Advances in Production Engineering

Michael J. Economides\textsuperscript{1}, Peter P. Valkó\textsuperscript{2} and Xiuli Wang\textsuperscript{3}
\textsuperscript{1}University of Houston, \textsuperscript{2}Texas A&M University, \textsuperscript{3}BP-Amoco

Summary

The purpose of this paper is to identify the most important advances in petroleum production engineering in the past decade. Of course, a review paper in the allotted space simply cannot do justice to all new technologies, especially those that are advances to established techniques. We then expound upon two technologies that we feel have made already or have the capacity of quantum impact on the petroleum industry. These are high-permeability fracturing (often referred to in the vernacular as \textit{frac-pack} and variants) and complex well architecture which deals with wells with a main or mother bore from which branches are drilled. At the end of this paper we have added a Bibliography section that includes several recent papers, which while not individually referenced in the text, we think as important contributions to the body of knowledge and experience in the two areas that we write about.

Introduction

Petroleum production is a mature engineering discipline where progress often comes from pushing the limits. One of the most obvious examples is the evolution of off-shore technology, first leaving on-shore, then going “deepwater” and now “ultra deepwater”. As subsea oil and gas developments reach ever deeper into the oceans (currently 2500 m) new challenges for topside, subsea and downhole equipment arise. In artificial lift, progressive cavity pumps have been successfully applied where emulsions and/or solids production makes ESP’s less reliable. Downhole separation (both gravity and cyclon based) of oil and water, and reinjection of the latter within the same wellbore is a major improvement, especially because the cost of water lifting, processing and disposal from the surface costs is ever increasing (Stuebinger and Elphingstone, 2000 and Bowers et al.,
Subsea flow assurance becomes a major constituent of production. Multiphase pumping becomes a viable option changing the economics of marginal off-shore locations.

If one can predict anything like long term impact, however, the most influential change is the evolution of real-time monitoring and control of both surface and downhole conditions (Kluth et al., 2000, Bøe et al., 2000, , Nyhavn et al., 2000). Multiphase metering systems offer a significant increase in functionality over traditional test separators.

The continuous monitoring of all produced fluids and the possibility of remote intervention are transforming the way how engineers do their job. Combining logging, imaging, and 3D visualization techniques with continuously available engineering data such as pressure, temperature, and saturation coming from permanent downhole instrumentation allow engineers to improve the management of their reservoirs and individual wells within it (Mjaaland et al., 2000).

Many of the improvements are driven by progress elsewhere. The most convincing example is the evolution of the technology of sensing and transmitting data, that is driven by consumer electronics (Jack, 2001). It is not surprising, that the price of an integrated circuit equivalent to yesteryear’s mainframe is only a couple of dollars, but it increases tremendously for every additional 10 degrees, and/or 100 psi-s temperature and/or pressure rating. The bottleneck for the newest technology to penetrate into our wells is reliability under high-temperature, high-pressure (HTHP) and chemically hostile conditions. Pressure and temperature are only the first things to look at. In the past few years, downhole video (Tague and Hollman, 2000) has emerged as a viable and cost-effective means for analyzing various wellbore problems (first of all corrosion), to image fluid entry and identify various wellbore plugging agents including scale and organic precipitation. Sensors are developed for signaling sand production, change in pH of the fluid or stress state of the rock matrix around the wellbore.

In the near future, the key issue will be not how to acquire and transfer data, but how to make sense of it. Intelligent or smart well systems make us to rethink how we understand optimization of well performance. Interestingly, the most advanced technology is applied
not necessarily because we want to produce more oil, but less water and – however surprising it sounds at times of high household gas bills, if gas handling capacity is limited – less gas, like in the Oseberg field (Erlandsen, 2000). Remote wellbore intervention (or rather “interventionless” change of the state of the well) becomes routine (Simonds et al., 2000, Storer et al., 1999).

