Propagation of Hydraulically Induced Fractures—a Continuum Damage Mechanics Approach

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The propagation of fluid-driven fractures during massive hydraulic fracturing treatments of hydrocarbon bearing formations is modelled using a continuum damage mechanics approach. The rationale for such an approach is that when only linear elastic fracture mechanics concepts are used certain inconsistencies are encountered such as difficult-to-explain treating pressures that are continuously increasing with time and are abnormally high. In this work, an expression for the fracture propagation rate is derived from Kachanov’s law of damage accumulation. The derived expression is used as a boundary condition in a Perkins–Kern–Nordgren type model (assuming constant height) and in one of its extensions (allowing for height migration). It is explained how the rupture process can retard the fracture propagation rate leading to much higher treating pressures than predicted from the original “fluid flow constrained” boundary condition. The continuum damage mechanics based model is applied to published examples manifesting abnormally high pressures. It is demonstrated how the parameters of the model can be identified from standard fracturing tests.

NOTATION

\( c_1 \) m factor of the dimensionless length;
\( c_2 \) s factor of the dimensionless time;
\( c_3 \) m factor of the dimensionless width;
\( c_4 \) Pa factor of the dimensionless net pressure;
\( C \) \( \frac{1}{(Pa \cdot s)} \) Kachanov parameter;
\( D \) [ ] damage variable;
\( E \) Pa Young’s modulus;
\( G \) Pa shear modulus;
\( h \) m fracture height;
\( i \) m/s injection rate (1 wing);
\( K_I \) Pa/m mode I stress intensity factor;
\( K' \) Pa-s\(^{n'}\) power law consistency index;
\( L \) m fracture length (1 wing);
\( l \) m average distance between microcracks (CDM scale);
\( m \) [-] power law flow behavior index;
\( n \) Pa net pressure;
\( r \) m distance from tip;
\( s \) s time;
\( u \) m/s fracture propagation rate;
\( w \) m fracture width;
\( x \) m lateral coordinate;
\( \eta \) [-] fluid efficiency;
\( k \) [ ] Kachanov exponent;
\( \mu \) Pa s fluid viscosity;
\( v \) [-] Poisson ratio;

\( \sigma \) Pa nominal (LEFM) stress;
\( \sigma_n \) Pa net section stress;
\( \sigma_h \) Pa minimum horizontal principal stress;
\( \tau \) s opening time.

Subscripts

D dimensionless.

INTRODUCTION

During massive hydraulic fracturing treatments, several thousand cubic meters of fluid are pumped into low-permeability hydrocarbon bearing formations in order to create a highly conductive propped fracture. In most cases, the created fracture has two wings extending in opposite directions from the well in the vertical plane perpendicular to the minimum horizontal stress [1]. Starting from the late 50’s, several propagation models have been proposed [2–10] and reviewed from time to time [11–15]. The proposed models usually incorporate the description of fluid flow in the fracture and into the formation, an elasticity relation between crack aperture and net pressure (the difference of pressure and the minimum principal stress), some \textit{a priori} restrictions on geometry and a crack propagation criterion.

Recently observed abnormally high fracturing pressures in tight gas sands [16] and in coal-bed [17] have
raised some doubts about the universal validity of existing propagation models. Further independent evidence for abnormally high pressures was presented in Ref. [18] where the pressure response of step-rate tests (injection tests with continuously increasing fluid injection rate) were investigated. In a thorough attempt to explain the two to nine times higher than predicted net pressures, the conclusion was drawn that high friction losses in possible constrictions are responsible for the phenomenon. "Apparent fracture-toughness effects" were not discounted but were described as a controversial basis for the explanation. Indeed, there are several arguments supporting the existence of very high friction pressures, including in situ measurements in a volcanic-tuff [19]. The aim of this paper is not to argue with the advocates of the constriction hypothesis, but to show that the abnormally high pressures can be also interpreted from a rock mechanics point of view in the framework of a consistent model. For this purpose, we first give a short overview of the inconsistencies inherent when linear elastic fracture mechanics (LEFM) principles are applied to hydraulic fracturing and then propose an alternative approach based on continuum damage mechanics (CDM) principles.

