Slopes Analysis of Frac & Pack Bottomhole Treating Pressures

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Abstract
A simple means has been developed to analyze frac & pack bottomhole treating pressures which gives insight to the fracture evolution and end-of-job proppant distribution. The method requires minimal input, is relatively model independent (as far as fracture propagation is concerned) and can be used in real time. This is not history matching, but rather an analysis or screening tool.

The key element of frac & pack treatments is the tip screen-out, a period of injection during which the fracture is inflated and the fracture extent remains relatively constant. The fracture may often extend in subsequent periods of regular fixed-area width growth with possible “jumps” and/or other irregular area extension periods between them. The slope of the increasing pressure curve during a “regular” period may be interpreted to obtain the “packing radius” of the fracture, characteristic for the given period. The progression of packing radiiuses can be combined with information on the proppant injection history to give final proppant distribution.

Introduction
Tip screen-out is considered to be the key element in most frac & pack operations. During tip screen-out, the fracture width is inflated while the area of the fracture faces remains constant. Consequently, a relatively short but wide propped fracture is obtained, approximating the optimum of the fracture-reservoir system characterized by a dimensionless fracture conductivity of order one.

Complete tip screen-out is expected to produce a distinct behavior in the treating pressure, i.e. the treating pressure should markedly increase with time. However, frac & pack treatments often exhibit numerous increasing pressure intervals which are interrupted by anomalous pressure decreases, most probably because fracture extension can still occur from time to time (in many cases, a single complete tip screen-out is not achieved). Therefore it seems quite realistic to consider a frac & pack treatment as a series arrested extension/width growth (regular) intervals interrupted by intervals of (irregular) fracture area extension.

This work provides a simple tool for examining such behavior. Treating pressure curves are analyzed to gain insight to the evolution of fracture extent and a plausible end-of-job proppant distribution.

In developing the tool, we intentionally imposed several design parameters: the method should require minimum user input beyond the real treatment data, it should be relatively independent of any fracture propagation model and it should not be a history matching procedure. In accordance with the basic requirement of model independence, the slopes analysis method is a screening tool based on simple equations and a well-defined (reconstructible) algorithm. Based on its simplicity, the tool lends itself well to real time use as well.

Treating Pressure Slopes Analysis
During tip screen-out, the fracture width is inflated while the area of the fracture faces remains theoretically constant. This phenomenon should manifest itself by a marked increase in the treating pressure. In practice, the increasing pressure intervals (“regular intervals”) may be interrupted by anomalous pressure decrease because fracture extension can still occur from time to time. Based on this rationale, the frac & pack treatment is considered as a series of regular arrested extension/width growth intervals interrupted by irregular fracture area extension intervals.

In this case, the treatment can be decomposed into sequential periods of constant fracture area separated by periods (possibly several) of fracture extension. The time periods are located by a simple processing of the treatment pressure curve.

If this vision of the treatment is accepted, then the slope of the increasing pressure curve during a width inflation period may be interpreted to obtain the “packing radius” of the
fracture at that point during the treatment, i.e. characteristic for the given period. Putting together a sequence of packing radii estimates we arrive at a scenario which - combined with additional information on the proppant injection history - yields the final proppant distribution.

In transforming the idea to a working algorithm, several assumptions must be made, both on fracture geometry and on the character of the leak-off process. We employ the following assumptions:

1. The created fracture is vertical with a radial geometry;
2. Fluid leakoff can be described by the Carter leakoff model in conjunction with the power-law type area growth used by Nolte;
3. Fracture packing radius may vary with time, being allowed to increase or decrease;
4. Hydraulic fracture radius (which defines leakoff area) cannot decrease and is the maximum of the packing radii which has occurred up to the given time;
5. During regular width-inflation periods, the pressure slope is defined by linear elastic rock behavior and fluid material balance with friction effects being negligible; and
6. Injected proppant is distributed evenly along the actual packing area during each incremental period of arrested extension/growth width.

The suggested method consists of several steps. First, we select those portions of the bottomhole pressure curve which show positive slope. The slope is then interpreted assuming that the pressure increase is caused by width inflation. The interpretation results in a packing radius which corresponds to a given time point. A step-by-step processing of the entire curve gives a history of the packing radius—though we still lack information on those intervals when the slope is negative. The history is made complete by interpolating between the known values.