Disciplines once considered less crucial – such as geomechanics – have become the frontiers of new thinking in petroleum production. A potential new completion technique: cavity like completions in weak sands – is based on a phenomenon traditionally we wanted to avoid.

**Hydraulic Fracturing for Production or Injection Enhancement**

In the early times of the practice, from the 1950s to the 1980s, fracturing was applied to low-permeability reservoirs found primarily in North America. After the substantial emergence of high-permeability fracturing in the late 1980s, with much smaller-volume treatments, the technique has expanded to cover any type of oil and gas wells.

Thanks to the evolution of field capabilities, there is now an overriding commonality in fracture design that transcends the value of the reservoir permeability. There is a strong theoretical foundation to this approach, which will be presented below. Hence a unified fracture design is now possible and the notion means both the connection between theory and practice but also that the design process cuts across all petroleum reservoirs and indeed it is common to all. Various diagnostic tools and methods, including well testing, net pressure analysis (fracture modeling), open-hole & cased-hole logging, surface & downhole tilt fracture mapping, microseismic fracture mapping [V4] can be applied then to compare the actual fracture to the design, and ultimately, to improve the performance of subsequently treated wells.
A Fresh Look at Hydraulic Fracturing

It is well known that the fracture length and the fracture conductivity are the two important variables that control the effectiveness of hydraulic fractures. The dimensionless fracture conductivity is a measure of the relative improvement of the fluid flow inside the fracture compared to outside. It is the ratio of the product of fracture permeability and fracture width, divided by the product of the reservoir permeability and fracture (by convention, half-) length. In low-permeability reservoirs, the fracture conductivity is de facto large, and a long fracture length is needed. A post-treatment skin can be as small as $-7$, leading to several folds-of-increase in well performance as compared to the unstimulated well.

For high-permeability reservoirs, a large fracture width is essential for adequate fracture performance. Hence, over the last few years, a technique known as tip screenout (TSO) has been developed which causes the deliberate arrest of the lateral growth of the hydraulic fracture, and the inflation of its width, exactly to affect a larger conductivity.

Dimensionless conductivity around unity is considered as physically optimum, i.e., the well will deliver the maximum production rate or accept the maximum injection rate, transcending any reservoir permeability. Larger values of the conductivity would mean relatively shorter-than-needed fracture lengths and, thus, the flow from the reservoir into the fracture would be restricted. Dimensionless conductivity values smaller than unity would mean less-than-optimum fracture width, rendering the fracture as a "bottleneck" to optimum production. Conductivity is then central to the entire idea of unified fracture design.

It must be emphasized here that the term optimum as used above means the maximization of the well production rate, which often is also the economic optimum. It is possible that in certain theaters of operation the economic optimum may be different than the physical optimum. In some rare cases the theoretically indicated fracture geometry may be difficult to achieve because of physical limitations that can be imposed either by the available equipment, limits in the fracturing materials or the mechanics of the rock to be fractured. However, aiming to maximize the well production or injection rate is an appropriate step to form the basis for fracture design.
Perhaps the best single variable to characterize the size of a fracturing treatment is the volume of proppant placed into the formation. Actual selection of the size of the fracturing treatment and the amount of proppant indicated for injection are primarily based on economics, the most commonly used criterion being the net present value (NPV).

As with most engineering activities, costs increase almost linearly with the size of the treatment, but after a certain point, the revenues increase only marginally. Thus, there is an optimum size of the treatment where the NPV of the incremental revenue, balanced against the treatment costs, becomes maximum (Balen et al., 1988).

Because the azimuth of hydraulic fractures is pre-ordained by the natural state of earth stresses, the azimuth of the drilled well must take this into account.

If the well azimuth does not coincide with the fracture plane, the fracture is likely to initiate in one plane and then twist, causing considerable tortuosity, *en route* to its final azimuth, which would be normal to the minimum stress direction. Examples of longitudinal agreements are vertical wells with vertical fractures, or perfectly horizontal wells drilled deliberately along the expected fracture plane. Perforations and their orientation may also cause a number of problems, including the highly undesirable multiple fracture initiation and, again, tortuosity effects.