**FRACTURE PROPAGATION RATE AND LEFM**

According to LEFM, the crack propagation law degenerates into an instability statement: when the stress intensity factor at the crack tip is below a critical value (fracture toughness), the crack growth is zero while in the opposite case the crack growth is unstable. To translate this statement into a time dependent quantitative model, two approaches have been proposed.

In constant height models [2-5] it is assumed that, once the fracture has exceeded a certain distance, the stress intensity factor is above the critical value. Therefore, fracture propagation is controlled by the fluid linear velocity and fracture toughness has no effect on the propagation rate.

In so-called 3D models, the instability statement of LEFM is translated into a numerical algorithm. Crack advance is accomplished in such a way that the stress intensity factor is kept "nearly equal" to the fracture toughness at every boundary node during crack extension [14]. Unfortunately, to obtain reasonable results for field cases, the fracture toughness used in simulation should be different from values determined in the laboratory [20]. Therivelin [21] enumerates several factors influencing the apparent fracture toughness. Effects of size, injection rate, confining pressure, propagation speed and heterogeneities are believed to increase its numerical value by one or more orders of magnitude.

Considerable success has been achieved with LEFM-based 3D models to describe laboratory experiments [22, 23]. In these experiments, the net injection pressure (the difference of fracturing pressure and the minimum stress) was decreasing with injection time. Indeed, the stress intensity is proportional to some average of the net pressure in the fracture and to the square root of the characteristic dimension; and hence, to keep it equal to a specified toughness value, the characteristic dimension and the pressure cannot increase simultaneously. However, in the interpretation of treating pressures in massive hydraulic fracturing, increasing treating pressure is common for a confined (nearly constant height) fracture and decreasing treating pressure trends are believed to be connected with height growth [24]. The 2D type Perkins–Kern–Nordgren (PKN) model [5] and its pseudo-3D variations can produce increasing pressure trends, while 3D models typically give decreasing pressure trends. As a consequence, there exists a considerable gap between the availability of highly sophisticated 3D computer codes and their usage in fracturing operation diagnostics.

Attempts to interpret abnormally high pressures during step-rate tests have made the picture even more controversial. In the PKN framework, the only means to interpret this phenomenon is to artificially increase the friction pressure loss, e.g. through increasing the viscosity of the fluid. In fracture toughness based 3D models, a large apparent fracture toughness can reproduce a high treating pressure at early times of fracture propagation. Yew and Liu [25] developed a process zone approach to predict the increase of fracture toughness with the increase of the hydrostatic pressure due to dilatant hardening near the tip. Unfortunately, a large (but constant) fracture toughness will result in decreasing computed pressures at later times even for the case of increasing injection rate while observations indicate pressure increase as shown on Fig. 1 adopted from the study of Palmer and Veatch [18]. Attempts to include a non-pressurized zone (fluid lag) between the fluid front and the tip might complicate the description without having a significant impact on the computed pressures since the length of the non-pressurized zone adapts itself to the conditions, as demonstrated by Gardner [26].

A mechanism retarding the fracture propagation with respect to the potential linear velocity of the fluid during the entire span of the fracturing process is lacking from the traditional models. In the following, continuum damage mechanics (CDM) principles are used to

![Fig. 1. Relative injection rate (exponential approximation), observed bottomhole pressures and 3D simulation results for the Red Oak #2 step-rate test adopted from Ref. [18].](image)
elaborate a fracture propagation law providing more consistency with experience.

FRACTURE PROPAGATION CRITERION FROM CDM

The applicability of CDM is not restricted to some specific material behavior but rather it is a matter of scale. Lemaitre [27] defines the "scale of mesomechanics" in terms of the characteristic dimension of the "Representative Volume Element" considered as a point in continuum mechanics. Our Table 1 is his scale extended for the massive hydraulic fracturing application.