Based on this history of packing radius evolution, the final proppant distribution is easily determined by superimposing real-time proppant injection data. Final proppant distribution (which implies fracture length and width) is the practical result of the proposed slopes analysis.

**Restricted Growth Theory**

Tip screen-out can be considered as inflating the fracture width while the area of the fracture face does not increase. If the average width is denoted by \( w \) and the fracture face area (one wing, one face) by \( A \), then

\[
\frac{dw}{dt} = \frac{1}{A} \left( i - q_L \right) \tag{1}
\]

where \( i \) is the injection rate (per one wing) and \( q_L \) is the fluid-loss rate (from one wing).

The basic notation is shown in Fig. 1.

![Fig. 1-Schematic of a radial fracture.](image)

We assume that the fracture is radial with radius \( R \). Then

\[
A = \frac{\pi R^2}{2} \tag{2}
\]

As a first approximation, assume that the pressure in the inflating fracture does not depend location, i.e. it is homogeneous. The net pressure (i.e. excess pressure above the minimum principal stress) is directly proportional to the average width:

\[
p_n = \frac{3\pi E'}{16R} w \tag{3}
\]

where \( E' \) is the plane-strain modulus, related to the Young’s modulus, \( E \), and Poisson’s ratio, \( \nu \), according to the equation \( E' = E/(1-\nu^2) \).

Substituting Eqs. 2 and 3 into Eq. 1, we obtain the time derivative of net pressure, \( p \), as

\[
\frac{dp}{dt} = \left( \frac{3\pi E'}{16R} \right) \left( \frac{2}{\pi R^2} \right) \left( i - q_L \right) \tag{4}
\]

where the subscript for net pressure is dropped because the derivative of bottomhole pressure and that of net pressure are equal.

Recording bottomhole pressure and injection rate provides the possibility of using Eq. 4 to determine \( R \). For this purpose, we need an estimate of \( q_L \).

Details of the Carter leakoff model are given in Ref. 3. The fluid lost through area \( A \) during time \( t \) is given by
where \( C_L \) is the leakoff coefficient and \( S_p \) is the spurt loss coefficient.

Assuming that the fracture has extended up to the given time \( t \) according to the power-law assumption of Nolte and is arrested at the given time instant \( t \), the leakoff rate \( q_{L,t} \) immediately after the fracture extension is arrested (at \( \Delta t = 0 \)) is given by

\[
q_{L,t} = 2A C_L \sqrt{t} \frac{1}{\sqrt{t}} \left( \frac{\partial g(\Delta t_D, \alpha)}{\partial \Delta t_D} \right)_{\Delta t_D = 0}
\]

where \( A \) is the current fracture area, \( \alpha \) is the power law exponent in Nolte’s assumption,

\[
\Delta t_D = \frac{\Delta t}{t}
\]

and the two-variable “g-function” is given by the following mathematical construction:

\[
g(\Delta t_D, \alpha) = 4\alpha \sqrt{\Delta t_D + 2(1 + \Delta t_D) \times F\left[ \frac{1}{2}, \alpha, 1 + \alpha; \left(1 + \Delta t_D\right)^{-1} \right]} - \frac{1}{1 + 2\alpha}
\]

The derivative of the g-function can be obtained from an analytical solution as

\[
\left( \frac{dg(\Delta t_D, \alpha)}{d\Delta t_D} \right)_{\Delta t_D = 0} = 1.91 \quad \text{(10)}
\]

The estimate of leakoff rate is reduced to

\[
q_{L,t} = 2A C_L \frac{1}{\sqrt{t}} \left( \frac{1}{1.91} \right)
\]

Eqs. 4 and 11 were developed explicitly in the text as they form the core basis for the slopes analysis method. Use of these relations is demonstrated in the following example.

**Slopes Analysis Example**

The restricted growth theory is combined with simple material balance computations to form the slopes analysis method as demonstrated below using a sample set of data provided by Shell E&P Technology Co.

**Selecting Intervals of Width Inflation.** Fig. 2 is the bottomhole pressure recorded during a frac & pack job.

Fig 2.-Bottomhole treating pressure from frac & pack treatment.

