APPENDIX A: DERIVATION OF Eq. 1 (DOWNWARD FLOW)

Foam is injected into a vertical pipe of diameter $A$ and length $L$ at $z=0$. The foam consists of gas ($g$) and incompressible fluid ($inc$). The mass rate of gas is $m_g$ and that of the incompressible fluid is $m_{inc}$. The mass fraction of gas is defined as

$$w_g = \frac{m_g}{m_g + m_{inc}} \quad (A-1)$$

The differential mechanical energy balance is

$$vdp + udu = -\left(\frac{2f_r}{D} - g\right) dz \quad (A-2)$$

where $v$ is the specific volume, $p$ is the external pressure, $u$ is the linear velocity of the foam, $f_r$ is the Fanning friction factor and $g$ is the gravitational acceleration.

The mechanical energy balance can be solved if the volumetric behavior of the foam is specified. Introducing the mass fraction $w_g$, the specific volume of foam can be expressed as

$$v = w_g v_g + (1-w_g) v_{inc} \quad (A-3)$$

where $v_{inc}$ is constant. For the gas it is suggested to use the virial equation truncated after the second term:

$$v_g = \frac{RT}{M_g} \left(1 + \frac{1}{p + p^0} + B \ldots \right) \quad (A-4)$$

where $p^0$ is the excess pressure in the foam because of the surface tension:

$$p^0 = \frac{4\sigma}{r} \quad (A-5)$$

$a$ is the surface tension and $r$ is the average bubble radius. In the case of elevated pressure in pipe flows the excess pressure can be neglected.

Therefore an equation of state for the foam is obtained:

$$v = w_g \frac{RT}{M_g} \left(1 + Br \ldots \right) + (1-w_g) v_{inc} = \frac{a}{p} + b \quad (A-6)$$

where

$$a = w_g \frac{RT}{M_g} \quad (A-7)$$

$$b = -w_g \frac{RTb^2}{M_g} + (1-w_g) v_{inc} \quad (A-8)$$

The linear velocity $u$ is easily obtained from the continuity equation and in terms of the specific volume it is:

$$u = cv = \frac{ac}{p} + bc \quad (A-9)$$

where

$$c = \frac{\alpha(w_g m_g + (1-w_g) m_{inc})}{D^2 \pi} \quad (A-10)$$

and hence

$$du = -\frac{ac}{p^2} \frac{dp}{dp} \quad (A-11)$$

Substituting Eqs. A-6, A-9 and A-11 into Eq. A-2 the differential mechanical energy balance takes the form

$$\frac{dp^0 + ap^2 + (-abcf_2)p + (-ac^2)}{(2f_r b^2c^2 - Dg)p^0 + 4f_r abc^2 p^2 + 2f_r c^2p} \frac{dp}{dz} = \frac{a}{D} \quad (A-12)$$

The solution of Eq. A-5 is obtained by integration from inlet point 1 to outlet point 2. The resulting pressure drop equation can be used to calculate the pressure at any point $z$, given its value anywhere else. The form of the solution depends on the values of the constants. The following cases could be distinguished:

Case 1: static state, when $c=0$

Case 2: special case, when $s_u=0$, where $s_u$ is defined as

$$s_u = 2f_r b^2 c^2 - Dg \quad (A-13)$$

(The physical meaning of this special case is a balance of gravitational and friction effects.)

Case 3: general case, when $s_u \neq 0$

Applying the method of partial fractions the solution of Eq. A-5 for the general case, when neither $s_u$ nor $b$ equal zero, can be obtained (see Eq. 1).

APPENDIX B: UPWARD FLOW

In the case of upward flow Eq. 1 takes the form:

$$K_1(p_1 - p_2) + K_2 \ln \frac{p_1}{p_2} + K_3 \ln \frac{p_1^2}{p_2^2} + \frac{u_1 p_1}{p_2} + u_2 + u_3 + kl(p_1 - p_2) + kl\ln \frac{p_1}{p_2}$$

$$+ \frac{2(K_4 - K_5 u_1)}{u_1} \left(\arctan \frac{2p_1 + u_1}{a} - \arctan \frac{2p_2 + u_1}{a}\right) = \frac{L}{D} \quad (B-1)$$

where

$$s_u = 2f_r b^2 c^2 + Dg \quad (B-2)$$

$$u_0 = \frac{2f_r a c^2}{s_u} \quad (B-3)$$

$$u_1 = \frac{4f_r abc^2}{s_u} \quad (B-4)$$

$$a = \frac{2ac}{s_u} \sqrt{2f_r Dg} \quad (B-5)$$

$$K_1 = \frac{b}{s_u} \quad (B-6)$$

$$K_2 = -\frac{1}{2f_r} \quad (B-7)$$

$$K_3 = \frac{2ac}{2s_u} - \frac{2f_r a b c^2}{s_u} + \frac{1}{4f_r} \quad (B-8)$$

$$K_4 = \frac{abc^2}{s_u} (s_u - 2f_r a) \quad (B-9)$$
In order to show the importance of the pressure drop equation, consider the following simulated example:

A mixture of nitrogen and water is injected into a pipe of 0.05 m (1.969 in.) inner diameter. The inlet pressure is $p_i = 2 \times 10^6$ Pa (290 psi), the inlet flow rates are $q_{W_i} = 0.3$ m$^3$/min (1.9 BPM) and $q_{W_i} = 600$ standard m$^3$/h (2.12 x 10$^4$ scf/h). It is assumed that the flow is isothermal with a temperature $T = 15$°C and because of pipe roughness the turbulent friction factor $f_t$ is constant. Solubility of nitrogen in water is neglected. Using the values presented in Table 1, the computed inlet quality is $r = 0.637$.

Eq. 1 was solved by bisection for $p_z$ with different Fanning friction factors. The results are presented in Table 2.

As it is seen from Table 2, the pressure profile changes dramatically with the friction factor and any attempt to use simple superposition for gravity and friction effects would be highly misleading. These sample calculations show that the measured downhole pressure in case of vertical foam flow is very sensitive to the friction factor and hence provides an improved way of addressing the friction phenomenon.

<table>
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<th>$z$ (m)</th>
<th>Static</th>
<th>$f_t = 0.005$</th>
<th>$f_t = 0.006$</th>
<th>$f_t = 0.007$</th>
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<td>3.550</td>
<td>1.580</td>
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**TABLE 1: Constants Used for the Computation in Appendix C**

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<th>Symbol</th>
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<th>Unit</th>
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<td>$M_2$</td>
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<td>kg/mol</td>
</tr>
<tr>
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<td>m</td>
</tr>
</tbody>
</table>

**TABLE 2: Pressure at Depth z Computed from Eq. 1 for Example in Appendix C**

Figure 1: Measurements Scheme

**Figure 2: Flow Loop Process Diagram**
Figure 3: Foam Generator

Figure 4: Fanning Friction Factors for Water Circulation Tests

Figure 5: Description of the 70-Q Nitrogen/Water Foam by the Power Law Model

Figure 6: Description of the 70-Q Nitrogen/Water Foam by the Bingham Plastic Model