The engineering (or macro-) scale in hydraulic fracturing is hundreds of meters. With respect to this scale, the damage due to microfracs can be considered continuous. The material damage at a given point is characterized by the dimensionless variable $D$. It varies between zero (undamaged state) and unity (total damage). The evolution of damage is described by a constitutive equation, the Kachanov law [28]. This law relates the damage growth, $dD/dt$, to the net section stress, $\sigma_n$. According to Rabotnov's interpretation [29], if a material point is partly damaged, only a $(1-D)$ fraction of an elementary section is able to carry the load and hence the net section stress is defined as:

$$\tau = (1-D)\sigma$$

where $\sigma$ is the nominal stress at the given material point. In terms of the net section stress, the Kachanov law is given by

$$\frac{dD}{dt} = C\sigma^{\kappa}$$

where $C$ and $\kappa$ are material constants (at the scale of interest). Wnuk and Kriz [30] use $\kappa$ values ranging from 0 to 2. For the sake of simplicity in the following, we restrict our treatment to the linear damage law, i.e. $\kappa = 1$. This simplification helps to prevent problems of overparametrization in massive hydraulic fracturing, where the individual effects of the parameters of the constitutive equation are somewhat masked in the pressure curve. In the case of the linear law of damage evolution, the combination of equations (1) and (2) yields

$$\frac{dD}{dt} = C\frac{\sigma}{1-D}.$$  (3)

Equation (3) was used by Wnuk and Kriz [30] in a local manner to describe macrocrack propagation in a damaged material. In the following, let $\sigma$ denote the component of the actual stress in the horizontal direction perpendicular to the minimum horizontal principal stress. For sake of brevity, we refer to $\sigma$ as actual stress. As the macrocrack approaches a given material point, the actual stress increases because the macrocrack acts as a stress concentrator. Meanwhile, the local damage is increasing according to the Kachanov law. The material point joins the macrocrack when the damage reaches its critical value, $D = 1$.

Suppose the damage starts at time $t_0$, i.e. $D(t_0) = 0$, and reaches the critical value at time $t_1$, i.e. $D(t_1) = 1$. Integrating equation (3) between these two limits, and using the equality

$$f_D = \int_0^1 (1-D) dD = \frac{1}{2}$$

the following local rupture criterion can be derived:

$$\frac{1}{2} = C \int_0^{t_1} \sigma [t] dt.$$  (5)

Equation (5) states that a material point ahead of the macrocrack will join it when the time integral of the actual stress reaches the critical value $1/(2C)$ [30]. In order to apply equation (5), we have to compute the stress distribution ahead of a moving macrocrack.

Stress ahead function

There is no general method available to compute the distribution of the stress concentration caused by the sharp variation of the geometry at the fracture tip [27]. This is especially true if the fracture, pressurized by a moving fluid, is embedded into a damaged environment with high compressive far field stress. Recently Wu and Chudnovsky [32] have developed a technique for solving crack-microcrack interaction problems if a detailed characterization of microcrack density, orientation and length distribution is available. In our case, such information is not available and hence we postulate a simple form of the stress ahead function (see Fig. 2):

$$\sigma(r) = \frac{K_I}{\sqrt{2\pi (r+L)}} \sqrt{\frac{1}{r} - \sigma_n}.$$  (6)

![Fig. 2. Damage affected stress distribution ahead of a hydraulically induced fracture in the presence of minimum horizontal principal stress, $\sigma_n$.](image-url)
where $K_1$ is the mode I stress intensity factor computed from LEFM, $r$ is the distance ahead of the macrocrack tip in the direction of crack propagation, $\sigma_h$ is the minimum horizontal stress, $\bar{T}$ is the average distance between the microcracks and $L$ is the fracture length (of one wing).