In low permeability reservoirs the fracture conductivity is naturally high, and therefore, the impact of the "choke" effects from the phenomena described above is generally minimized, and to avoid tortuosity, point-source fracturing is frequently employed.

The fracture-to-well connectivity is considered today a critical point in the success of high permeability fracturing, often dictating the well azimuth (e.g. drilling S-shape vertical wells) or indicating horizontal wells drilled longitudinal to the fracture direction. Perforating is revisited and alternatives, such as hydrojetting of slots, are considered by the most advanced practitioners.
Key Issues in High-Permeability Fracturing (HPF)

The rapid ascent of high permeability fracturing from a few isolated treatments before 1993 (Martins et al., 1992; Grubert, 1991; Ayoub et al., 1992) to a widely practiced technique in the United States by 1996 (Tiner et al., 1996), suggests that HPF has become a dominating optimization tool for integrated well completion and production. The role of fracturing thanks to HPF has now expanded considerably (see Table 1).

Table 1. Fracturing Role Expanded

<table>
<thead>
<tr>
<th>Permeability</th>
<th>Gas</th>
<th>Oil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>$k&lt;0.5$ md</td>
<td>$k&lt;5$ md</td>
</tr>
<tr>
<td>Moderate</td>
<td>$0.5&lt;k&lt;5$ md</td>
<td>$5&lt;k&lt;50$ md</td>
</tr>
<tr>
<td>High</td>
<td>$k&gt;5$ md</td>
<td>$k&gt;50$ md</td>
</tr>
</tbody>
</table>

Fundamental modeling and field evidence have suggested that HPF treatments are primarily effective by virtue of bypassing near-well damage (DeBonis et al., 1994; Grubert, 1991; Hannah et al., 1993; Hunt et al., 1994; Martins et al., 1992; Montagna et al., 1995; Monus et al., 1992; Mullen et al., 1994; Patel et al., 1994; Reimers and Clausen, 1991; Smith et al., 1987; Stewart et al., 1995a and 1995b; Wong et al., 1993). This is both the controlling and the necessary mechanism for appreciable production enhancements from HPF jobs.

Complex Well/Fracture Configurations

Today, vertical wells are not the only candidates for hydraulic fracturing. Horizontal wells using conventional or especially high permeability fracturing with the well drilled in the expected fracture azimuth (accepting a longitudinal fracture) appear to have, at least conceptually, a very promising prospect. However, a horizontal well intended for a longitudinal fracture configuration would have to be drilled along the maximum horizontal stress. This, in addition to well-understood drilling problems, may contribute to long-term stability problems.
Figure 1 illustrates two advanced multi-fracture configurations. A rather sophisticated conceptual configuration involves the combination of HPF with multiple-fractured vertical branches emanating from a horizontal “mother” well drilled above the producing formation. Of course, horizontal wells, being normal to the vertical stress, are generally more prone to wellbore stability problems. Such a configuration would allow for placement of the horizontal borehole in a competent, non-producing interval. Besides, there are advantages to fracture treating a vertical section over a highly deviated or horizontal section: multiple starter fractures, fracture turning, and tortuosity problems are avoided; convergence-flow skins (“choke” effects) are much less of a concern; and the perforating strategy is simplified.

Figure 1. Multibranche and multiple-fracture configurations for horizontal wells

**Productivity Index Increase Because of Fracturing**

In the case of a propped fracture there are several ways to incorporate the stimulation effect into the productivity index. One can use the pseudo-skin concept:
or, one can just provide the dimensionless productivity index as a function of the fracture parameters:

\[ J_D = \text{function of drainage-volume geometry and fracture parameters} \]  

Both options give exactly the same results (if done coherently). The last option is the most general and convenient, especially if we wish to consider fractured wells in more general (not necessarily circular) drainage areas.