The suggested method consists of selecting those portions of the bottomhole pressure curve which show positive slope. Straight lines are fitted to the points corresponding to each such interval.

Using a simple algorithm (discussed in the Appendix) one can select points satisfying the criterion of restricted fracture growth. Straight lines are fitted to the individual series to arrive at the plot shown in Fig. 3.
The slope of the straight line gives an average pressure derivative corresponding to the given time interval of restricted growth.

**Determining the Packing Radius Corresponding to a Width Inflation Period.** Once the restricted growth intervals have been selected and the slopes have been determined, we apply Eq. 4. Using the estimate of the leakoff-rate, our basic equation takes the form

\[
m(t) = \left( \frac{3\pi E' a}{16R} \right) \left( \frac{2}{\pi R^2} \right) i - 2 \left( \frac{\pi R^2}{2} \right) C_L \frac{1}{\sqrt{t}} 1.91 \tag{12}\]

Rearranging Eq. 12, we obtain the following:

\[
R^3 + R^2 \left( \frac{3 \times 1.91 \pi E' C_L}{8m \sqrt{t_e}} \right) - \left( \frac{3E' a}{8m} \right) = 0 \tag{13}
\]

Knowing the slope, \( m \), and the injection rate, \( i \), at a given time, \( t \), Eq. 13 can be solved for \( R \). Since the equation is cubic, an explicit (analytical) solution can be given.

**Fig. 4** shows the analysis of the example Shell data which suggests that after a certain period of pumping time (approx. 25 mins.), the packing radius began to decrease, i.e. fracture packing back toward the well. This is not surprising; it indicates that near the end of the treatment only the near-wellbore part of the fracture is “packed.” This is consistent with the treatment objectives for this well (decreased injection rate and high proppant concentration at the end of the treatment.)

**Interpolation between known values of the packing radius.**

Since the packing radius is obtained only in those selected intervals where width-inflation can be assumed, we need a simple tool to fill-in the “gaps.” A simple logarithmic interpolation is used to estimate the packing radius in between the known values.

**Calculation of the final areal proppant concentration (APC) in the fracture.**

Final proppant concentration (proppant distribution) in the fracture can be derived in a relatively straightforward fashion from the packing radius curve and knowledge of the bottomhole proppant concentration as a function of time. (The standard job record typically includes this information.)

Calculation of the final areal proppant concentration (APC) in the fracture follows the simple scheme:

1. For every time interval, \( \Delta t \), determine the mass of proppant entering the fracture.
2. Assume this mass to be uniformly distributed inside the packing radius corresponding to the given time step.
3. Obtain the mass of proppant in a “ring” between radius \( R_1 \) and \( R_2 \) by summing up (accumulating) the mass of proppant placed between corresponding times \( t_1 \) and \( t_2 \).
4. Repeat Step 3 for all rings to obtain the areal proppant concentration as a function of location \( R \).

Application of the above scheme to the example data results in **Fig. 5** below.

Areal proppant concentration, \( c_p \), lb/ft²
Treatment Scenario Reconstruction

An approximate scenario of the frac & pack job can be reconstructed using simple material balance considerations.

The estimate of the hydraulic fracture extent can be taken as the radius of the leak off area. Knowing this area, one can calculate the volume leaked off according to Ref. 5. The actual fracture volume at time \( t \) can be estimated as the injected volume minus the leakoff volume, if we assume that there is no spurt loss.

The fluid efficiency, defined as the ratio of the created fracture volume to the injected volume (calculated without spurt loss) is shown in Fig. 6. The calculated efficiency decreases quickly in the first period where the fracture extension is fast, then increases in the intervals of restricted growth, with possible “fall-backs” in between.

The actual fluid efficiency might be less if there is a spurt loss. It is informative to present the results in the form of different hypothetical widths. (See Fig. 7.) The hypothetical injection width is calculated without leakoff. The hydraulic width is the actual width during the treatment and is less than the injection width. Finally the propped width is the width after the fluid totally leaks off and closes on the proppant.

All these curves are obtained point by point using the hydraulic fracture extent at time \( t \). For the packed width we use the density and porosity data of the proppant.