Equation (6) gives $-\sigma_h$ when $r \to \infty$, implying that a material point far enough from the crack does not "feel" its presence. The stress is infinite at $r = 0$ and obeys the inverse square root law in the vicinity of the tip. The factor $\bar{T}/(\bar{T} + L)$ is less than unity for a finite fracture length. It accounts for the fact that the fracture is opened in a damaged environment and hence the net pressure acting along the fracture faces cannot be totally concentrated. The more microcracks are crossed by the fracture faces, the less efficient is the stress concentration mechanism. The factor $\bar{T}/(\bar{T} + L)$ can be interpreted as the reciprocal number of the microcracks crossed by an elongated fracture. Note that equation (6) reduces to the basic result of LEFM if $\bar{T} \to \infty$ and $\sigma_h = 0$.

Equation (6) expresses a kind of coupling between damage and stress for massive hydraulic fracturing in a highly simplified form. It should be considered as a working postulate reflecting the two main known features of the stress ahead function: (i) the effect of damage is the more pronounced the larger is the ratio of macrocrack length to the characteristic scale associated with the damage; and (ii) the damage does not affect the stress field far from the damaged area.

Quasi-steady-state propagation rate

Since compressive stress (negative $\sigma$) has been acting on the rock for a very long time, we can assume that damage starts to develop only when the stress changes sign. This happens when the fracture arrives at a certain distance, $r_0$, from the given material point. Equating the stress to zero in equation (6) we obtain

$$r_0 = \left( \frac{K_1 \bar{T}}{2\pi \sigma_h (\bar{T} + L)} \right)^2. \quad (7)$$

Let $t_0$ denote the time when the fracture tip is at distance $r_0$ from a given material point and $t_1$ the time of rupture. Suppose that the fracture propagation speed $u$ can be considered constant in this short time interval (quasi-steady-state approximation), then

$$u = \frac{r_0}{t_1 - t_0}. \quad (8)$$

Replacing $r$ by

$$r = r_0 - (t - t_0)u \quad (9)$$

in equation (6), we obtain the stress as a function of time which can be further introduced into equation (5). Hence the rupture criterion can be restated as

$$\frac{1}{2C} = \int_{t_0}^{t_1} \frac{K_1 \bar{T}}{\sqrt{2\pi (\bar{T} + L)}} \times \sqrt{\frac{1}{r_0 - u(t - t_0) - \sigma_h}} dt. \quad (10)$$

where time dependence is taken into account only if explicitly indicated. The integral in equation (10) was evaluated using the formula manipulation software, Mathematica [31]. (Note that the integrand becomes infinite at $t_1$.) Rearranging the result for the unknown $u$, we obtain the basic equation of the fracture propagation rate:

$$u = \frac{C t^2}{\pi \sigma_h (\bar{T} + L)^2 K_1^2}. \quad (11)$$

The exponent, 2, is the result of the linear damage law. As we mentioned earlier, the use of a non-linear damage law would be difficult to justify in massive hydraulic fracturing, where the propagation rate is not a directly observable variable. Otherwise the result, that the fracture propagation rate depends on the stress intensity factor raised to a certain power, is not surprising. For example Kemény [33], describing non-linear rock deformation under compression, postulates a similar relation for sub-critical crack growth rate. The difference is that our CDM derivation gives explicit dependence on the minimum horizontal stress and expresses an additional size effect through the parameter $\bar{T}$.

Equation (11) controls the propagation rate only if the computed speed of fracture propagation is smaller than the linear velocity of the fluid would be in the same fracture geometry, i.e. when the rupture process retards the fracture propagation.