**Well-Fracture-Reservoir System**

We consider a fully penetrating vertical fracture in a pay layer of thickness \( h \). The relation between the drainage area \( A \), the drainage radius \( r_e \) and the drainage side length, \( x_e \) is given by

\[ A = r_e^2 \pi = x_e^2 \]  

For a vertical well intersecting a rectangular vertical fracture that fully penetrates from the bottom to the top of the formation, the performance is known to depend on the \( x \)-directional penetration ratio:

\[ I_x = \frac{2x_f}{x_e} \]  

and on the dimensionless fracture conductivity, which was shown by Prats (1961) that it could encompass all the variables affecting fracture performance:

\[ C_{JD} = \frac{k_f w}{k x_f} \]  

where \( x_f \) is the fracture half length, \( x_e \) is the side length of the square drainage area, \( k \) is the formation permeability, \( k_f \) is the proppant pack permeability, and \( w \) is the average (propped) fracture width.
Proppant Number

The key to formulating a meaningful technical optimization problem is to realize that the fracture penetration and the dimensionless fracture conductivity (through width) are competing for the same resource: the propped volume. Once the reservoir and proppant properties and the amount of proppant are fixed, one has to make the optimal compromise between width and length. The available propped volume puts a constraint on the two dimensionless numbers. To handle the constraint easily we introduce the dimensionless proppant number:

\[ N_{prop} = I^2 C_{fl} \]  

(6)

The proppant number as defined above, is just a combination of the other two dimensionless parameters: penetration ratio and dimensionless fracture conductivity. Substituting the definition of the penetration ratio and dimensionless fracture conductivity into Eq. 6 we obtain

\[ N_{prop} = \frac{4k_f x_w}{k x_w^2} \frac{4k_f x_w h}{k x_w^2 h} = \frac{2k_f V_{prop}}{k V_{res}} \]  

(7)

where: \(N_{prop}\) is the proppant number, dimensionless, \(k_f\) is the effective proppant pack permeability, \(md, k\) is the formation permeability, \(md, V_{prop}\) is the propped volume (two wings, including the extra void space between the proppant grains, but accounting only for proppant contained in the pay layer), \(ft^3\) and \(V_{res}\) is the drainage volume (i.e. drainage area multiplied by pay thickness), \(ft^3\). (Of course any other coherent unit can be used, because the proppant number involves only the ratio of permeabilities and the ratio of volumes.)

Equation 7 reveals the real significance of the proppant number: it is the weighted ratio of propped fracture volume (two wings) to reservoir volume, with the weight being twice the permeability contrast. Note that only that part of the proppant counts into the propped volume that reaches the pay. If, for instance, the fracture height is three times the net pay thickness, then \(V_{prop}\) can be estimated as the bulk volume of the injected proppant in a closely packed state divided by 3. In other words, the “packed” volume of the injected proppant multiplied by the volumetric proppant efficiency yields the \(V_{prop}\) going into the proppant number.
The dimensionless proppant number, $N_{\text{prop}}$, is by far the most important parameter in the unified fracture design.

Figures 2 and 3 show that, for a given value of $N_{\text{prop}}$, the maximum productivity index is achieved at a well-defined value of the dimensionless fracture conductivity. Since a given proppant number represents a fixed amount of proppant reaching the pay, the best compromise between length and width is achieved at the dimensionless fracture conductivity located under the peaks of the individual curves.
Fig. 2. Dimensionless productivity index as a function of dimensionless fracture conductivity with proppant number as a parameter (for $N_{prop} \leq 0.1$)

Fig. 3 Dimensionless productivity index as a function of dimensionless fracture conductivity with proppant number as a parameter (for $N_{prop} > 0.1$)
One of the main results seen from Figs. 2 and 3 is, that at proppant numbers less than 0.1, the optimal compromise occurs at $C_{fD_{opt}} = 1.6$. When the propped volume increases, the optimal compromise happens at larger dimensionless fracture conductivities, because the penetration cannot exceed unity and hence the width has to increase. This effect is shown on Fig. 3. From that figure an absolute maximum of the achievable dimensionless productivity index can also be read. It is 1.909 (this value, equal to $6/\pi$ is the productivity index for perfect linear flow in a square reservoir.)

