Key

PETE 301

Final Exam

December 14, 2004

(Do not turn in your 4 cheat sheets. Show your work. Show the units, if appropriate.)

1. (10 points) The measured side length of a metal cube is $a = 2.50$ ft $\pm$ 0.1 ft.

Its mass (measured with a weighing scale that has ± 1 % accuracy) is $m = 1.60 \times 10^3$ lb_m.

Calculate the density (in lb_m/ft^3) and indicate its relative error in %.

-3 $$\rho = \frac{m}{V} = \frac{m}{a^3} = \frac{1{,}600}{(2.5)^3} = \boxed{102.4 \ \#/ft^3}$$

-3 $$d\rho = \left|\frac{\partial \rho}{\partial m} dm\right| + \left|\frac{\partial \rho}{\partial a} da\right|$$

$$= \left|\frac{1}{a^3} dm\right| + \left|m(-3a^{-4})\, da\right|$$

$$= \frac{1}{2.5^3}(0.01)(1600) + (1600)(3)(2.5)^{-4}(0.1)$$

$$= 1.024 + 12.288$$

-3 $$= 13.312 \ \#/ft^3$$

-3 $$\text{rel. error} = \frac{13.312}{102.4} \times 100\% = \boxed{13.0\ \%}$$

2. (10 points) Consider the following equations for the Newton-Raphson method. (a) What is the Jacobian matrix, (b) what are the initial residuals, assuming initial values of $x_1 = x_2 = 1$, and (c) perform one Newton-Raphson iteration, showing the new values for x_1, x_2, r_1, and r_2.

$$10x_1 + x_2^2 = 1$$
$$x_1 + 10x_2 = 5$$

-3 (a)

$$\begin{bmatrix} 10 & 2x_2 \\ 1 & 10 \end{bmatrix}$$

-3 (b)

$$10(1) + (1)^2 - 1 = r_1 = 10$$
$$(1) + 10(1) - 5 = r_2 = 6$$

(c)

$$\begin{bmatrix} 10 & 2x_2 \\ 1 & 10 \end{bmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \end{pmatrix}$$

$$\begin{cases} 10\,\Delta x_1 + 2\,\Delta x_2 = 10 \\ 1\,\Delta x_1 + 10\,\Delta x_2 = 6 \end{cases}$$

Eq. 2

$$\cancel{1 - \tfrac{10}{10}\,\Delta x_1} \to 0 \quad + \left(10 - \tfrac{2}{10}\right)\Delta x_2 = 6 - \tfrac{10}{10} = 5$$

$$9.8\,\Delta x_2 = 5$$

$$\Delta x_2 = \frac{5}{9.8} = \boxed{0.510} \qquad \boxed{x_2 = 1 - 0.510 = 0.49}$$

Eq. 1

-3

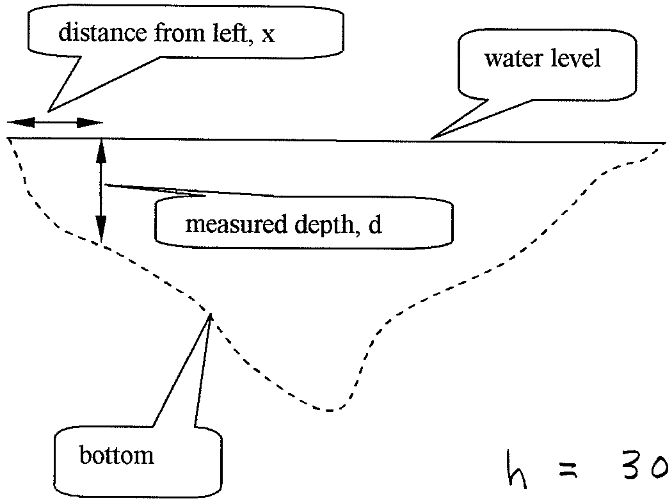
$$10\,\Delta x_1 + 2(0.510) = 10$$

$$\Delta x_1 = \frac{10 - 1.020}{10} = \boxed{0.898} \qquad \boxed{x_1 = 1 - 0.898 = 0.102}$$

-3

$$r_1 = 10(0.102) + (0.49)^2 - 1 = \boxed{0.260}$$
$$r_2 = (0.102) + 10(0.49) - 5 = \boxed{0.002}$$

3. (10 points) The depth of a 240 ft wide river is measured from a boat (see the sketch). The results are shown in the table. Calculate (a) the cross-sectional area of the river, and (b) the *water flow* down the river (in ft^3/sec) if the water velocity is known to be 0.7 ft/sec. (Note: Because of the equidistant measurements and correct number of panels, use repeated Simpson's rule.)