Tip boundary condition from CDM

The CDM propagation criterion [equation (11)] can be used in conjunction with any model, including 2D and even 3D finite element formulations. In the following, we restrict our consideration to constant height fractures. Figure 3 shows the usual fracture geometry for the PKN model [3, 5] (constant height, elliptic vertical cross section, elongated shape), but in our case the fracture width at $x = L$ is not zero. Applying the vertical plane strain approximation first introduced by Perkins and Kern [3], Nordgren [5] derived a non-linear second order partial differential equation for the (maximum) fracture width $w(x, t)$.

$$\frac{G}{64(1 - \nu)\mu h} \frac{\partial^2 w}{\partial x^2} (w^4) = \frac{8C_L}{\pi \sqrt{t - \tau(x)}} + \frac{\partial w}{\partial t}. \quad (12)$$
The Perkins–Kern–Nordgren concept is based on the plane strain assumption applied independently in successive vertical planes along the fracture. While this assumption is certainly a simplification, it provides a unique possibility to obtain a one-to-one correspondence between width and pressure and hence to work with one variable and one partial differential equation. The derivation of equation (12) has been presented also in this journal by Charlez et al. [34]. The initial condition is trivial. The wellbore boundary condition is given by:

$$-\frac{\pi G}{256(1-v)\mu} \frac{\partial}{\partial x} w^*(x=0,t) = i(t).$$  \hspace{1cm} (13)

Kemp [35] has shown that the Nordgren equation is a moving boundary Stefan problem and hence two boundary conditions are needed at the tip. One of them (missing in Ref. [5]) is the Stefan boundary condition:

$$-\frac{G}{64(1-v)\mu h w} \frac{\partial}{\partial x} w^*(x=L,t) = \frac{dL}{dt}. \hspace{1cm} (14)$$

The physical meaning of equation (14) is that the linear velocity of the fluid at the fluid front and the tip propagation rate are equal.

The other tip condition of the Nordgren equation states that the net pressure is zero at the tip. In fact there is a small unwetted zone between the fluid front and the fracture tip, but the fracture has zero width in this zone and its presence is irrelevant from the point of view of the fluid. The zero net pressure condition of the Nordgren model implies that the fracture propagation is not retarded by any tip effect, the fracture is opened “automatically” by the arrival of the fluid.

In our treatment, the “zero net pressure at the tip” assumption is not used. Instead the tip propagation rate is explicitly given by the condition:

$$\frac{dL}{dt} = u \hspace{1cm} (15)$$

where $u$ is the quasi-steady-state tip propagation rate determined from equation (11). As a consequence, at the fluid front the net pressure will not be zero and the fracture will have a finite width. In this content $L$ denotes the location of the fluid front. The region beyond the fluid front is not described by the model.

The effect of the retardation of the tip propagation is manifested in a high net pressure at the tip. As a consequence, the pressure distribution is much more uniform along the fracture relative to the original Nordgren–Kemp model. We use this property to approximate the LEFM stress intensity factor $K_I$ from the tip net pressure and from the wetted length:

$$u = \frac{C_I^2}{\sigma_h (I + L)^2} L p_{n,x=L}. \hspace{1cm} (16)$$

Using the relation between width and net pressure [5], we obtain the final expression of the tip propagation rate:

$$u = \frac{C_I^2 G^2}{\sigma_h (1-v)^2 h^2 (I + L)^2}. \hspace{1cm} (17)$$

Equations (12)–(15) and (17), hereafter called the CDM-PKN model, are derived for possibly varying injection rate, $i(t)$.

**COMPUTATIONAL RESULTS FOR CONSTANT HEIGHT**

The dimensionless variables, $x_D$, $t_D$, $w_D$, and $L_D$, first introduced by Nordgren [5]

$$x = c_1 x_D, \hspace{1cm} t = c_2 t_D, \hspace{1cm} w = c_3 w_D, \hspace{1cm} p_n = c_4 w_D, \hspace{1cm} L = c_5 L_D \hspace{1cm} (18)$$

where

$$c_1 = \pi \left[ \frac{(1-v)\mu i^2}{256 C_i h G} \right]^{1/3}, \hspace{1cm} c_2 = \pi^2 \left[ \frac{(1-v)\mu i^2}{32 C_i h G} \right]^{2/3},$$  