In “medium and high” permeability formations, that is above 50 md, it is practically impossible to achieve a proppant number larger than 0.1. For frac-and-pack treatments, typical proppant numbers range between 0.0001 and 0.01. Therefore, for medium to high permeability formations the optimum dimensionless fracture conductivity is always 1.6 .

In “tight gas” it is possible to achieve large dimensionless proppant numbers, at least in principle. If one calculates the proppant number with a limited drainage area and does not question whether the proppant really reached the pay layer, dimensionless proppant numbers of the order 1 or even 10 can be calculated.

The above result provides a deeper insight into the real meaning of the dimensionless fracture conductivity. The reservoir and the fracture can be considered as a system working in series. The reservoir can deliver more hydrocarbons if the fracture is longer, but (since the volume is fixed) this means a narrower fracture. In a narrow fracture, the resistance to flow may be significant. The optimum dimensionless fracture conductivity corresponds to the best compromise between the requirements of the two subsystems.

The most important implication is that there is no theoretical difference between low and high permeability fracturing. In all cases there exists a physically optimal fracture which should have a $C_{fD}$ near unity. While in low permeability formations this requirement results in a long and narrow fracture, in high permeability formations a short and wide fracture provides the same dimensionless conductivity. Solely the proppant number determines the productivity index that can be realized by the optimum placement. For the case of $N_{prop} \leq 0.1$ the optimum dimensionless productivity index can be calculated from
\[ J_D = \frac{1}{0.99 - 0.5 \ln N_{\text{prop}}} \]  

(8)

For all proppant numbers, the optimum dimensions can be obtained from

\[ x_f = \left( \frac{k_f V_{\text{prop}} / 2}{C_{\beta \text{Dop}} k h} \right)^{1/2} \quad \text{and} \quad w = \left( \frac{C_{\beta \text{Dop}} k V_{\text{prop}} / 2}{k_f h} \right)^{1/2} \]  

(9)

It is indispensable to use realistic “equivalent” values in Eq 7, 8 and 9. For instance, if non-Darcy flow effects (Gidley, 1990) are present in the fracture, \( k_f \) should be reduced by a factor in order to represent the actual pressure drop. Similarly, in the case of significant proppant embedment (Lacy et al., 1997), \( V_{\text{prop}} \) should be reduced by a factor.

**Fracturing Gas Condensate Reservoirs**

Gas condensate reservoirs, especially in higher-permeability formations in offshore locations are emerging as prime exploitation candidates. In gas-condensate reservoirs a situation emerges very frequently that is tantamount to fracture-face damage. Because of the pressure gradient that is created normal to the fracture, liquid condensate is formed which has a major impact on the reduction of the relative permeability-to-gas. Such a reduction depends on the phase behavior of the fluid and the penetration of liquid condensate, which in turn, depends on the pressure drawdown imposed on the well. These phenomena cause an apparent damage, which affects the performance of all fractured wells irrespective of the reservoir permeability but the effect is far more pronounced in high-permeability formations.

Wang et al. (2000) presented a model that predicts the fractured well performance in gas-condensate reservoirs, quantifying the effects of gas permeability reduction. Furthermore they presented fracture treatment design for condensate reservoirs. The distinguishing feature primarily affects the required fracture length to offset the problems associated with the emergence of liquid condensate.

In the Wang et al. (2000) study, gas relative permeability curves were derived by using a pore-scale network model and are represented by a weighted linear function of immiscible and miscible relative permeability curves.
Cinco-Ley and Samaniego (1981) provided an expression of the fracture face skin effect that becomes additive to the dimensionless pressure for the finite conductivity fracture performance. The skin is

\[
s_{fs} = \frac{\pi b_s}{2x_f} \left( \frac{k}{k_s} - 1 \right)
\]  

(10)

where \( b_s \) is the penetration of damage and \( k_s \) is the damaged permeability.