x, ft	d, ft
0	0.00
30	2.54
60	9.29
90	4.83
120	0.00

$h = 30$ ft,

-8 (a) $A = \frac{h}{3}\left(f_0 + 4f_{30} + f_{60} + f_{60} + 4f_{90} + f_{120}\right)$

$= \frac{30}{3}\left(0 + 4(2.54) + 9.29 + 9.29 + 4(4.83) + 0\right)$

$= 10\,(48.06) = \boxed{480.6\ ft^2}$

-3 (b) Flow.-

$480.6\ ft^2 \times 0.7\ ft/sec = \boxed{336.4\ ft^3/sec}$

4. (10 points) Remember that we developed formulas for calculating **b** and **m** for Least Squares fitting of a straight line. We chose **b** and **m** to minimize the sum of the residuals $[\Sigma(y-y_i)^2]$. But, remember that we want to have **b** fixed and fit only **m** for determining OGIP from our p/z plot. So, we just need a formula for determining **m** when **b** is given. Derive the formula for determining **m** for this case. [Assume you have **n** data point of y_i and x_i.]

$$y = b + mx$$

−5 $$\Sigma r^2 = \sum[(b + m x_i) - y_i]^2$$

−4 $$\frac{d\,\Sigma r^2}{dm} = \sum \cancel{2}[b + m x_i - y_i]\, x_i = \sum(\cancel{2}b x_i + \cancel{2}m x_i^2 - \cancel{2}y_i x_i) = 0$$

$$b\sum x_i + m\sum x_i^2 - \sum x_i y_i = 0$$

$$\boxed{m = \frac{\Sigma x_i y_i - b\,\Sigma x_i}{\Sigma x_i^2}}$$

−4

5. (10 points) Write a VBA subroutine named "VBATest()" that sums up the numbers (1, 2, 3, 4 ,…, 100) and writes the result into cell B3.

-2

```
Sub VBATest ( )
Dim sum as double, i as integer
-2
sum = 0
-2
For i = 1 To 100
-2
sum = sum + i
Next
-2
. sheets ("sheet 1"). cell (3, 2) = sum
End Sub
-2
```

6. (10 points) Consider the ordinary differential equation $y' = x + e^x$ with an initial value of $y = 2.6$ at $x = 0$. Calculate the value for y at x = 2 using the non-iterative Heun method (h=2).

step 1 – Euler

$$y_2 = y_0 + \frac{dy}{dx} h = y_0 + (x + e^x) h$$

-5

$$= 2.6 + (0 + 1)(2) = 2.6 + 2 = 4.6$$

step 2 – Huen

$$y_2 = y_0 + \frac{1}{2}(f_0' + f_2') h$$

$$f_0' = 0 + 1 = 1$$

$$f_2' = 2 + e^2 = 9.39$$

-5

$$y_2 = 2.6 + \frac{1}{2}(1 + 9.39)(2) = 2.6 + 10.39$$

$$= \boxed{12.99}$$

7. (10 points) Suppose you have the following data:

x	f(x)
1.3	3.6
1.4	3.9
1.5	4.2
1.6	4.7

Use three different Taylor's series approximations (forward, backward, and central) to estimate f ' (1.5). For each method (a) write the formula with proper notation, (b) calculate the value, and (c) state the order of accuracy. Clearly identify each method.

-3 -4

Forward: $f' = \dfrac{f_{i+1} - f_i}{\Delta x} = \dfrac{4.7 - 4.2}{0.1} = 5 \qquad O(\Delta x)$

-3 -4

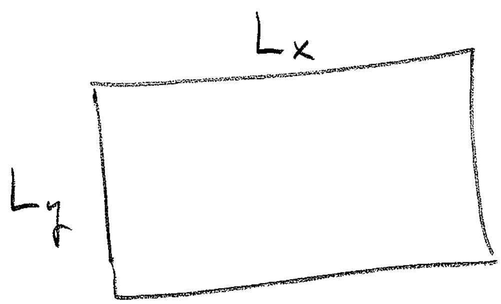
Backward: $f' = \dfrac{f_i - f_{i-1}}{\Delta x} = \dfrac{4.2 - 3.9}{0.1} = 3 \qquad O(\Delta x)$

-3 -4

Central: $f' = \dfrac{f_{i+1} - f_{i-1}}{2\Delta x} = \dfrac{4.7 - 3.9}{0.2} = 4 \qquad O(\Delta x^2)$

8. (10 points) For the following Gassim data: (a) calculate the total pore volume of the reservoir (b) calculate the total cumulative production for the run. Show the units with your answers.