$$c_3 = \left[ \frac{16 (1-v) \mu i^2}{C_i ^2 G} \right]^{1/3}, \hspace{1cm} c_4 = \left[ \frac{16 G^2 i^2}{C_i ^3 (1-v) h} \right]^{1/3} \hspace{1cm} (19)$$

have the advantage of describing the model behavior in a concise manner. The dimensionless counterparts of the two CDM parameters are defined by the relations:

$$I = c_1 t_D, \hspace{1cm} C = c_3 c_4 C_i. \hspace{1cm} (20)$$

The CDM-PKN model can be solved by a finite-differences shooting algorithm. Figures 4 and 5 show computational results obtained for constant injection rate. The effect of the CDM parameters can be summarized as follows [36]:

1. When $I_D \ll L_D$, the dimensionless solution path is affected only by the combined parameter $C_D I_D^2$. In this case:

   If $C_D I_D^2 > 1$, then the fracture propagation is not restricted by the rupture process and the solution path does not differ from the original Nordgren–Kemp one. In other words, the CDM model automatically reduces to its ancestor model if the combined parameter is large enough.
If $C_D\tau_D < 1$, then the fracture propagation is retarded by the rupture process and hence the length will be less, the wellbore width and efficiency (ratio of the fracture volume to the fluid volume injected) will be greater than the corresponding Nordgren-Kemp value. As a consequence, the net treating pressure will be a multiple of the Nordgren-Kemp value as shown for unit dimensionless time in Fig. 4. The treating pressure versus time curve is increasing with time (see Fig. 5).

(2) When $\tau_D$ cannot be neglected with respect to $L_D$, then the corresponding $C_D\tau_D$ pressure–time solution path is reached asymptotically for late times while, at early times, the curve has a sharp decreasing character (see Fig. 5).

As seen from the figures, the combined parameter $C_D\tau_D^2$ can be used to describe the "abnormally high pressures" during the whole fracturing treatment, while the parameter $\tau_D$ can be used, if necessary, to match a higher break-down pressure followed by a pressure decrease, sometimes observed at early treatment times. In most field cases, however, it is reasonable to suppose that $\tau_D \ll L_D$, and hence the only parameter to identify is $C_D\tau_D$. (The computer code is written to accept $C_D\tau_D^2$ and $\tau_D$ as input parameters, where $C_D\tau_D^2$ is a positive number while $\tau_D$ can be set to zero.)

If the (dimensionless) combined parameter is relatively high, the tip propagation rate is controlled by the viscosity of the fluid (similarly to the classical fracturing models). When the (dimensionless) combined parameter is low, the propagation is retarded by the damage evolution and hence the fluid viscosity has very limited effect on the propagation rate and shape.

**PSEUDO 3 DIMENSIONAL CDM MODEL**

The system of equations (12)–(15) and (17) is written for constant height. It can be used if the fracture is perfectly contained in one geological layer. A more realistic extension of the model allows for the growth of the average height $h$ (being only the function of time). In this case, equation (12) takes the form

$$
\frac{G}{64(1-\nu)h} \frac{\partial^2}{\partial x^2} (w^4) = \frac{8C_L}{\pi \sqrt{t - \tau(x)}} + \frac{\partial w}{\partial t} + \frac{w}{h} \frac{dh}{dt}.
$$

Since the height is opened gradually, the opening time, $\tau(x)$, should be interpreted as an average at location $x$ which varies with time (note the overbar in equation 21). To obtain a closed system, an additional equation is needed to determine the height penetration rate. The great advantage of the CDM approach is that the same concept can be used in the vertical direction that is used for the lateral extension:

$$
\dot{u}_h = \frac{C \tau^2}{\pi \sigma_u (l + h)^2} (K_p^*)^2
$$

where $u_h$ gives the height penetration rate into the upper and lower zones, respectively, if the stress intensity factor is computed for the upper or lower tip. The asterisk indicates that different Kachanov parameters may be used in the lateral and vertical directions if the lithology justifies such a distinction. Height growth is controlled first of all by the stress variations (a vertically varying $\sigma_u$ is used in the computation of the vertical stress intensity factors). The effect of the variation of $\sigma_u$ may be further amplified or dumped by the lithological properties reflected in the Kachanov parameter. No attempt is made to compute different height migration rates for different lateral locations since the height is treated as a lumped parameter. The computational realization of the pseudo 3 dimensional CDM model allows for varying viscosity computed, for instance, from a power law relation.