The point where the hydraulic width and propped width curves come closest together defines the maximum possible value of the spurt loss coefficient. The maximum spurt loss possible corresponds to 1/2 of the closest distance between the curves (i.e. in this case, 0.6 inch which corresponds to a spurt loss coefficient of 0.37 gal/ft².)

A possible scenario of the treatment can be derived from the figures. From Figs. 4, 6 and 7, it is realistic to assume that at \( t = 10 \) mins. the fracture growth was arrested because of “tip screen-out.” At this time, the fracture extent was already quite large, approximately 44 ft. The fast fracture growth might be a consequence of the sizable minifrac which was carried out just before the main treatment.

Fig. 4 shows that the restricted fracture extension (packing) intervals started at \( t = 10, 12, 20, 23, 28, 33 \) and 44 mins. The packing intervals were separated by short intervals when the fracture was extending, possibly in smaller “jumps.” Up to time \( t = 22 \) mins., the extension of the fracture lead to the simultaneous increase of the packed area. The proppant packing radius then decreased in the subsequent packing stages, in spite of the larger available hydraulic extent. This phenomenon is caused by the high slurry proppant concentration and the reduced injection rate.

Summary and Discussion

Eq. 7-59 in Reservoir Stimulation⁴ gives the slope of the pressure curve during restricted extension as

\[
m = \left( \frac{3E'}{8R^3} \right) \eta_i \quad \text{.........................................................(14)}
\]
where \( \eta \) is the fluid efficiency and \( i \) is the injection rate (for one wing). Since the actual fluid efficiency at time \( t \) is not known, Nolte and Smith used unit efficiency to obtain an upper bound on the extent of the fracture.

Comparing Eq. 14 to our basic equation (Eq. 4), it is seen that Eq. 7 estimates the leak-off rate as

\[
q_L = \left(1 - \eta\right)i \tag{15}
\]

Eq. 14 relates the fluid leakoff rate to the instant value of the fluid injection rate and the fluid efficiency. It can be justified only if the fluid injection rate and fluid efficiency are approximately constant for the whole treatment (and there is no spurt loss). While these assumptions might be reasonable for a massive hydraulic fracturing treatment, for frac & pack jobs they are unrealistic.

Besides the underlying restrictive assumptions, Eq. 14 brings about a technical difficulty as well. To use the equation, one has to estimate the fluid efficiency during execution of the frac & pack treatment. While model parameters such as leak-off coefficient may be transferable from a minifrac test to the main job, a state variable such as fluid efficiency is not transferable.

Our suggested procedure improves the original technique in two respects. First, the leak-off rate is estimated implicitly, depending on the computed radius. Only the leakoff coefficient (a system parameter) is used in the computation; efficiency, which is specific to the treatment procedure, is not needed.

Second, the estimation of fracture extent is repeated for all time intervals where restricted extension is likely. From the series of estimated “packing radius” the final areal proppant distribution and the likely scenario of the treatment are reconstructed. Of course, leakoff coefficient should be determined from a method consistent with material balance.

The proposed method for evaluating pressure behavior of frac & pack treatments is not based on specific fracture mechanics and/or proppant transport models. Rather, it takes the pressure curve “as it is” and processes it using minimum additional data. Using the usual data records of a job (slurry injection rate, bottomhole proppant concentration and bottomhole pressure) one can estimate fracture extent and the distribution of proppant in the fracture. The only other additional input parameters necessary for the analysis are plane-strain modulus and leak off coefficient.

Success of the procedure depends on the validity of the key assumption that positive slopes observed in the bottomhole pressure curve are caused by restricted fracture extension. If there is no time interval satisfying the criteria of restricted extension or other phenomena involved masks the effect (e.g. pressure transients caused by sharp changes of the injection rate or dramatic changes in friction pressure due to proppant concentration changes) the estimated packing radius might be in considerable error. Nevertheless, the suggested procedure is considered as a first step in the analysis of frac & pack treatment pressure data.