An analogy can be made readily for a hydraulically fractured gas condensate reservoir. Liquid condensate dropout, normal to the fracture face, can also result in a skin affect reflecting the reduction of the relative permeability to gas. The penetration of damage would be the zone inside which liquid condensate exists, i.e., at the boundary the pressure is the dew point pressure. The permeability ratio reduces to the ratio of the relative permeabilities and because at the boundary \( k_{rg} \) is equal to 1, then Eq. 10 becomes simply

\[
s_{fs} = \frac{\pi b_s}{2x_f} \left( \frac{1}{k_{rg}} - 1 \right)
\]  

(11)

Hydraulic Fracture Geometry Optimization in Gas Condensate Reservoirs

In gas condensate reservoirs the fracture performance is likely to be affected greatly by the presence of liquid condensate, tantamount to fracture face damage. An assumption for the evaluation is that at the boundary of this “damaged” zone the reservoir pressure must be exactly equal to the dew point pressure.

For any fracture length and a given flowing bottomhole pressure that is known to be inside the retrograde condensation zone of a two-phase envelop the pressure profile normal to the fracture phase and into the reservoir will delineate the points where the pressure is equal to the dew point pressure. From this pressure profile the fracture face skin distribution along the fracture face is determined. Using Eq. 11 (the modified Cinco-Ley and Samaniego expression) the depth of the affected zone is determined.

Using this technique Wang et al. (2000) have shown that the optimum fracture length in gas condensate reservoirs should be considerably larger than the calculated value when ignoring the effects of condensate.
The impact is far more pronounced in high-permeability reservoirs. For example, in a 200-md reservoir the optimization for the fracture dimensions with gas condensate damage, showed an optimum half-length equal to 45 ft (a 30% increase over the zero-skin optimum of 35 ft.) The corresponding dimensionless productivity index would be $J_D = 0.171$ in contrast to the optimistic value calculated without the effect of condensate: $J_D = 0.210$.

Here the impact of gas condensate damage on the productivity index expectations and what it would be needed to counteract this effect is serious. The required proppant number would be about 0.003 or, putting it differently, this would mean a required mass of proppant about 6 times the originally contemplated one. Obviously such fracture execution would be virtually impossible and the expectations from well performance would need to be pared down considerably.

**Complex Well Architecture in Petroleum Production Engineering**

In the past several years, slowed for a period of time because of the oil price collapse in 1998-1999, complex well architecture has entered the petroleum industry. Invariably, but somewhat incorrectly, labeled as “multilateral”, these wells were a natural extension from horizontal wells. The latter, although known for a long period of time, were introduced in the early 1980s and became commonplace by the early 1990s.

This section will deal briefly with horizontal wells but because they are already commonplace much of the writing will deal with the salient characteristics of complex wells. We will use a few terms, which have been used by others, but we may distinguish our definitions as follows:

*Multilateral* wells which, in our thinking, imply wells emanating from the same vertical trunk and reach targets at roughly the same depth. These wells can be extended reach, regular diameter holes and can reach several thousand feet of horizontal displacement.

*Multibranch* wells which are branches emanating from a horizontal trunk and are often short-radius, relatively small-diameter holes. They can be drilled with coiled tubing drilling, while the main, “mother” hole may be drilled with conventional drilling assemblies.
Multilevel wells which as the name suggests are horizontal branches drilled off of a vertical trunk but targeting either distinct layers on top of each other or, different locations along the vertical height of a very thick reservoir. This configuration may also lend itself to injection/production schemes, using the same vertical well for both.