The CDM parameters can be determined from the pressure response of standard tests involving constant and/or variable injection rates, shut-in periods and possible flow-back periods. (These tests are also used to determine the minimum principal stress and the leakoff coefficient.) Once the CDM model parameters are identified, the fracture propagation during the main treatment can be simulated. The CDM model can be used not only to explain observed pressures *a posteriori* but to predict fracture growth *a priori* (even if during the main fracturing job the pumping parameters are different from the ones in the test). One of the main arguments supporting the CDM approach is that the combined parameter identified from abnormally high treating pressures corresponding to very different geographical, lithological and engineering environments is surprisingly consistent, being in a narrow range around $C\tau^2 = 10^{-3}$ m$^2$/Pa-s.

**MODEL APPLICATIONS**

In the following, we present the results of the parameter fitting procedure for the Red Oak #2 calibration test of Ref. [18], already introduced in our Fig. 1. In this step-rate test, the fluid injection rate into the sandstone formation was varied approximately...
Table 2. Formation and pumping parameters for the Red Oak #2 step-rate test [18]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumed fracture height, ( h )</td>
<td>58 m</td>
</tr>
<tr>
<td>Young modulus, ( E )</td>
<td>34.5 GPa</td>
</tr>
<tr>
<td>Poisson ratio, ( \nu )</td>
<td>0.2</td>
</tr>
<tr>
<td>Leakoff coefficient, ( C_l )</td>
<td>( 7.9 \times 10^{-5} ) m/( \mu ) ( \text{m/s} )</td>
</tr>
<tr>
<td>Assumed minimum horizontal stress, ( \sigma_h )</td>
<td>35.5 MPa</td>
</tr>
<tr>
<td>Fluid viscosity, ( \mu )</td>
<td>( 10^{-3} ) Pa s</td>
</tr>
<tr>
<td>Injection rate at start of propagation (for 2 wings)</td>
<td>0.07 m(^3)/s</td>
</tr>
<tr>
<td>Max injection rate (for 2 wings)</td>
<td>0.021 m(^3)/s</td>
</tr>
<tr>
<td>Time elapsed from start of propagation to end, ( t_{end} )</td>
<td>7.6 min</td>
</tr>
</tbody>
</table>

The wellbore treatment pressure solution of the Nordgren-Kemp model (no retardation of the tip propagation) for these parameters is shown in Fig. 6. In spite of the slightly increasing character of the computed curve, the agreement is far from satisfactory. To match the constant height CDM-PKN model, we assumed \( f \ll L \) and found the optimal \( C_T \) value \( 6 \times 10^{-8} \) m\(^2\)/Pa-s (0.0045 ft\(^2\)/psi-s). The corresponding computed pressure curve, reproducing not only the final value but also the character of the observed one, is shown in Fig. 6. No other parameters were assumed or changed relative to the original publication [18]. While the fit is satisfactory, it is in order to note that the resulting length, \( L = 3.4 \) m is an order of magnitude less than the height and hence the vertical plane strain assumption inherent in the PKN-based models might be questioned.

Our next example, taken from a recent case study of fracturing coal-bed [37], is presented here to reveal the controversial state of the art of fracturing pressure interpretation.

