

Key

PETE 301

Exam C

November 22, 2004

(Turn in your 3 cheat sheets with your exam. Show your work. Show the units, if appropriate.)

1, (20 points) Describe each of the following matrix problems as a, b, c, or d.

- (a) unique solution
- (b) no solution
- (c) infinite number of solutions
- (d) ill-conditioned

$$\begin{bmatrix} 3.1 & -2.8 \\ 9.3 & -8.4 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2.3 \\ 6.9 \end{pmatrix} \quad \text{(c)} \quad -5$$

$$\begin{bmatrix} 3.1 & 2.8 \\ 3.101 & 2.8 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2.3 \\ 5.2 \end{pmatrix} \quad \text{(d)} \quad -5$$

$$\begin{bmatrix} 3 & 2 \\ 4 & -5 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2.3 \\ 5.2 \end{pmatrix} \quad \text{(a)} \quad -5$$

$$\begin{bmatrix} 3.1 & -2.8 \\ 9.3 & -8.4 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2.3 \\ 6.8 \end{pmatrix} \quad \text{(b)} \quad -5$$

2. (15 points) If a subroutine for solving a system of linear equations returns with the error message "singular coefficient matrix", then most probably

- 10 a) the system you want to solve has one unique solution
- 10 ✓ b) the system you want to solve has no solution
- ✓ c) the system you want to solve has infinitely many solutions
- 10 ✓ (d) either b) or c)
- e) none of the above

3. (15 points) What is the Jacobian matrix of the Newton-Raphson method for the following system of equations.

$$3x_1 + e^{x_2} = 5$$

$$2x_1^2 + x_2 = 3.1$$

$$\begin{bmatrix} 3 & e^{x_2} \\ 4x_1 & 1 \end{bmatrix}$$

4. (15 points) Suppose we have a sand packed core container 20 ft long. We pressure it with air at 100 psia. Then we open valve on both ends to the atmospheric pressure, 14.7 psia. Consider the finite difference equation for this flow problem in the following form: $p_{i-1}^{n+1} - 2p_i^{n+1} + p_{i+1}^{n+1} = \alpha (p_i^{n+1} - p_i^n)$.

Suppose we are using 5 grid points with a grid point on each end. Fill in the following table of the matrix coefficients and right hand side for *the first timestep*. Include the proper boundary conditions. Use exact values where you can, but use the math symbols elsewhere.

i	a_i	b_i	c_i	d_i
1		1	0	14.7
2	1	$-(2+\alpha)$	1	$-\alpha p_2^n$
3	1	$-(2+\alpha)$	1	$-\alpha p_3^n$
4	1	$-(2+\alpha)$	1	$-\alpha p_4^n$
5	0	1		14.7

-3

5. (15 points) What is the maximum timestep that we can take with the explicit FDE for the following data:

Φ , fraction	0.15
μ , cp	1.2
c_t , psi ⁻¹	7×10^{-6}
k , md	30
Δx , ft	16

$$\alpha = \frac{\rho u c_t}{0.00633 k} \frac{\Delta x^2}{\Delta t} = 2$$

$$\Delta t = \frac{\rho u c_t \Delta x^2}{2 (0.00633) k} = 0.000849 \text{ days}$$

6. (20 points)

The following program multiplies matrix A times matrix B to yield the resulting C matrix.

1. What are the dimensions of matrix C?

Program:

2. Read matrix (A) from data sheet
3. Read matrix (B) from data sheet
4. Write the resulting matrix C in "Output" sheet
5. Complete the routine, including the arguments

```
Sub Matrix_multiplication()  
Dim a(5, 4) As Double, b(4, 4) As Double, c(..., ...) As Double  
Dim i As Integer, j As Integer, s As Double, k As Integer
```

```
With Worksheets("Data")
```

```
.....  
.....  
.....  
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.....
```

```
.....  
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.....  
.....  
.....
```

```
For i = 1 To ...  
  For j = 1 To ..  
    s = 0  
    For k = 1 To ...  
      s = s + a(..., k) * b(..., ...)  
    Next k  
    c(i, j) = s  
  Next j  
Next i
```

```
.....  
.....  
.....  
.....  
.....
```

```
End .....
```

```
Sub Matrix_multiplication()  
Dim a(5, 4) As Double, b(4, 4) As Double, c(5, 4) As Double  
Dim i As Integer, j As Integer, s As Double, k As Integer  
With Worksheets("Data")
```

```
For i = 1 To 5.  
For j = 1 To 4  
a(i, j) = .Cells(i, j)  
Next j  
Next i
```

```
For i = 1 To 4  
For j = 1 To 4  
b(i, j) = .Cells(i, 4 + j)  
Next j  
Next i  
End With
```

```
For i = 1 To 5  
For j = 1 To 4  
s = 0  
For k = 1 To 4  
s = s + a(i, k) * b(k, j)  
Next k  
c(i, j) = s  
Next j  
Next I
```

```
With Worksheets("Output")
```

```
For i = 1 To 5.  
For j = 1 To 4  
.Cells(i, j) = c(i, j)  
Next j  
Next i
```

```
End With  
End Sub
```