Some of the obvious applications from complex wells are (Economides at al., 1998)

- Draining multiple zones or lenticular sands with dedicated horizontal wells of each zone
- Preventing water and/or gas coning by providing both drawdown control and by positioning branches away from the oil-water contact.
- Improving the usability of slot-constrained platform structures
- Improving waterflood and enhanced oil recovery efficiency, especially in heavy crude reservoirs
- Intersecting vertical fractures

To allow for appropriate zonal isolation and well management the junctures between branches become an important issue. Thus, different types of junctures have evolved, all assigned a “level”.

- Level 1: The juncture is open hole to open hole
- Level 2: The juncture is cased hole to open hole
- Level 3: The juncture is cased hole to a slotted liner or other type of completion but without cementing the connection
- Level 4: The juncture is cased hole to cased hole with cemented connection. Fluids from the branches are commingled.
- Level 5: Same as with Level 4 but fluids from branches are isolated with appropriate tubing and packer connections.
- Level 6: Mechanical isolation of the branches using hardware that allow the deployment of two branches that are connected metal-to-metal.

Production From Horizontal and Complex Wells

A comprehensive multi- and single-well productivity or injectivity model has been introduced that allows arbitrary positioning of the well(s) in anisotropic formations
(Economides et al., 1996). This flexible, generalized model can be used for the study of several plausible scenarios, especially the economic attractiveness of drilling horizontal and multilateral wells.

The work had some notable predecessors. Borisov (1964) introduced one of the earliest models, which assumed a constant pressure drainage ellipse in which the dimensions depended on the well length. This configuration evolved into Joshi’s (1988) widely used equation, which accounted for vertical-to-horizontal permeability anisotropy. It was adjusted by Economides et al. (1991) for a wellbore in elliptical coordinates.

Using vertical well analogs, Babu and Odeh (1989) grouped their solution into reservoir/well configuration shape factors and a (horizontal) partial-penetration skin effect.

Kuchuk et al. (1988) used a uniform flux solution to predict the performance of horizontal wells, including wells that were not vertically centered.

The Economides et al. (1996) solution obtains dimensionless pressures for a point source of unit length in a no-flow boundary “box”. Using a line source with uniform flux, it integrates the solution for the point source along any arbitrary well trajectory. Careful switching of early- and late-time semianalytical solutions allows very accurate calculations of the composite dimensionless pressure of any well configuration.

The productivity index, \( J \), is related to the dimensionless pressure under transient conditions (in oilfield units):

\[
J = \frac{q}{\bar{p} - p_{wf}} = \frac{\bar{k}x_e}{887.22 B \mu \left(p_D + \frac{x_e}{2\pi L} \sum s\right)}
\]

where \( \bar{p} \) is the reservoir pressure (psi), \( p_{wf} \) is the flowing bottomhole pressure (psi), \( \mu \) is the viscosity (cp), \( B \) is the formation volume factor, \( p_D \) is the calculated dimensionless pressure, and \( \bar{k} = \sqrt[3]{k_x k_y k_z} \) is the average reservoir permeability (md), \( \sum s \) is the sum of all damage and pseudoskin factors. Dimensioned calculations are based on the reservoir length, \( x_e \); \( L \) is the horizontal well length.
The generalized solution to the dimensionless pressure, $p_D$, starts with early-time transient behavior and ends with pseudosteady state if all drainage boundaries are felt. At that moment, the three-dimensional (3D) $p_D$ is decomposed into one two-dimensional (2D) and one one-dimensional (1D) part,

$$p_D = \frac{x_C C_H}{4\pi h} + \frac{x_C}{2\pi L} s_x$$

(13)

where $C_H$ is a “shape” factor, characteristic of well and reservoir configurations in the horizontal plane, and $s_x$ is the skin accounting for vertical effects. The expression for this skin effect (after Kuchuk et al., 1988) is

$$s_x = \ln\left(\frac{h}{2\pi r_w}\right) + \frac{h}{6L} + s_e$$

(14)

and $s_e$, describing eccentricity effects in the vertical direction, is

$$s_e = h\left[\frac{2z_w}{h} - \frac{1}{2} \left(\frac{2z_w}{h}\right)^2 - \frac{1}{2}\right] - \ln\left[\sin\left(\frac{\pi z_w}{h}\right)\right]$$

(15)

which is negligible if the well is placed near the vertical middle of the reservoir.