**Nomenclature**

- \( h_f \) = fracture height, L, m, ft
- \( k \) = reservoir permeability, \( L^2 \), m², md
- \( k_f \) = fracture permeability, \( L^2 \), m², md
- \( F_{CD} \) = dimensionless frac. conductivity
- \( x_f \) = fracture (half-) length, L, m, ft
- \( w \) = fracture width, L, m, in

**SI Metric Conversion Factors**

- \( \text{cp} \times 1.0 \times 10^{-03} = \text{Pa} \cdot \text{s} \)
- \( \text{ft} \times 3.048 \times 10^{-01} = \text{m} \)
- \( \text{in} \times 2.54 \times 10^{+00} = \text{m} \)
- \( \text{md} \times 9.869 \times 10^{+03} = \mu \text{m}^2 \)
- \( \text{psi} \times 6.894 \times 10^{+03} = \text{E}^{-04} = \text{KPa} \)
- \( \text{bbl} \times 1.589 \times 10^{+03} = \text{E}^{-01} = \text{m}^3 \)

*conversion factor is exact

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**References**


**Appendix**
The following algorithm comprises the slopes analysis method. It can be used to analyze frac & pack treatment data in order to give the evolution of fracture extent and an estimate of final areal proppant distribution.

**Algorithm:**

1. Input the three treatment curves: bottomhole pressure, bottomhole injection rate and bottomhole proppant concentration.
2. Input the plane-strain modulus, leakoff coefficient and the density and porosity of the proppant.
3. Input the number of points averaged to obtain a smoothed data point. (If treatment data is recorded at every second, the suggested value is 30.)
4. Calculate smoothed injection rate, injected proppant concentration, injected slurry volume, injected proppant mass, and bottomhole pressure points.
5. Select time intervals of restricted growth. Subsequent points are selected if the injected proppant concentration is nonzero and the time derivative of pressure (calculated by central difference) is positive. If a negative derivative is met, discontinue processing the given time interval and search for a new interval of restricted growth.
6. For every selected time interval fit a straight line to the pressure. Denote the slope at time \( t \) by \( m(t) \).
7. Inside the selected time interval and for every time point, calculate the packing radius from the following cubic equation:

\[
R_p = -\frac{a}{3} + \frac{0.419974a^2}{c} + 0.264567c
\]

where

\[
a = \frac{3 \times 1.91 \times \pi \times E' \times C_L}{8 \times m \sqrt{t}}
\]

\[
b = -\frac{3 \times E' \times i}{8 \times m}
\]

\[
c = \left( -2a^3 + \sqrt{-4a^6 + \left(2a^3 + 27b\right)^2 - 27b}\right)^{1/3}
\]

The slope, \( m \) and the injection rate \( i \) (given for one wing) vary with time, \( t \).
8. The obtained radius, \( R_p \) is called the packing radius. At this point of the algorithm it is specified only in the selected intervals. Estimates of the packing radius for the time points which were excluded are obtained as follows:

Before the first interval, when \( t < t_1 \), interpolate according to:

\[
R_p(t) = \left(\frac{V_{inj,t}}{V_{inj,t_1}}\right)^{4/9} R_p(t_1)
\]

where \( t_1 \) is the first time included into a “restricted fracture extension” time interval. Between two intervals, where the packing radius could not be obtained, use logarithmic interpolation.
9. Calculate the proppant placed in every time step, distributing the mass injected during the time step along the area determined by the corresponding packing radius.
10. Cumulate the deposited proppant in every “ring” between two packing radius values and divide it by the base area of the ring. The result is the areal proppant concentration distribution as a function of the radius.
11. Additional material balance calculations:

   The hydraulic radius, \( R_h \), is defined as the maximum of the packing radius up to a time \( t \). \( V_{inj} \) is the total volume injected and \( M_p \) is the total proppant mass injected up to time \( t \). Using the hydraulic radius calculate the average widths corresponding to the injected volume:

\[
w_{inj} = V_{inj} \left(\frac{R_h^2 \pi}{L}\right)
\]

the average widths corresponding to the injected minus leaked off volume:

\[
w_h = \frac{V_{inj} - V_{Lost}}{R_h^2 \pi}
\]

The average widths corresponding to the dry-proppant volume:

\[
w_{dryprop} = \frac{M_p}{R_h^2 \pi (1 - \phi) \rho_p}
\]

The estimate of the maximum possible spurt loss using the minimal difference between the hydraulic width curve and the dry-proppant width curve is:

\[
S_{p,max} = 0.5 \times \min_t \left( w_h - w_{dryprop}\right)
\]