IMAX	16					
JMAX	6					
SWAT	0.26					
CROC	3.00E-06					
GRAV	0.7					
PREF	4500					
T	610					
END						
CMNT	grid	data	section----			
DELX	120					
DELY	-1					
	30	40	50	50	45	40
KX	0.10					
KY	0.10					
H	53					
PHI	0.21					
POI	4500					
END						
CMNT	schedule	data	section----			
NAME	1	1	1	0		
QG	1	50000				
ALPH	1.5					
WELL	2					
PMAP	2					
DELT	0.1					
DTMX	50					
TIME	1					
TIME	10					
TIME	50					
TIME	100					
TIME	500					
TIME	1000					
END						



(a) $V_p = \phi V = \phi L_x L_y h$

$= (0.21)(120 \times 16)(30+40+50+50+45+40)(53)$

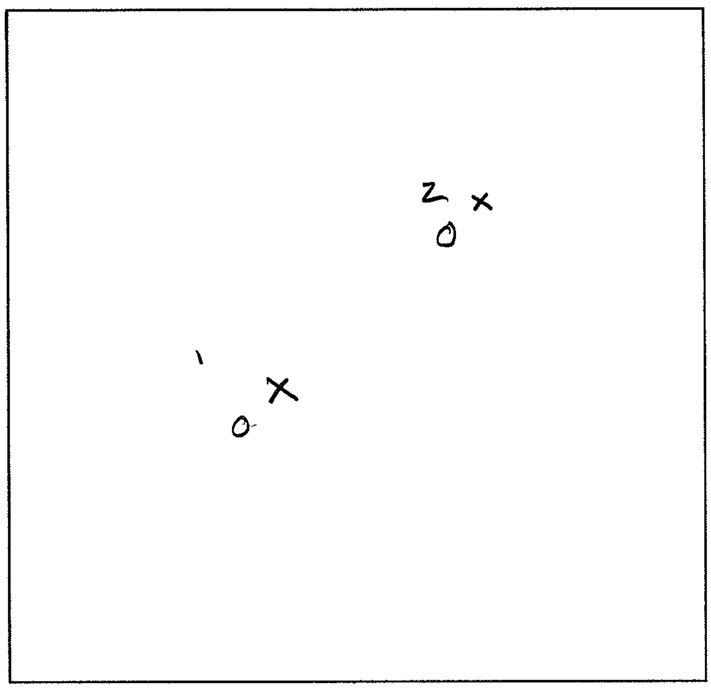
$= \boxed{5{,}449{,}248 \ ft^3}$

-5

(b) $50{,}000 \ scf/D \times 1{,}000 \ days = \boxed{50 \times 10^6 \ scf/D}$

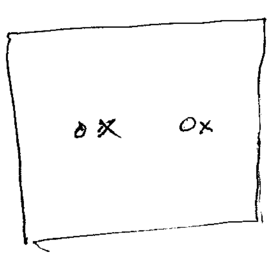
-5

9. (10 points) Suppose you have a square shaped reservoir. You want to maximize production over a 10 year period. (a) indicate with **O** where you would drill two wells if you drilled them at the same time. Label them well 1 and well 2. (b) Now assume you drilled well 1 at the beginning and cannot drill well 2 until the second year. Mark these optimal locations with **X**.



O – Equally spaced, far from boundaries,

X – X_1 closer to center
X_2 farther from center



– 4

10. (10 points) Answer the following questions by putting true (**T**) or false (**F**)

1- We can insert function code inside sub code (F)
2- The VBA function should start with the word function and end by words end sub (F)
3- A sub can return either one or many values (T)
4- VBA program without Option Explicit use default declaration for all the variables as double. (T)
5- We can write the output on the spreadsheet without using with sheets keyword. (T)
6- While ....Wend loop does not work if the condition is true and the For...Next loop does the same (F)
7- Which of the following statement is correct:-
   a. If x=10 then sum= x end if)
   b. If x=10 then sum= x
   c. If x=10 then
   sum= x
   end if

8- Which of the following statement is correct:-

a) Sum = 6
b) Sum = 15
c) Sum = 7

```
Option Explicit
Sub sumprogram()
Dim n As Interior, sum As Double, i As Integer
sum = 0
i = 6
Do
  i = i + 1
  sum = sum + i
Loop While (sum = 7)
End Sub
```