Shape factors for various reservoir and well configurations, including multilateral systems, are given in Table 2.
Table 2. Shape factors for single horizontal and multibranch wells.

<table>
<thead>
<tr>
<th>$x_e=y_e$</th>
<th>$L/x_e$</th>
<th>$C_H$</th>
<th>$x_e=4y_e$</th>
<th>$L/x_e$</th>
<th>$C_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>3.77</td>
<td></td>
<td></td>
<td>0.25</td>
<td>2.66</td>
</tr>
<tr>
<td>0.5</td>
<td>2.09</td>
<td></td>
<td></td>
<td>0.5</td>
<td>1.36</td>
</tr>
<tr>
<td>0.75</td>
<td>1.00</td>
<td></td>
<td></td>
<td>0.75</td>
<td>0.69</td>
</tr>
<tr>
<td>1</td>
<td>0.26</td>
<td></td>
<td></td>
<td>1</td>
<td>0.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_e=2y_e$</th>
<th>$L/x_e$</th>
<th>$C_H$</th>
<th>$x_e=y_e$</th>
<th>$L/x_e=0.4$</th>
<th>$L/L_y=1$</th>
<th>$L/L_y=2$</th>
<th>$L/L_y=0.5$</th>
<th>$C_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>3.19</td>
<td></td>
<td></td>
<td>0.25</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.49</td>
</tr>
<tr>
<td>0.5</td>
<td>1.80</td>
<td></td>
<td></td>
<td>0.5</td>
<td>1.48</td>
<td>1.48</td>
<td>1.48</td>
<td>1.49</td>
</tr>
<tr>
<td>0.75</td>
<td>1.02</td>
<td></td>
<td></td>
<td>0.75</td>
<td>1.48</td>
<td>1.48</td>
<td>1.48</td>
<td>1.49</td>
</tr>
<tr>
<td>1</td>
<td>0.52</td>
<td></td>
<td></td>
<td>1</td>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Example Application for a Horizontal Well and a Multibranch Well

A reservoir has the following dimensions: $x_e = 3000$ ft, $y_e = 3000$ ft, and $h = 100$ ft. First assume that $k_x = k_y = k_z = 10$ md, the well is in the vertical middle (i.e., $z_w = 50$ ft) and that $r_w = 0.328$ ft, $B = 1.15$ res bbl/STB, and $\mu = 0.8$ cp. A horizontal well with length $L = 1200$ ft is drilled in the $x$-direction.

Since the well is in the vertical middle, $s_e = 0$ (otherwise Eq. 13 should have been used, which accounts for eccentricity effects). From Eq. 12, $s_x = 3.89$

From Table 2, noting that $x_e = y_e$ and $L/x_e = 1200/3000 = 0.4$, the shape factor $C_H$ is obtained. It is equal to 2.64. Then from Eq. 11, $p_D = 7.86$.

Finally, from Eq. 10 based on the assumption of no skin damage, the productivity index, $J$, is 4.7 STB/d/psi.

[Note: For vertical-to-horizontal permeability anisotropy ($k_z = 1$ md), the average permeability $\bar{k} = \frac{1}{3}(10)(10)(1) = 4.6$ md and the reservoir dimensions $x_e$, $y_e$, $h$, the well length must be adjusted accordingly as shown in Economides et al., 1996. The productivity index calculated in this exercise would be reduced by more than 30% for such permeability anisotropy.]
For a six-arm multibranch well the shape factor is (from Table 2) 1.33. Thus, $p_D = 4.73$ and the productivity index becomes 7.8 STB/d/psi, an increase of 66% percent over that of a single horizontal well.
References


Bibliography

Complex Well Architecture


High-Permeability Fracturing