The observed surface pressure behavior is shown on Fig. 7. Also shown is the pressure curve computed from a "finite element multiple fracture simulator" [37]. While the fit is convincing, those authors made some arbitrary assumptions:

1. An apparent fracture toughness which is approximately two orders of magnitude larger than the laboratory-measured value was used in the simulation.
2. It was assumed that in the first 2.6 min a vertical fracture was extending in the lateral direction and afterwards a horizontal fracture was extending radially (at the upper or lower coal-barrier interface) while the vertical fracture was arrested. The rationale behind this assumption was that the stress computed from the overburden was less than the observed minimum pressure necessary to start fracture propagation. Note that the minimum principal horizontal stress was still assumed to be lower than the vertical principal stress.

Without denying the possibility of the interpretation of those authors we show here that the pseudo 3 dimensional CDM model (assuming no stress barrier and no difference between the lateral and vertical CDM parameters) can reproduce a similar pressure behavior. The data shown in Table 3 were used for the simulation. The optimal CDM parameters are: \( C_T = 2.5 \times 10^{-7} \) m\(^2\)/Pa-s and \( L = 4 \) m. Note that a nonzero \( f \) is necessary to describe the decreasing character of the pressure curve, that is the deteriorating effect of damage on the stress ahead function is building up gradually during the treatment and becomes significant only after several meters. The CDM pressure response cannot describe the pressure behavior in the first 2 min when, according to our interpretation, the fluid is partly leaking off directly to the formation and is partly compressed (i.e. the observed pressure behavior is more the continuation of the pre-propagation stage than characteristic for the propagation). Since with a nonzero \( f \) the pressure curve is always sharply decreasing at early times, we omitted this part of the curve in Fig. 7. The complex phenomenon of initiation is beyond the scope of our work aimed to describe quasi-steady-state propagation. In the remaining interval, the pressure response computed from the CDM model complies with
the observed curve. The combined parameter suitable to describe the retardation of the tip propagation has a typical value found by us in several other applications.

The main advantage of the CDM interpretation is that only one fracture (with two wings) is assumed. Although underground observations have revealed the possibility of horizontal or even more complex fracture configurations in shallow coalbeds [38], such phenomena are less likely in this case since the minimum horizontal stress is considerably less than the vertical stress [37] and the pressure curve itself is certainly not enough to prove such a complex behavior. The details of the fracture extension process obtained from the CDM model are shown in Fig. 8. The CDM model results in a fracture height $h = 55$ m, fracture (half) length $L = 26$ m and average width $w = 14$ mm. (The extension in the lateral and vertical direction is virtually the same because no stress contrast is assumed.) The CDM results can be appreciated if we compare them with the "two perpendicular fractures" scenario proposed in Ref. [37] where a vertical fracture of half length, $L = 88$ m is created in the first 2.6 min followed by the extension of a 25 m radius horizontal radial fracture during the remaining 22.5 min.

CONCLUSIONS

Abnormally high treating pressures observed in massive hydraulic fracturing have been explained in the literature on the basis of elevated friction pressures due to fluid flow constrictions, high apparent fracture toughness, fluid lag, etc. In this paper, a rock mechanics explanation was provided based on the principles of continuum damage mechanics. Starting from Kachanov's law of damage accumulation and assuming a reasonable functional form for the stress field ahead of the macrocrack, a simple relation was obtained for the quasi-steady state fracture propagation rate. The relation was applied to replace the "fluid flow restriction" boundary condition in Nordgren's constant height propagation model and in its pseudo 3-dimensional version. The combined parameter $C_\theta$ was shown to be responsible for retarding the fracture propagation rate and hence causing much higher treating pressures than predicted by known models. This parameter can be identified from standard tests. In cases when a lucid pressure decrease can be observed at early times, the other CDM parameter, $\tilde{\omega}$, should be also identified, as demonstrated in our second example. Once the CDM parameters are known, the model can be used to predict fracture growth for any set of pumping parameters. The CDM model, while not attempting to submerge into the microphysics of rock behavior, can provide a physically sound basis for describing the main features of hydraulic fracture propagation.

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